# Counting Cubes

**A-E Strand(s): Algebra. Sample Courses: Middle School Course 2 and Middle School One-Year Advanced Course.**

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**Topic/Expectation**

    A.A.2 Functions

        b.   Represent and interpret functions using graphs, tables, words and symbols.

**Other Topic/Expectation(s)**

    A.A.1 Variables and expressions

        d.   Identify and transform expressions into equivalent expressions.

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## Rationale

This task asks students to create a pattern using multiple representations (pictures, tables, graphs, and algebraic rules). Students engaged in the task will generate various ways to describe the pattern depending on how they visualize the situation, which will lead to equivalent, but different, expressions.

## Instructional Task

Use the building pattern below to answer the questions that follow.
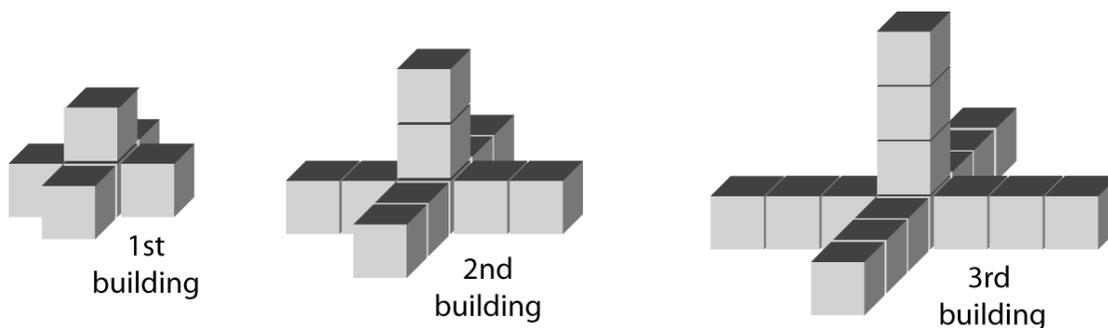


1.  Describe in words the pattern that you see for adding cubes to each new building as it is constructed.

2.  Using the same building pattern shown above, create a table showing the number of cubes that the 4th, 5$^{th}$, and 7th buildings would have.

3.  Use the table to find the relationship between the building number and the number of cubes in that building. Use the process column in the table to help you describe how to determine the number of cubes in each building. (This will be helpful later as you further explore this pattern.)

| Building Number | Process *(how you determine the number of cubes using the building number)* | Number of Cubes |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 7 | | |
| 10 | | |
| 25 | | |
| *n* | | |

4.  Write a function rule that expresses the relationship between the building number, *n*, and the number of cubes in the building, *f(n)*. The process column from the table may be helpful.

5.  Give a verbal description of the functional relationship between the building number and the number of cubes in the building.

6.  How many cubes would be in the 18th building? Explain your process for determining the solution.

7.  If you continued to construct buildings using your pattern, would there be a building that uses exactly 110 cubes? Explain your reasoning.

8.  Using your graphing calculator, make a scatter plot of the data from the table. Then graph the function *f(n)* over the scatter plot. How does the graph of the function differ from the graph of the scatter plot?

9.  What do the points between the data points on the graph of the function represent? Are they appropriate for this function? Why or why not?

## Discussion/Further Questions/Extensions

Mathematically, three examples are not enough to define a pattern. However, these examples can strongly suggest a possible pattern, which is the assumption upon which this task is structured. It is important to note, however, that students may generate various correct solutions based on

patterns different from the one given. The most important consideration is whether the student can justify his/her pattern mathematically. In such cases, the teacher will have to evaluate the student's particular response.

Teachers may want to ask students for different ways to visualize the task, then select one to explore further. In this case, students might respond that five additional cubes are added to a building to generate the next building. They might say that one cube is added to each side and to the top to produce each new building. Since this task is aimed at representing functions, teachers should push students beyond the idea of "adding on" (iteration) as they develop a generalized function rule (i.e., given any building number, find the number of cubes needed).

# Sample Solutions

1. Describe in words the pattern that you see for adding cubes to each new building as it is constructed.

   *In tasks like this one, a student's process for counting the number of cubes in a building determines his or her function rule. For example, if students see Building 1 as a center block plus 1 block added to each side and 1 block added on top of the center block, then the pattern would dictate that Building 2 have a center of 1 block plus 2 blocks added to each side and 2 blocks added on top of the center block. The general rule, therefore, would be 1 block (the center) plus 5 times the building number, i.e., $1 + 5n$, where n is the building number.*

   *Another student may see Building 1 as the base structure, composed of 6 blocks, and see the pattern for constructing each new building as adding a block to each side and to the top, making the rule $6 + 5(n - 1)$, where n is the building number.*

2. Using the same building pattern shown above, create a table showing the number of cubes that the 4th, 5$^{th}$, and 7th buildings would have.

   *Following the pattern above, the 4th building would have 21 cubes, the 5th would have 26, and the 7th would have 36 cubes. Note that if the student generates a different rule (as described in the Discussion above), the teacher may choose to accept different answers.*

| Building Number | Number of Cubes |
|:---:|:---:|
| 1 | 6 |
| 2 | 11 |
| 3 | 16 |
| 4 | 21 |
| 5 | 26 |
| 7 | 36 |

3. Use the table to find the relationship between the building number and the number of cubes in that building. Use the process column in the table to help you describe how to determine the number of cubes in each building. (This will be helpful later as you further explore this pattern.)

*The following might appear in the table for this particular building pattern. Either expression in the process column is acceptable, as are other possible variations.*

| Building Number | Process *(how you determined the number of cubes using the building number)* | Number of Cubes |
|---|---|---|
| 1 | *1 + 5 or 6 + 5(0)* | 6 |
| 2 | *1 + 5(2) or 6 + 5(1)* | 11 |
| 3 | *1 + 5(3) or 6 + 5(2)* | 16 |
| 4 | *1 + 5(4) or 6 + 5(3)* | 21 |
| 5 | *1 + 5(5) or 6 + 5(4)* | 26 |
| 7 | *1 + 5(7) or 6 + 5(6)* | 36 |
| 10 | *1 + 5(10) or 6 + 5(9)* | 51 |
| 25 | *1 + 5(25) or 6 + 5(24)* | 126 |
| n | *1 + 5n or 6 + 5(n − 1)* | 5n + 1 |

4. Write a function rule that expresses the relationship between the building number, *n*, and the number of cubes in the building, *f(n)*. The process column from the table may be helpful.

   *The function rule f(n) = 1 + 5n or f(n) = 6 + 5(n – 1), or some other equivalent rule is acceptable.*

5. Give a verbal description of the functional relationship between the building number and the number of cubes in the building.

   *For the pattern described by the function f(n) = 1 + 5n, we might say,*

   *"The number of cubes in a building is one more than five times the number of the building."*

   *Variations are possible.*

6. How many cubes would be in the 18th building? Explain your process for determining the solution.

   *There would be 91 cubes in the 18th building. Using the function f(n) = 1 + 5n,*

   *f(18) = 1 + 5(18)*

*f(18)= 91*

*Note that a student might explain this in words or with the rule.*

7.  If you continued to construct buildings using your pattern, would there be a building that uses exactly 110 cubes? Explain your reasoning.

    *There could not be a building with exactly 110 cubes. Using this function (f(n) = 1 + 5n), there is always one more (cube) than a multiple of five. The number 110 is one more than 109, which is not a multiple of 5. (A student might also suggest that the number of cubes always ends with a '1' or a '6,' which is an acceptable answer if the student adequately justifies how he or she knows this.)*

8.  Using your graphing calculator, make a scatter plot of the data from the table. Then graph the function *f(n)* over the scatter plot. How does the graph of the function differ from the graph of the scatter plot?
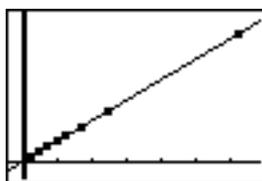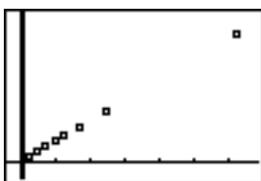
    

    *The graph of the function includes points that are not represented in the scatter plot, for example, (20.5, 103.5).*
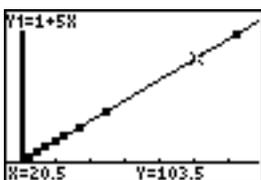
    

9.  What do the points between the data points on the graph of the function represent? Are they appropriate for this function? Why or why not?

    *The points between the data points are supposed to represent buildings. The points in between the dots on the scatter plot don't make sense for this situation because you can't have a 'two-and-a-half' building. The buildings are numbered with ordinal numbers: 1st, 2nd, 3rd, etc., which means that they appear as discrete points on the scatter plot, not as continuous points. This means that the function f(n) is a model for this situation that is meaningful only for whole number values of n.*