Cones Launch

A-E Strand(s): Geometry. Sample Courses: Middle School Course 1, Middle School One-Year Advanced Course, Integrated 1, Integrated 2, Integrated 3, Algebra I, and Geometry.

Topic/Expectation

G.A.4 Length, area and volume
  d.   Know and apply formulas for the surface area of right circular cylinders, right circular cones and spheres.

G.B.4 Circles
  c.   Know and apply formulas for the circumference and area of a circle.

G.B.7 Visual representations
  a.   Relate a net, top-view, or side-view to a three-dimensional object that it might represent.

G.C.1 Geometry of a circle
  c.   Recognize, verify and apply statements about the relationships between central angles, inscribed angles, and the circumference arcs they define.

Other Topic/Expectation(s)

N.B.1 Estimation and approximation
  d.   Determine a reasonable degree of precision in a given situation.

G.A.4 Length, area and volume
  e.   Know and apply formulas for the volume of right prisms, right pyramids, right circular cylinders, right circular cones, and spheres. (Extension)

Rationale

This task allows students to connect algebra and geometry by using geometric formulas and spatial skills to solve a problem in a real-world context. It allows for varying levels of difficulty and can be accessible to a wide range of students. The task provides the opportunity for students to apply what they know about ratios and percents to solve equations in a rich geometric context.

Instructional Task

You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice cream cones in the freezer section of area grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones.
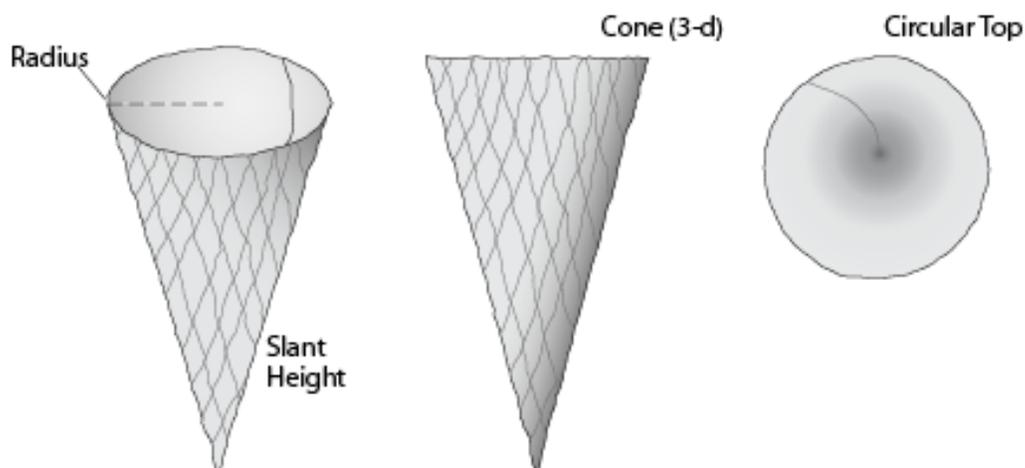
Use a real cone or the dimensions of an ice cream cone to complete the following tasks (see Discussion).

1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.

2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone.

3. Provide the ice cream company owner with a single formula that he can use to find the surface area, SA, of a wrapper (including the lid) for any size cone if he knows the radius, r, of the base and the slant height, s, of the cone. Use the formula from #2, above, for the lateral area of the cone, as well as your previous knowledge about the area of a circle, to develop this new formula.

4. Use a ruler to find the radius of the base and the slant height of the particular cone you are using. Use these measurements to calculate the total surface area of your cone.

5. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

Discussion/Extension/Notes

Students should have had experience constructing nets (two-dimensional representations of the surfaces of three-dimensional solids) at the elementary or middle school level. If they have not had such experience, a brief introduction or review of nets may be necessary.

Ideally, students should have a real cone for this task. If actual ice cream cones are not available, students could be given the dimensions of a cone. A standard sugar cone has a radius of

approximately 2.5 cm and a slant height of approximately 11.5 cm. (Note: A real ice cream cone is required in #4. If actual cones are unavailable, simply provide students with the approximate dimensions and move on to #5.)

This task can allow for varying levels of difficulty, depending on the level of the class and the particular instructional goal(s). Students can be given the formula for the area of a circle, $A = \pi r^2$, and the formula for the surface area of a cone (including its circular base), $SA = \pi rs + \pi r^2$, where r is the radius of the circular base and s is the slant height. Or students can derive the surface area formula based on what they already know about circles, including area and circumference. Determining the central angle of the circular sector of the net for the cone can provide an excellent opportunity for a rich discussion and problem-solving exercise in its own right, since this problem will require students to apply what they know about the number of degrees in a circle, percents, and equation solving.

Extensions:

- Ask students to determine the volume of the cone to see how much ice cream it could hold.

- Students could compare surface areas and volumes of two or more different sizes of cones to make a recommendation about which size might make the most efficient use of paper and/or hold the most ice cream.

- Ask students to create an innovative lid design that uses a shape other than a flat circular disk (for example, a three-dimensional lid with a circular base) and then develop a formula for calculating its surface area.

Sample Solution

1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.

    Check that students have created an accurate net for the situation. One possible representation is shown below. Note that the placement of the circle may vary.
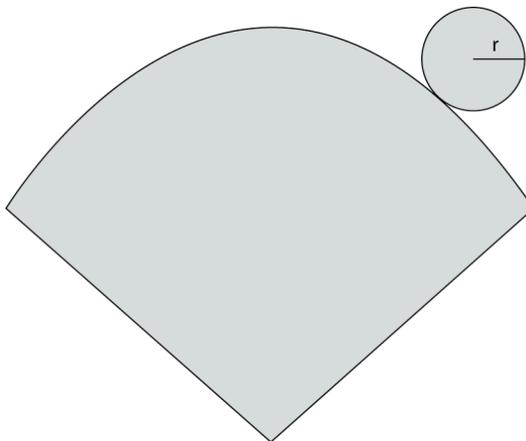
2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone.

    To generate a formula for the lateral area of a cone, sketch a circle with center P (vertex of the cone) and radius s (slant height of the cone).

    The circumference of the circular top of the cone, where r is the radius of the base of the cone, is 2πr. This is also the length of the arc of the sector.

    2∏r

    s   s

    p
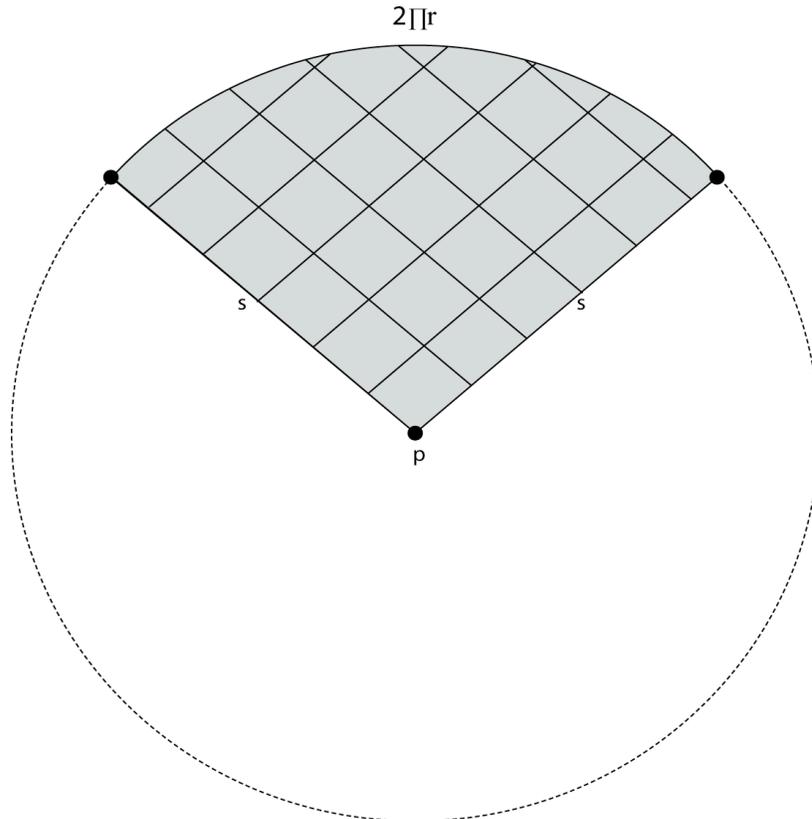
    s = slant height of the ice cream cone

    To find the lateral area of the ice cream cone (a sector of unknown area A), we can set up a proportion of corresponding ratios:

    $$\frac{area\ of\ sector}{total\ area\ circle} = \frac{arc\ of\ the\ sector}{total\ circumference\ of\ circle}$$

    $$\frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

    $$\pi s^2 \bullet \frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s} \bullet \pi s^2$$

    $$A = \pi rs$$

3. Provide the ice cream company owner with a single formula that he can use to find the surface area, *A*, of a wrapper (including the lid) for any size cone if he knows the radius, *r*, of the base and the slant height, *s*, of the cone. Use the formula from #2, above, for the lateral area of the cone, as well as your previous knowledge about the area of a circle, to develop this new formula.

   As found above, the formula for the lateral area, A, of the cone is $A = \pi rs$. To find the area of the circular top portion (the lid) of the cone use the formula for the area of a circle, $A = \pi r^2$.

   To calculate the total surface area (SA) of a cone, combine the two areas by adding the lateral and the area of the base; the formula, then, is $SA = \pi rs + \pi r^2$.

4. Use a ruler to find the radius of the base and the slant height of the particular cone you are using. Use these measurements to calculate the total surface area of your cone.

   Measurements for a standard sugar cone are approximately 11.5 cm for slant height and 2.5 cm for radius. (Answers will vary if a different cone size is used and according to the level of measurement precision.) The total surface area can be found using the measured values for r and s.

   Total surface area: $SA = \pi rs + \pi r^2$

   $$SA = \pi (2.5)(11.5) + \pi(2.5)^2$$
   $$= 110.0 \; cm^2$$

5. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

   Answers may vary.

   Students may decide to first lay out one net, then another, to explore a pattern for estimating the number of complete wrappers that could be cut from the large piece of paper.

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