Grade 5 Clarifying Lessons:
Testing for Tessellations

OLD Resources. These resources have NOT yet been updated to align with the revised elementary mathematics TEKS. These revised TEKS were adopted by the Texas State Board of Education in 2005, with full implementation scheduled for 2006–07. These resources align with the original TEKS that were adopted in 1998 and should be used as a starting point only.

What is a Clarifying Lesson?

A model lesson teachers can implement in their classroom. Clarifying Lessons combine multiple TEKS statements and may use several Clarifying Activities in one lesson. Clarifying Lessons help to answer the question "What does a complete lesson look like that addresses a set of related TEKS statements, and how can these TEKS statements be connected to other parts of the TEKS?"

TEKS Addressed in This Lesson

Geometry and spatial reasoning: 5.7A, B; 5.8A, B

Underlying processes and mathematical tools: 5.14A, D; 5.15A, B; 5.16A, B

Materials

- overhead projector and overhead pattern blocks
- tagboard
- pens and markers
- tape
- scissors
- quilt or wallpaper samples (optional)

Lesson Resources

Giganti, Jr., P., and Cittadino, M. J. The art of tessellation, *Arithmetic Teacher*, (March, 1990), NCTM.

Related Resources

Books about Islamic art, M. C. Escher, and Leonardo da Vinci

Lesson Overview

Students cover the plane with polygons (and other shapes) to create tesselations.

Mathematics Overview

Students use formal geometric language to describe polygons (and other shapes) that will tessellate the plane and those that will not. Students make generalizations about the characteristics of a polygon (or other shape) that will tessellate the plane and identify the transformations used to create the congruent images that form the tessellations.

Set-up (to set the stage and motivate the students to participate)

1. Show designs of M. C. Escher and ask students to discuss what they notice. Ask students if they have seen other "patterns that cover the plane" in their environment (e.g., bricks on a patio, certain wallpapers, tiles in the bathroom and kitchen). (5.14A)

2. Tell students that the repeated use of simple, closed curves (like rectangles or triangles) to completely cover a plane without gaps or overlapping is called a tessellation. (5.15B)

3. Give students several of one shape from the pattern blocks. Have students explore to see how they can arrange their figures to cover a surface completely. Ask them to share arrangements and discuss the pattens they see. (5.7B, 5.8B, 5.14D, 5.15A, 5.16A))

4. Ask students to try to find shapes that cannot be used to cover the plane without gaps or overlapping. Have them share ideas about why these shapes and arrangements do not work. (For example, the angles that meet don't complete a circle so they leave a gap, or they make more than a circle and overlap.) (5.7A, B; 5.15A, B; 5.16A)

5. Give each student just one pattern block. Have each student place this polygon in the middle of a blank piece of paper and trace around it. Have students continue to trace their polygons again, lining them up with any side of a previously traced image. Students should continue until they have covered the entire page. Teachers can also demonstrate their arrangements using pattern block pieces on the overhead projector. (5.8A, 5.14D)

6. After practicing making tessellations with the pattern blocks, have students create their own geometric shapes and cut out 15 of them (a possible homework assignment). Students should test their shapes to see if they will tessellate. Make and post a "will work/won't work" wall chart on which students can paste up examples of shapes that will tessellate and shapes that will not tessellate. (5.8A, 5.14D, 5.16A)

7. See "The 'Nibble' Technique" on p. 9 of the referenced article in *Arithmetic Teacher* to explain how to alter polygons and create irregularly shaped figures that also tessellate. Give students many opportunities to try all the techniques of making a shape that tessellates. (5.8A, B; 5.14D)

8. Students should create poster-sized tessellations using shapes they like. See "The Art in Tessellations" on pp. 11-12 of the article for lesson suggestions. (5.8A, B; 5.14D)

Teacher Notes (to personalize the lesson for your classroom)

Guiding Questions (to engage students in mathematical thinking during the lesson)

- What is a tessellation? (5.15B)

- What is a polygon? (5.15B)

- What are some polygons that will tessellate? How do you know? (5.7A, B; 5.14D)

- What are some polygons that will not tessellate? How do you know? (5.7A, B; 5.14D; 5.16A)

- What do you notice about the polygons that do tessellate? (5.7A, B; 5.8B; 5.15A; 5.16A)

- What do you notice about the polygons that don't tessellate? (5.7A, B; 5.15A)

- What is a rotation? A translation? A reflection? (5.8B, 5.15B)

- How are you using rotations, translations, and reflections to create tessellations? (5.8A, B; 5.14D)

- How did you use rotations, translations, or reflections to make your own shape to tessellate? (5.8A, B; 5.14D)

Teacher Notes (to personalize the lesson for your classroom)

Summary Questions (to direct students' attention to the key mathematics in the lesson)

To encourage students to make generalizations about tessellations from the examples and nonexamples they have observed, ask questions such as:

- Which shapes tessellated? How did you know? (5.7B, 5.14D, 5.15A)

- What did the shapes that tessellated have in common? (5.7A, 5.15A, 5.16A)

- Which shapes did not tessellate? How did you know? (5.7B, 5.14D, 5.15A)

- What did the shapes that did not tessellate have in common? (5.7A, 5.15A, 5.16A)

- How did you make shapes that would tessellate? (5.8A, B; 5.14D)

- What attributes of a shape can tell you whether it will tessellate or not? (5.7A, 5.16A)

To highlight the role played by transformations in creating tessellations, ask questions such as:

- How is the position of this shape in the tessellation related to the position of this other shape? (It is a translation, rotation, or reflection.) (5.8B, 5.15B)

- Which of the tessellations on display used translations? reflections? rotations? How can you tell? (5.8B, 5.16B)

- Which transformations did you use to make your tessellation? (5.8A, B)

- Which transformations did you use to make your original shape to tessellate? (5.8A, B)

- How would you describe to someone a translation? a rotation? a reflection? (5.15A, B)

Teacher Notes (to personalize the lesson for your classroom)

Assessment Task(s) (to identify the mathematics students have learned in the lesson)

- Have each student create a poster-sized tessellation and write a short description of their process, using appropriate geometric vocabulary.

- Have students record in their journals, using appropriate geometric vocabulary, what they have discovered about tessellations, e.g. how they can tell whether a shape will tessellate or not.

Teacher Notes (to personalize the lesson for your classroom)

Extension(s) (to lead students to connect the mathematics learned to other situations, both within and outside the classroom)

- Students can make a scrapbook of tessellations found in the world around them (e.g. using pictures in magazines and snapshots of examples).

- Students can make a shape resembling one of the states in the U.S. that will tessellate.

Teacher Notes (to personalize the lesson for your classroom)