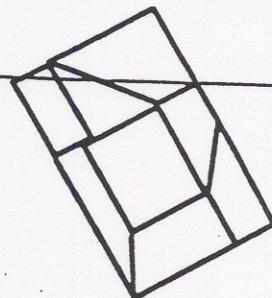# Perplexing Puzzle

Institute Notes

Concept: Investigate the properties and characteristics of a proportional relationship.

TEKS Focus: 6.3—The student solves problems involving proportional relationships.
7.3—The student solves problems involving proportional relationships.
8.3—The student identifies proportional relationships in problem situations and solves problems.

Overview: The puzzle activity will set the tone for the entire institute by providing investigations into proportional relationships and their characteristics. Participants will explore how they can build a larger puzzle by using a variety of strategies. Through a debriefing of the activity, characteristics of a proportional relationship will be discussed:

- comparing with ratios vs. comparing with differences,
- using a table to look for patterns that form equivalent ratios,
- using equivalent ratios to find the constant of proportionality ($\frac{y}{x} = k$) in both fraction and decimal form,
- using this constant of proportionality, k, to write an equation of the form $y = kx$, and
- graphing the ordered pairs in the table to see that they form a straight line through the origin with the equation of the form $y = kx$.

Throughout this activity, multiple representations (verbal, concrete, pictorial, tabular, graphical, and algebraic) will be used to provide depth of understanding for this "big idea" of proportional relationships.

Materials: Puzzle cut into pieces in envelope (1 per group of 4-6)
Construction paper (4 sheets same color per group)
Construction paper of a different color (1 sheet per group)
Scissors   Metric rulers   Graphing calculators
1" graph paper   Masking tape
Scratch paper   Glue sticks
Markers and/or peel-and-stick dots

Procedure: 1. Participants will work in groups of 4-6 on this activity.

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2. Each member of the group is to select one or more of the pieces of the puzzle. The group is to work together to arrange the pieces to make a rectangle.
3. Once the rectangle is made with all eight pieces, each member will retrieve their piece(s) of the puzzle and carefully measure each side in centimeters.
4. The task of each group is to make a puzzle larger than the one in the envelope using this clue: "The side measuring 4 cm in a figure must measure 6 cm in the larger puzzle." (Show Transparency 1.)
5. Each member of the group is to work independently to enlarge their puzzle piece.
6. When everyone in the group has completed making their enlargement, the group will work together again to arrange the larger pieces to complete the puzzle.

7. If any pieces of the enlarged puzzle do not fit, participants should discuss the strategies used to enlarge each piece.
8. After each group has had time to assemble the puzzle using the enlarged pieces, begin the debriefing of this activity. This is the most important part of the activity as strategies for solving are discussed. These strategies will be used throughout the institute and be explored in depth.

Debriefing: Depending upon your observations of the groups, choose one of the following two options for debriefing.

Option One:
1. Ask each group to explain their strategy for enlarging the puzzle. At least one of the groups will probably use equivalent fractions and the "cross-product" method. Ask them how many different equations they used to find the larger dimensions for their puzzle pieces. Ask them what was the same for each equation used and why.
2. Ask participants what the $\frac{6}{4}$ represents in the equation. What is another way to write each equation using decimals? They should respond: $y = 1.5(7)$, $y = 1.5(9)$, $y = 1.5(8)$.

Math Notes:
There are many ways to put the puzzle together into a rectangle.

If participants work on the puzzle in a group before trying it independently, the misconceptions that the puzzle can bring out about proportionality will be corrected too early by the group. The important contrast between "comparing with addition and subtraction" and "comparing with multiplication and division" will not occur.

Participants are likely to add 2 to 4 to get 6 and likewise add 2 to each measurement to get the larger measurement. However, the resulting puzzle pieces will not fit together. By doing this, they are thinking of an "additive process" rather than a proportional "multiplicative process".

By examining a pattern in each of these equations, help participants develop an equation that can be used to find the dimensions of any puzzle piece according to the rule for enlargement.
Example:

$$\frac{4}{6} = \frac{7}{y} \qquad \frac{4}{6} = \frac{9}{y}$$

$$4y = 6 \cdot 7 \qquad 4y = 6 \cdot 9$$

$$y = \frac{6 \cdot 7}{4} \qquad y = \frac{6 \cdot 9}{4}$$

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3. Ask participants to write an equation for enlarging any puzzle piece using the given condition.
4. Discuss the significance of $\frac{6}{4}$ or 1.5 as the constant of proportionality. Ask participants to compare their method of finding the larger sides of their puzzle piece using "cross-products" vs. using the equation, $y = 1.5x$.
5. Ask participants to help you complete the table of values in Activity 1 and discuss the questions.
6. Next, have participants graph the data from the table on 1" graph paper using markers and peel-and-stick dots.
7. Record the data on Transparency 2 and have participants discuss the questions.

Option Two:
1. Instead of asking participants how they solved the puzzle, begin with having them help you complete the table in Activity 1 for original and new lengths of the puzzle pieces.
2. Next, have them observe any patterns in the table and note the ratio of new length to original length for each measurement in the table.
3. Ask participants to rewrite the equation $\frac{y}{x} = k$ so that it is not in fractional form.
   They should write the equation $y = kx$.
4. Next, have participants use 1" graph paper, markers, and peel-and-stick dots to graph the data in the table.
5. Record the data on Transparency 2 and have participants discuss the questions.

Extension: Have participants make a puzzle which represents a reduction of the larger puzzle created in the original activity according to the following condition: The side of the enlarged puzzle piece which measures 9 cm must

Have participants generalize by writing $y = 1.5x$, where x is the length of the side to be enlarged and y is the new length. Ask participants to verbalize the meaning of the terms of the equation $y = 1.5x$.

Discuss the use of technology in solving the problem. The form $y = 1.5x$ can be used more readily with a calculator or as part of a computer program than can the cross-product method.

Participants should observe that there is a set of equivalent ratios of the general form $\frac{y}{x}$ which equals a constant k. Thus, we can write an equation $\frac{y}{x} = k$ to express this relationship. For this activity, the equation becomes $\frac{6}{4} = k$ or 1.5 = k in decimal form. The constant k is called the constant of proportionality.

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measure 5.4 cm in the smaller puzzle piece.

Assessment:
a. Provide participants with a set of tangrams and have them enlarge the set so that the side measuring 5 cm will be 9 cm in the larger puzzle. Ask them to use the characteristics of a proportional relationship to help them complete the task. Also, have participants graph the ordered pairs (original length, new length) and compare to their first graph in this activity.

b. Have participants write a journal entry for the activity by completing the following statements: "I used _____ (strategy) to enlarge my puzzle piece initially because _____. Now that I am aware of the characteristics of a proportional relationship, I plan to use _____ to enlarge or reduce a figure because _____."

Notes:

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Transparency 1

We are going to make a larger puzzle
the same shape as the smaller puzzle,
so that the edge of a piece that
measures 4 cm in the old puzzle
measures 6 cm in the new puzzle.

Enlarge your piece of the puzzle
so that it will fit into
the new larger puzzle.

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Activity 1

Original Length (in cm)	Process New Length (in cm)	New Length (in cm)
2	1.5(2)	3.0
3	1.5(3)	4.5
4	1.5(4)	6.0
5	1.5(5)	7.5
6	1.5(6)	9.0
7	1.5(7)	10.5
8	1.5(8)	12.0
11	1.5(11)	16.5
X	1.5(x)	Y

a. What is the ratio of new length to original length for each row? What does this ratio represent?

b. If a piece of the original puzzle had a side with length 14 cm, what would be the length of the side of the new puzzle piece?

c. If the new puzzle piece has a side of length 22.5 cm, what was the length of the side of the original puzzle piece?

d. If you wanted to make a third puzzle where the original piece with a side of 4 cm becomes 7 cm, how would you determine the lengths of the other sides of the third puzzle pieces?

e. If your new puzzle piece has a length of 12 cm and you wanted to make a reduction of the piece to 5 cm, how would you determine the lengths of the other sides?

f. If you wanted to make a new puzzle where one side of the new puzzle is 2.5 times as long as a side of the original puzzle, what equation could you use to determine the lengths of the other sides?

TEXTEAMS Rethinking Middle School Mathematics: Proportionality   Activity-8

Reason and Communicate:
a. Participants should observe that these ratios are equivalent ratios. Ask if anyone used equivalent ratios to find the larger dimensions of their piece(s). Also have participants note that in a proportional relationship, the ratio of y/x is constant and can be seen easily in a tabular format.

Answers and Math Notes:

a. $\frac{6}{4}$, $\frac{3}{2}$, or 1.5; It is the constant of proportionality.

b. 21 cm

c. 15 cm

d. Multiply each side of the original by 1.75.

e. Multiply each side of the original by $\frac{5}{12}$ or its decimal equivalent.

Or, use the equation $y = \frac{5}{12} x$,

where y represents the new length and x the original length.

f. $y = 2.5 x$, where y represents the new length and x the original length.
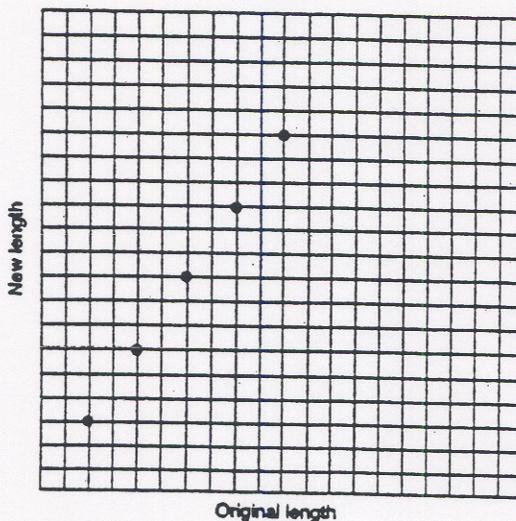
Perplexing Puzzle

Activity 1

Original Length (in cm)	Process New Length (in cm)	New Length (in cm)
2		
3		
4		
5		
6		
7		
8		
11		
X		Y

a. What is the ratio of new length to original length for each row? What does this ratio represent?

b. If a piece of the original puzzle had a side with length 14 cm, what would be the length of the side of the new puzzle piece?

c. If the new puzzle piece has a side of length 22.5 cm, what was the length of the side of the original puzzle piece?

d. If you wanted to make a third puzzle where the original piece with a side of 4 cm becomes 7 cm, how would you determine the lengths of the other sides of the third puzzle pieces?

e. If your new puzzle piece has a length of 12 cm and you wanted to make a reduction of the piece to 5 cm, how would you determine the lengths of the other sides?

f. If you wanted to make a new puzzle where one side of the new puzzle is 2.5 times as long as a side of the original puzzle, what equation could you use to determine the lengths of the other sides?

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Transparency 2

New length

Original length

a. Describe the graph.
b. What does the ordered pair (8,12) mean in this problem?
c. If we are looking for y in the ordered pair (12, y), state what this means in words.
d. Compare the ratios of $\frac{y}{x}$ for each ordered pair graphed.
   What is significant about these ratios? Explain.
e. What is another name for 1.5 or 3/2 in this setting?
f. Use a graphing calculator to graph this line, y=1.5x, and set up a table of values. Use the table of values to answer these questions:
   1) If the length of a side of the new puzzle piece is 34.5 cm, how long was a side of the original puzzle piece?
   2) If a side of the original puzzle piece is 29 cm, how long would a side of the new puzzle piece be in cm?

TEXTEAMS Proportional Relationships Institute   9   Transparency

Answers:
a. Participants can recognize $k = 1.5$ as the slope of the line by showing that a change of 1 in x results in a change of 1.5 in y, e.g. from (1, 1.5) to (2,3).
b. Possible answers: A piece of the original puzzle whose length is 8cm becomes a piece in the new puzzle whose length is 12cm. A length of the new piece is 1.5 times the length of the original piece or 12 is 1.5 times 8. Some participants may observe that (8, 12) represents "an ordered pair of measurements."
c. We are looking for a length of the new piece that is 1.5 times the length of 12. Or, we are looking for the value of y so that the ratio of y to 12 is equivalent to $\frac{3}{2}$.
d. These ratios are equivalent and name the number 1.5 or $\frac{3}{2}$.
e. constant of proportionality
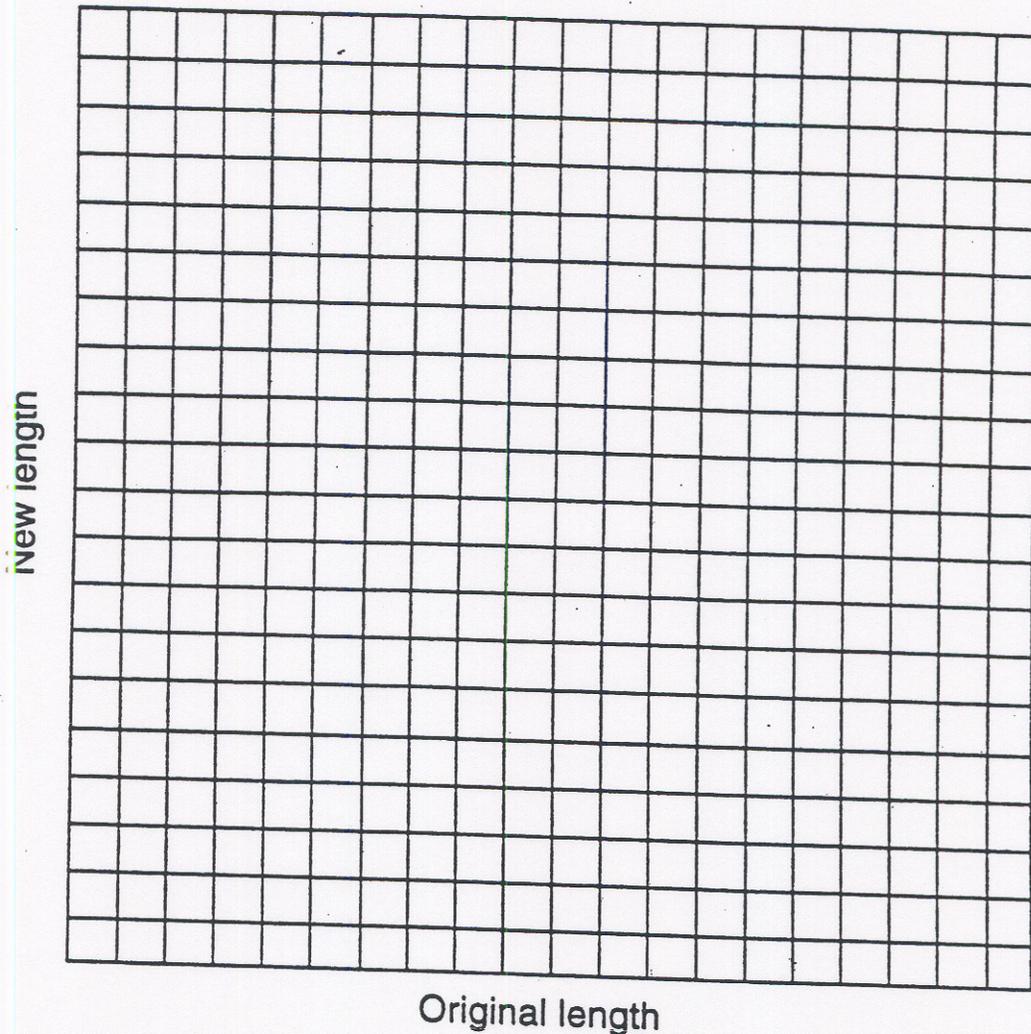f. 1) 23 cm
   2) 43.5 cm

Math Notes:
a. They should explain that the graph of their data is a straight line that passes through the origin. Ask them to keep this in mind as we examine other proportional relationships to see if there is a pattern in the graphs.

d. Lead participants to make the connection between the ordered pairs in their table with the constant ratio 6/4. They should realize that a ratio can be interpreted as "an ordered pair of measurements."

Perplexing Puzzle

Transparency 2

New length

(grid)

Original length

a. Describe the graph.
b. What does the ordered pair (8,12) mean in this problem?
c. If we are looking for y in the ordered pair (12, *y*), state what this means in words.
d. Compare the ratios of $\frac{y}{x}$ for each ordered pair graphed.

   What is significant about these ratios? Explain.
e. What is another name for 1.5 or $\frac{3}{2}$ in this setting?
f. Use a graphing calculator to graph this line, *y* = 1.5*x*, and set up a table of values. Use the table of values to answer these questions:
   1) If the length of a side of the new puzzle piece is 34.5 cm, how long was a side of the original puzzle piece?
   2) If a side of the original puzzle piece is 29 cm, how long would a side of the new puzzle piece be in cm?