

# Archaeological Similarity

A-E Strand(s): Geometry. Sample Courses: Middle School Course 2, Middle School One-Year Advanced Course, Integrated 1, and Geometry.

## Topic/Expectation

G.B.6 Similarity and Congruence

- Interpret the definition and characteristics of similarity for triangles in the plane.
- Apply similarity in practical situations.
- Identify and apply conditions that are sufficient to guarantee similarity of triangles.

G.B.3 Pythagorean theorem

- Apply the Pythagorean theorem and its converse to solve problems.

## Other Topic/Expectation

D.C.2 Mathematical reasoning

- Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexample in mathematical reasoning.

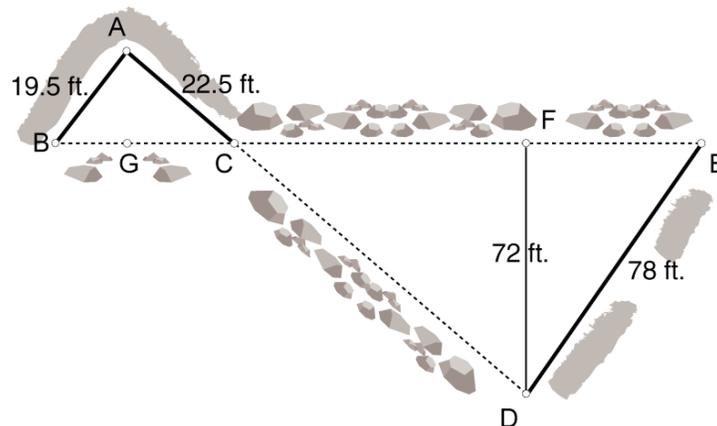
## Rationale

This task allows students to use their knowledge of similar triangles, the Pythagorean theorem, and the converse of the Pythagorean theorem to solve a historically and culturally interesting problem while at the same time developing students' understanding of mathematical proofs.

## Instructional Task

A group of archaeologists about to start a new excavation fly over a remote area in the interior of Mexico and see what appear to be the ruins of an ancient ceremonial temple compound. They observe that some of the buildings are triangular in shape and that some of the walls remain intact. The archaeologists know that some ancient civilizations used geometry in interesting ways when designing their buildings and communities. Before excavating the site, they need to construct an accurate scale drawing of the existing temple compound.

Based on their flight altitude, they estimate measurements and generate a drawing of the ruins.



On the drawing, they decide to mark walls appearing to be intact as solid segments and any walls that have crumbled as dashed segments. They note that segment AB appears to be parallel to segment DF and that segment DE appears to be perpendicular to segment EF.

1. The archaeologists suspect triangle ABC was the ceremony preparation room, and triangle CDF was a two-room building in which the ceremonies were held. What relationship appears to be true about triangles ABC and CDF? Justify your conclusion.
2. They estimate that it will require about forty-five minutes to excavate each linear foot of the exterior walls of the temple site, regardless of the condition of the walls. Approximately how long will it take to excavate all the exterior walls? Explain how you determined this.
3. If the entrance to the ceremony preparation room is opposite vertex B and its position along wall AC corresponds to point E's position along wall FC, what is the distance from B to the preparation room entrance?
4. Are any of the walls of the preparation room (triangle ABC) perpendicular to each other? Justify your answer.

## Discussion/Further Questions/Extensions

In this task, students may be asked to provide any level of reasoning from informal justification to formal proof, depending on their experience and the goals of the teacher. Because of the nature of the task, a group discussion on “How do we know something is true or valid in mathematics?” can be valuable, focusing on what we know, what we assume, and what we make an educated guess about. The goal might be for students to get at the notion of what it means to prove something in mathematics.

This task may provide an opportunity to discuss other archaeological findings that have mathematical bases, such as the pyramids in Egypt or the circular pits used by some Native American groups.

## Sample Solutions

1. The archaeologists suspect triangle ABC was the ceremony preparation room, and triangle CDF was a two-room building in which the ceremonies were held. What relationship appears to be true about triangles ABC and CDF? Justify your conclusion.

*Triangles ABC and FDC appear to be similar.*

*Since line AB appears to be parallel to line DF, angle ABC and angle FDC are congruent because they are alternate interior angles within these two parallel lines. For the same reason, angle BAC and angle DFC are congruent. Two triangles with two congruent angles are similar according to the angle-angle similarity theorem, so it therefore appears that  $\triangle ABC \cong \triangle FDC$ .*

2. They estimate that it will require about forty-five minutes to excavate each linear foot of the exterior walls of the temple site, regardless of the condition of the walls. Approximately how long will it take to excavate all the exterior walls? Explain how you determined this.

*To solve this problem you must first find the perimeter of the exterior walls.*

*If  $\triangle ABC \cong \triangle FDC$ , then*

$$\frac{BC}{BA} = \frac{DC}{DF} \text{ therefore } \frac{22.5}{19.5} = \frac{DC}{78} \text{ and } DC = 90, \text{ and}$$

$$\frac{DF}{FC} = \frac{BA}{AC} \text{ therefore } \frac{78}{84} = \frac{19.5}{AC} \text{ and } AC = 21.$$

*Since triangle DEF appears to be a right triangle,  $78^2 - 72^2 = EF^2$ , and  $EF = 30$ .*

*In addition, it appears that triangle DEC is also a right triangle, and therefore  $90^2 - 72^2 = CE^2$ , and  $CE = 54$ .*

*Since  $FC = EF + CE$ ,  $FC = 30 + 54 = 84$ .*

*The total perimeter of the two triangles can be calculated as follows:*

$$P = AB + BC + AC + DF + DC + FC$$

$$P = 19.5 + 22.5 + 21 + 78 + 90 + 84 = 315 \text{ feet}$$

*Forty-five minutes to excavate one linear foot would mean that it would take about 14,175 minutes, or 236 hours and 15 minutes, to excavate all the exterior walls.*

3. If the entrance to the ceremony preparation room is opposite vertex B and its position along wall AC corresponds to point E's position along wall FC, what is the distance from B to the preparation room entrance?

*If P = the location of the preparation room entrance and BP = the distance from B to the preparation room entrance, then due to triangle similarity,*

$$\frac{DE}{BP} = \frac{DC}{BC} \text{ and } \frac{72}{BP} = \frac{90}{22.5}, \text{ therefore } BP = 18 \text{ feet}.$$

4. Are any of the walls of the preparation room (triangle ABC) perpendicular to each other? Justify your answer.

*If triangle ABC is a right triangle, side BC would be the hypotenuse since it is the longest side. Therefore, using the Pythagorean theorem  $19.5^2 + 21^2$  must equal  $22.5^2$ . Since  $821.25$  is not equal to  $506.25$ , triangle ABC is not a right triangle. Therefore, no walls of the preparation room are perpendicular.*