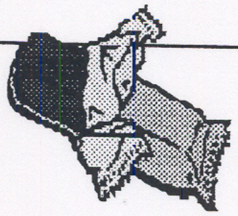
Divide and Conquer!



Institute Notes

Concept: Build a conceptual understanding of division of fractions.

TEKS Focus: **7.2a-** Represent multiplication and division simulations involving fractions and decimals with concrete models, pictures, words, and numbers.

7.2.b- Use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals.

Overview: Given problems involving whole numbers, participants will analyze student responses to two different types of division problems and discuss how the inclusion of the language of sharing and grouping will help students recognize situations that require division. Participants will solve a sequence of problems organized to provide scaffolding for understanding of an algorithm for division of fractions. Emphasis will be placed on using pictures or diagrams and written descriptions of the thought processes involved to support problem solutions and development of an algorithm. The content of this activity was adapted from material developed by Glenda Lappan, author for the Connected Mathematics Project. Her final work will be included in Bits and Pieces II. Ideas for the format of the activity were adapted from work done by Linda Sams and Denise Kubecka of the Cypress Fairbanks ISD. The richness of this activity is due to their willingness to share their work.

Materials: Blank transparencies, Transparency pens, One pad of easel paper or butcher paper, Markers, Masking tape

Procedure:

1. Have participants work in pairs to discuss the thought processes that may have led to each diagram or drawing in problems 1 & 2 of Activity 1. Briefly discuss their conclusions. Use Activity 2 as the springboard to a whole-group discussion of the language of sharing and grouping and the important role it plays in teaching division of whole numbers and fractions.

Divide and Conquer!

2. Have participants work alone for a few minutes to solve the problems in Activity 3. Then have them pair with a partner to discuss their thoughts and work through the problems. Have participants share their answers to problems 1 & 2 on the overhead with an emphasis on how they thought about it, pictured it, and processed it. Alternately, assign groups a problem to write on chart paper and then do a gallery tour to look at all the solutions. Conduct a whole-group discussion of questions 3-5.

3. Follow the same procedure for Activity 4 and 5.

4. If time permits, discuss Extension #2.

Extensions:

1. How is the answer to $20 \div \frac{1}{5}$ related to the answer to $20 \div \frac{3}{5}$? Explain your answer.

2. Can the answer to a division problem be larger than, smaller than or between the two numbers you are dividing? In each case, explain why or why not and show an example to illustrate your answer.

Assessment: You considered two different kinds of situations that call for division -sharing and grouping. Write a story problem for each kind of situation and explain why you think it is a sharing or grouping problem.

Notes:

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Divide and Conquer!

Activity 1

Read problems 1 and 2. Study the diagrams and discuss the thought processes that may have led to each diagram.

1. Emily has three good friends. The four friends go on a hike together. Emily's dad sends along a small bag of caramel candy bars. There are 12 candy bars in the bag. How much does each get if they share them equally?

Diagram 1

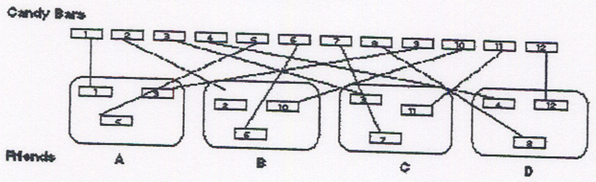

Diagram 2

Distributing Twelve Candy Bars

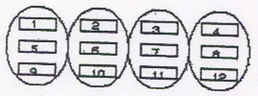

Diagram 3

12 (bars) – 4 (one for each person) = 8 (bars left)
8 – 4 = 4
4 – 4 = 0
We subtracted three groups of four so each person gets three candy bars.

Reason and Communicate:

- What does the answer to the problem tell us in context of the situation? *Problem 1 involves candy bars per friend.*

- What kind of division does the problem illustrate? Does the problem represent a sharing or grouping problem? Explain why you think so. *Problem 1 is a sharing problem. In problem 1 each friend's share is 3 candy bars.*

- How would the answer to problem 1 be expressed if the number of candy bars to be shared is 15? *Each friend would still get 3 whole candy bars, but there would be 3 extras. We would have to decide how to divide these. Should we cut each of them into 4 pieces and give each friend 3 pieces? Or should we give the 3 extra candy bars back to Emily's dad?*

Math Notes:

It is important to use what students know about whole number division to help stimulate their thinking about division situations that involve these new kinds of numbers - fractions. To help review their experiences with whole numbers and division, problems like those in Activity 1, which focus entirely on whole numbers, can be used to set the stage for division of fractions. These problems should not take long, but the discussion of such problems is crucial.

In diagram 1, the student has arranged the 12 candy bars in a row and distributed them one by one to the circles representing the four friends. This is like dealing out the candy bars one by one among the four friends until they are all gone.

In diagram 2, the student has made an array that is four columns long to represent the four students. Then she made as many rows of four candy bars as she could. Then she circled each friend's share.

The student who created diagram 3 used repeated subtraction to find the answer. The student may have reasoned something like this: We have 12 candy bars and four friends, so it takes four bars to give each person one candy bar. We can just keep subtracting four until we have nothing left.

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Activity 1

Read problems 1 and 2. Study the diagrams and discuss the thought processes that may have led to each diagram.

1. Emily has three good friends. The four friends go on a hike together. Emily's dad sends along a small bag of caramel candy bars. There are 12 candy bars in the bag. How much does each get if they share them equally?

Diagram 1

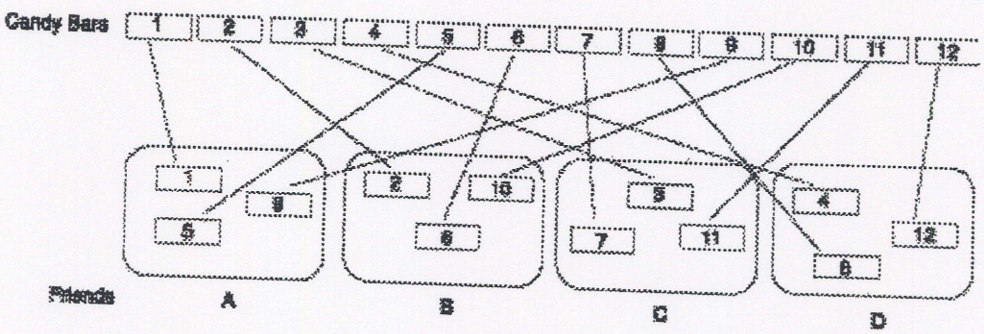

Diagram 2

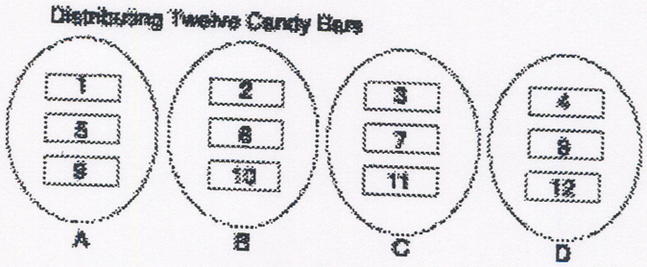

Diagram 3

12 (bars) – 4 (one for each person) = 8 (bars left)

8 – 4 = 4

4 – 4 = 0

We subtracted three groups of four so each person gets three candy bars.

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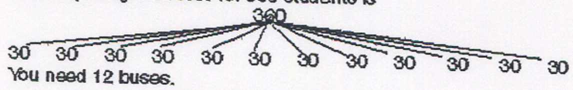
Activity 1, cont.

2. There will be 360 students going on a field trip and each school bus carries 30 students. How many buses will be needed?

Diagram 1

One way to figure buses for 360 students is



You need 12 buses.

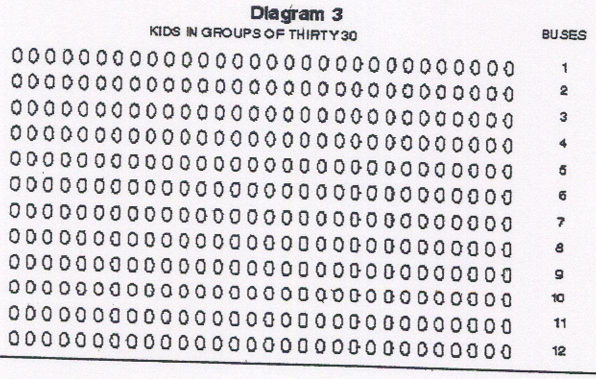
Diagram 2

```
 360
-300  10 X 30
  60

  60
 -60   2 X 30
   0
```

makes 12 buses

Diagram 3



Reason and Communicate:

- What does the answer to the problem tell us in context of the situation? *Problem 2 is number of buses needed.*

- What kind of division does the problem illustrate? Does the problem represent a sharing or grouping problem? Explain why you think so. *Problem 2 is a grouping problem. Here you are finding that you can create 12 groups of 30 students. So you need 12 buses. The number of groups is 12.*

- How would the answer to problem 2 be expressed if the number of students to be placed on buses is 376? *You will have a remainder when you divide 376 by 30, but all the students need a bus so you would have to order an extra bus for the 16 students left over when you divide by 30.*

Math Notes:

In diagram 1, the student used repeated subtraction to find the answer. He or she may have reasoned something like this: We have 360 students and each bus will hold 30, so we can just keep subtracting 30 until we have nothing left. Then we count to see how many buses we need.

In diagram 2, the student made sense of the problem using multiplication facts and subtraction. He knows 10 times 30 is 300, but that is not enough buses. He needs two more buses to carry the remaining 60 students. Then he adds the number of buses needed.

The student who created diagram 3 drew 360 circles in groups of 30 to represent the students on the buses.

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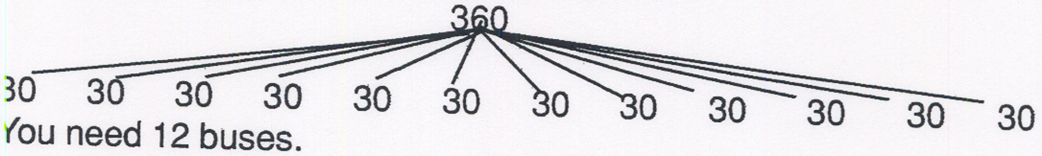
Activity 1, cont.

2. There will be 360 students going on a field trip and each school bus carries 30 students. How many buses will be needed?

Diagram 1

One way to figure buses for 360 students is



You need 12 buses.

Diagram 2

$$\begin{array}{r} 360 \\ \underline{-300} \\ 60 \end{array} \quad 10 \times 30$$

$$\begin{array}{r} 60 \\ \underline{-60} \\ 0 \end{array} \quad 2 \times 30$$

makes 12 buses

Diagram 3

KIDS IN GROUPS OF THIRTY 30	BUSES
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	2
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	3
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	4
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	5
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	6
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	8
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	9
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	10
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	11
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	12

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Math Notes:

There are many kinds of situations that call for division whether with whole numbers or fractions. The two problems in Activity 1 are designed to allow you to have a conversation with the participants and students about two major types of division problems - sharing problems and grouping problems.

In a sharing situation, you have to *share* some known quantity (amount) equally among a known number of entities (people, boxes, packages, etc.). What is not known in a sharing situation is the amount of the given quantity per share. In problem 1, 4 friends are sharing 12 candy bars. In this problem you know the amount to be shared - 12 candy bars - and the number of entities who will share the candy - 4 friends. So we are looking for the share for each person. This would be structurally the same as sharing 120 cookies among 24 scout members. This kind of problem may be solved using division to find the fair share. In problem 1, the quotient from the division means *candy bars per friend*. In the scout situation - 120 cookies and 24 scout members - the quotient would mean *cookies per scout*. To summarize, in a sharing situation you know the amount to be shared and the number of entities that will receive a share. What the answer to the division, the quotient, represents is the amount per share or *size of each share*.

In a grouping situation, you find the number of groups of a given size that you can make from a given quantity (amount). In problems that call for grouping what is known is the size of the groups you are making and what quantity (amount) you have to use to make the groups. What is not known is how many groups of the given size can be made from the quantity given. In problem 2, you know the total number of students is 360 and you know that the group size is 30 students per bus. You are asked to find the number of groups you can make with each group representing a busload. This is a grouping problem much like having 24 students and making groups (teams) of four. How many groups (teams) can you make? In each kind of problem, grouping or sharing, division may be used to determine the numerical answer, but the context tells what the numerical answer means. In problem 2, the quotient means number of buses needed. In the making groups or teams example above, the answer to 24 divided by 4 is 6. We have to give the answer a label to tell what it means. Here it means 6 teams. In a grouping situation, the answer tells *how many groups* of the specified size can be made from the given quantity.

Once the language of *sharing* and *grouping* is introduced in the whole number problems similar to those in Activity 1 and 2 continue to use this language and ask for such analyses in the fraction division problems as well. This language can be very helpful in understanding when division is an appropriate operation.

Answers:

1. Both involve quantities (candy, students) to be shared equally among a number of entities (people, buses). Division can solve both problems.

   Problem 1 is a **sharing** problem. We know the quantity (12 bars) to be shared and the number of entities (4 people) who will share them. We do not know the **size of the share**.

   Problem 2 is a **grouping** problem. We know the quantity (360 students) to be shared and the size of each share (30 students on each bus). We do not know **how many groups** will be required.

2. 3 pieces of candy per 1 student

3. 12 groups of 30 students

4a. The share is 5 miles per 1 student so each student will need to swim 5 miles to reach the goal.
 b. 9 groups of 30 will be formed so we need 9 math teachers for the 7th grade.
 c. The share is 30 boxes per 1 student to sell 600 boxes.
 d. 6 groups of 4 students will be formed.

5. For some students this language is very helpful in understanding when division is an appropriate operation.

Divide and Conquer!

Activity 2

1. How are problems 1 and 2 alike? How are they different?

2. In problem 1, we think of the situation as 12 candy bars to be shared by 4 students, so the quotient means ___________ per _______________.

3. In problem 2, we know 360 students will be placed on buses in groups of 30 so the quotient means _________ groups of _____________.

4. In each of the following problems, use the language in problems 2 & 3 above to state the answer as a sharing or grouping problem.

   a. The 24 members of the school swim team planned to raise money by getting pledges for miles in a swim marathon. If the team goal is to swim 120 miles, how many miles should each swimmer swim?

   b. There are 270 students in the 7th grade at Tippitt Middle School. If mathematics classes are limited to 30 students, how many mathematics teachers are needed for the 7th grade?

   c. Members of the school band plan to sell 600 boxes of cookies in the fund-raising project. There are 20 members in the band. How many boxes should each member sell to reach the goal if all sell the same number of boxes?

   d There are 24 students in Ms. Phillips 5th period mathematics class. She is planning an activity that requires teams of four students. How many teams will be formed?

5. Why should we use the language of sharing and grouping when teaching division?

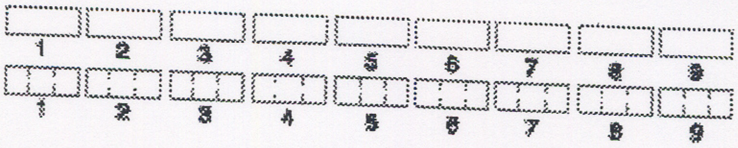
Divide and Conquer!

Math Notes:

Encourage participants to use the think about it column to record their thoughts on the problems. Example: How many pizzas can she make with 9 bars of cheese if each takes 1/3 bar of cheese? *How many 1/3s are in 9? What do I multiply 1/3 by to get 9? What do I get when I divide 9 by 1/3?*

Encourage participants to draw pictures or diagrams to help them think about the problems. Example:

Student One



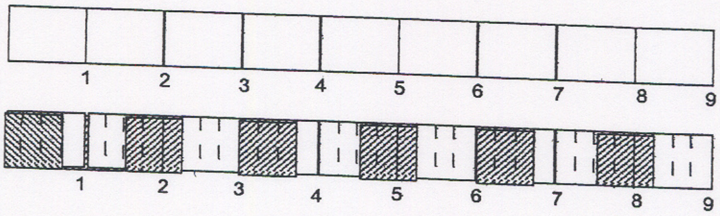
I need to find out how many 1/3's are in 9. This will tell me how many pizzas I can make. I need to divide 9 by 1/3 to find how many pizzas. So 9 ÷ 1/3 = 27, which means that I can make 27 pizzas with 9 bars of cheese.

Others may draw the above diagram and said, "I have 9 bars of cheese. Since each bar has three thirds, I can multiply 9 x 3 to find how many thirds in 9. So 27 pizzas can be made with 9 bars of cheese when each pizza takes 1/3 cup of cheese.

Some participants may use common denominators and reason like this:

$9 \div \frac{1}{3}$ is the same as $\frac{27}{3} \div \frac{1}{3}$. This means how many 1/3s are in 27 thirds, so the answer is 27.

A diagram for the fourth situation in problem 1 is a bit different. Here the numerator of the divisor is not one.



I need to find how many 3/4 's are in 9. This will tell me how many pizzas I can make. I divide 9 by 3/4 to find how many pizzas. I can mark off 3/4 size groups on the 9 bars of cheese and count to see that there are 12. So 9 ÷ 3/4 = 12, which means that I can make 12 pizzas with 9 bars of cheese.

(Some students may write this as 9 x 4 ÷ 3 = 12.)

Reason and Communicate:

- What kind of situation does this problem represent, sharing or grouping? *It represents a grouping problem because we know the quantity to be shared (9 bars of cheese) and the size of the share (1/3 of a bar), but we don't know how many groups can be made.*

Ask participants to justify their answers and share the pictures they drew to make sense of the problem. Be sure participants verbalize that what you do is find the number of the fractional parts that are in the whole number, for example, 1/3s in 9. So you find how many in one whole and multiply by 9. Looking at the questions that involved division by a unit fraction, the pattern is clear:

Problem 1:

a. 9 ÷ 1/3 = 27 or 9 X 3 = 27
b. 9 ÷ 1/6 = 54 or 9 X 6 = 54
c. 9 ÷ 1/4 = 36 or 9 X 4 = 36

Divide and Conquer!

Problem 2: a. 12 ÷ 1/5 = 60 or 12 X 5 = 60

c. 12 ÷ 1/8 = 96 or 12 X 8 = 96

In each case the answer is the same as the product shown. This makes sense because there are twenty-seven 1/3's in 9 and so on.

- How might we write a rule for the pattern we have found in these problems? *When you divide a whole number by a unit fraction, you multiply the whole number by the denominator to find the answer.*

Using the pattern, ask participants to do additional problems of the same type.

- What would be the answer to these problems: 6 ÷ 1/2? *(12)* 5 ÷ 1/9? *(45)* 7 ÷ 1/3? *(21)*

- What about the problems that involved division by a non-unit fraction? Can we find a pattern for them that makes sense?

Problem 1. d. 9 ÷ 3/4 = 12

Problem 2. b. 12 ÷ 3/5 = 20

d. 12 ÷ 5/8 = 19 1/5

- Why do these answers make sense? Look back at our reasoning when the numerator was a one. In Problem 1, part d, if the problem was 9 ÷ 1/4, we would say that there are 4 fourths in each whole. So we would multiply 9 X 4 to get 36. However, the problem is 9 ÷ 3/4. It takes three 1/4ths to make a pizza.

- How will this affect the answer? *We will have to divide by three to get the final answer since it takes three times as many 1/3s to make a pizza.*

- Does the same reasoning work on the other two problems? In problem 2b, we are asked how many 3/5-pound bags can be made from 12 pounds of coffee. One way to think of the problem is to ask how many 3/5s are there in 12 wholes. How could we answer this question? *First find the number of 1/5s in 12, which is 12 = 60. Then divide by 3 because it takes 3 of the 1/5s to fill the bag and 60 ÷ 3 is 20, so you get 20 bags of coffee.* This is the same as multiplying by 5/3. 12 X 5/3 = 20

For Problem 2d, we need to divide 12 pounds of coffee into smaller bags containing 5/8 of a pound. The problem can be written 12 ÷ 5/8. How could we determine the number of bags? *Find out how many eighths are in one whole and multiply by the 12 wholes. Then we have to divide by the number of eighths needed for each bag of coffee. This is the same as multiplying by the denominator and dividing by the numerator of the divisor. So if we are dividing by 5/8 we can get the answer by multiplying by 8/5. This is the same as multiplying by 8 and dividing by 5.* The remainder in this problem could cause confusion. There is an extra 1/8, but that is 1/5 of what you need to fill another bag. Be sure to discuss this carefully.

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Activity 3

Use the chart to organize your thoughts about problems 1 & 2. Then use what you observe to answer questions 3, 4 & 5.

1. Naylah plans to make small cheese pizzas to sell at a school fund-raiser. She has 9 bars of cheddar cheese. How many pizzas can she make if each takes:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)
a. $\frac{1}{3}$ bar of cheese?				
b. $\frac{1}{6}$ bar of cheese?				
c. $\frac{1}{4}$ bar of cheese?				
d. $\frac{3}{4}$ bar of cheese?				

Divide and Conquer!

Activity 3, cont.

2. How many smaller bags of coffee can be made from a twelve-pound bag if each of the smaller bags contains:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. $\frac{1}{5}$ pound?					
b. $\frac{3}{5}$ pound?					
c. $\frac{1}{8}$ pound?					
d. $\frac{5}{8}$ pound?					

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Divide and Conquer!
Activity 3

3. Use ideas from your work on the questions about cheese pizzas and coffee bags to complete the following calculations.

a. $12 \div \frac{1}{4}$ b. $12 \div \frac{1}{3}$

c. $12 \div \frac{2}{3}$ d. $15 \div \frac{5}{3}$

e. $18 \div \frac{5}{6}$ f. $21 \div \frac{2}{3}$

g. Explain in words why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$. How are these two calculations related? Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

4. Describe a procedure that seems to make sense for dividing any whole number by any fraction

5. Write a story problem that can be solved by the division $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

Math Notes:
The problems for Activity 3 were written to allow participants to explore division of fractions in an organized way. The analysis of whole number division in Activity 1 and 2 reminded them of the different ways division can be pictured and reviewed them on the ideas of sharing and grouping. In this activity, the chart is intended to help them picture the problems, write about their thinking, and eventually identify **why** the algorithm they teach works.

Answers:
1a. 27 b. 54 c. 36 d. 12
2a. 60 b. 20 c. 96 d. 19 1/5 (Note that this is 1/5 of the answer to c.)
3a. 48 b. 36 c. 18 d. 9 e. 21 3/5 f. 31 1/2
g. There are 3 thirds in each whole, so there are 8 X 3 thirds in 8. This gives 24. Since 8 ÷ 2/3 asks how many 2/3s there are in 8, we know there would be half as many as the number of 1/3s, so the answer will be half the answer to 8 ÷ 1/3. This would give 24 ÷ 2 or 12 two-thirds in 8.
4. See discussion on page 140.
5. One example: Sam has 12 cups of milk. He is making individual custards that take 2/3 of a cup of milk each. How many can he make? *To solve the problem we need to find out how many 2/3s there are in 12. To do this we need to divide 12 by 2/3. This is a grouping problem.*

Divide and Conquer!

Activity 3

3. Use ideas from your work on the questions about cheese pizzas and coffee bags to complete the following calculations.

a. $12 \div \frac{1}{4}$

b. $12 \div \frac{1}{3}$

c. $12 \div \frac{2}{3}$

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e. $18 \div \frac{5}{6}$

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g. Explain in words why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$. How are these two calculations related? Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

4. Describe a procedure that seems to make sense for dividing any whole number by any fraction

5. Write a story problem that can be solved by the division $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

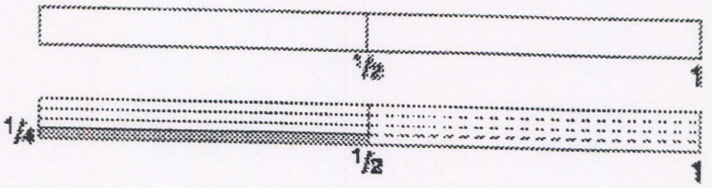
Divide and Conquer!

Math Notes:

This activity is intended to build on the experiences participants have had with the problems calling for division of a whole number by a fraction in Activity 3. In exercise 6, participants will be dealing with contexts in which division of a fraction by a whole number is encountered. The problems are organized to develop a pattern that will lead to an algorithm for division of fractions if either the dividend or the divisor is a whole number. Problems 6 and 7 give a different situation with several subparts. In problem 6, all of the problems are of the form - a unit fraction divided by a whole number. Problem 7 deals with situations where a non-unit fraction is divided by a whole number. Remind participants to use the chart to organize their thinking.

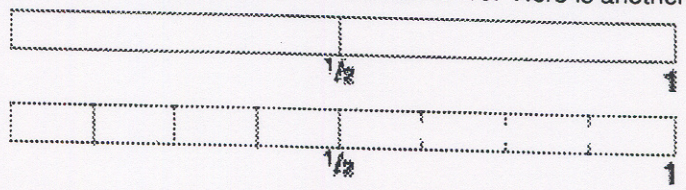
Reason & Communicate:

- How are the situations in problems 6 and 7 different from the ones in problems 1 and 2? *In problem 1 and 2, we divided a whole number by a fraction, but in problems 1 & 2 we are dividing a fraction by a whole number.*
- What are the numbers in the problem and what do they represent? What do you need to find? What does the answer tell you? Are the problems grouping or sharing problems? *These are sharing problems because we know the whole amount (pounds of jellybeans, length of chocolate bar) and the number of groups (students who will share the jellybeans or chocolate bar), but we don't know the size of the share each student will receive.*
- What does the answer tell you? Show me a diagram for problem 1a and explain how it answers the question.



I drew a fraction strip to show 1/2. Then I needed to divide this amount into four parts. This lets me find 1/4 of the amount because that is what each will get. Then I had to name the small part that shows 1/4 of 1/2. The picture shows that 1/4 x 1/2 = 1/8, since we have eight equal parts in the whole. So each person gets 1/8 of a pound of jellybeans.

This diagram shows that 1/2 ÷ 4 is the same as 1/2 x 1/4 which is 1/8. Here is another possible diagram:



First I made a drawing of 1/2. Then I divided 1/2 into four parts because there are four people to share. Now I can see that each person gets 1/8th of a pound of jellybeans. So 1/2 ÷ 4 = 1/8. This makes sense because you will have smaller pieces than the original 1/2 and they will be four times smaller or 1/4 as large. I guess we could do this by seeing that 1/2 ÷ 4 = 1/2 x 1/4 = 1/8.

- How does problem 7 differ from problem 6? *The part to be shared is a non-unit fraction.*

Ask participants to share how they solved problem 7. Some may have used drawings; others may have found common denominators. As you share strategies, encourage participants to verbalize an algorithm for division involving fractions and whole numbers. Build on what they found out in problems 1 and 2. By this time they should have reasonable ways of thinking about the computations. Some will see that multiplying by the reciprocal makes sense and works in both kinds of problems they have encountered so far. Others will see this as multiplying by the denominator and dividing by the numerator of the divisor. Some will still need to draw pictures to help think through a problem.

Problem 6:

a. 1/2 ÷ 4 = 1/8 or 1/2 x 1/4 = 1/8
b. 1/4 ÷ 3 = 1/12 or 1/4 x 1/3 = 1/12
c. 1/3 ÷ 3 = 1/9 or 1/3 x 1/3 = 1/9
d. 1/5 ÷ 2 = 1/10 or 1/5 x 1/2 = 1/10

Divide and Conquer!

Activity 3, cont.

6. Ms. Phillips brought jars of jellybeans to be shared by members of the student groups winning each game at a Family Math Night. How much of a pound of candy will each student get when:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. the jellybeans weigh 1/2 pound and there are 4 students on the winning team?					
b. the jellybeans weigh 1/4 pound and there are 3 students on the winning team?					
c. the jellybeans weigh 1/3 pound and there are 3 students on the winning team?					
d. the jellybeans weigh 1/5 pound and there are 2 students on the winning team?					

Divide and Conquer!

Activity 3, cont.

7. A candy store in town donated very long chocolate bars that were used for prizes in a team competition. What fraction of a long bar did each team member get when:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. a two-person team won $\frac{3}{4}$ of a bar as a prize and share it equally?					
b. a four-person team won $\frac{7}{8}$ of a bar and shared it equally?					
c. a four-person team won $1\frac{1}{2}$ bars and shared it equally?					

Answers:

a. $3/4 \div 2 = 3/8$ or $3/4 \times 1/2 = 3/8$
b. $7/8 \div 4 = 7/32$ or $7/8 \times 1/4 = 7/32$
c. $1\,1/2 \div 4 = 3/8$ or $1\,1/2 \times 1/4 = 3/8$ or $3/2 \times 1/4 = 3/8$

Math Notes:

In each case the answer is the same as the product shown. This makes sense because when you divide 1/2 into four equal parts each part is 1/8 of the whole and so on.

Reason and Communicate:

- How might we write a rule for the pattern we have found in these problems? *When you divide a fraction number by a whole number, you multiply the fraction by the reciprocal of the whole number to find the answer.*

Divide and Conquer!

Activity 3, cont.

7. A candy store in town donated very long chocolate bars that were used for prizes in a team competition. What fraction of a long bar did each team member get when:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. a two-person team won $\frac{3}{4}$ of a bar as a prize and share it equally?					
b. a four-person team won $\frac{7}{8}$ of a bar and shared it equally?					
c. a four-person team won $1\frac{1}{2}$ bars and shared it equally?					

Divide and Conquer!

Activity 3, cont.

8. Use ideas from your work on the questions about jellybeans and chocolate bars to complete the following calculations.

a. 1/2 ÷ 4   b. 3/2 ÷ 2

c. 2/5 ÷ 3   d. 4/5 ÷ 4

e. 7/10 ÷ 2   f. 9/5 ÷ 3

9. Describe a procedure that seems to make sense for dividing any fraction by any whole number.

10. Write a story problem that can be solved by the division 8/3 ÷ 4. Explain why the calculation matches the story.

Answers:

8a. 1/8 b. 3/4 c. 2/15 d. 4/20 e. 7/20 f. 9/15

9. Multiply the dividend by the reciprocal of the divisor. Or multiply by the denominator of the divisor and divide by the numerator.

10. Answers will vary. One possibility is: Four brothers are sharing pizza. Their grandmother made three pizzas and ate 1/3 of one before the boys got home. They shared the rest equally. How much did each brother get? Here you have 8/3 pizza to share among four brothers, but you have to realize that 3 whole pizzas minus the 1/3 the grandmother ate leaves 8/3 to share.

Divide and Conquer!

Activity 3, cont.

8. Use ideas from your work on the questions about jellybeans and chocolate bars to complete the following calculations.

a. $\frac{1}{2} \div 4$

b. $\frac{3}{2} \div 2$

c. $\frac{2}{5} \div 3$

d. $\frac{4}{5} \div 4$

e. $\frac{7}{10} \div 2$

f. $\frac{9}{5} \div 3$

9. Describe a procedure that seems to make sense for dividing any fraction by any whole number.

10. Write a story problem that can be solved by the division $\frac{8}{3} \div 4$. Explain why the calculation matches the story.

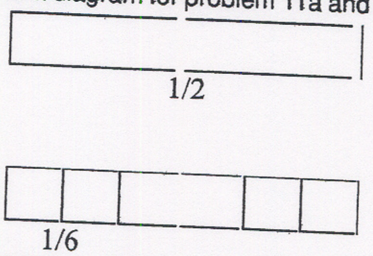
Divide and Conquer!

Math Notes:

This activity is intended to build on the experiences participants have had with the earlier problems and extend their thinking to include problems where a fraction is divided by a fraction. While this is conceptually and procedurally more difficult, the basis for understanding has been built through the problems 1 and 2, and 6 and 7. Problem 11 deals with a unit fraction divided by unit and non-unit fractions and a mixed number. Problem 12 deals with a non-unit fraction divided by a non-unit fraction and a mixed number. Remind participants to use the chart to organize their thinking.

Reason & Communicate:

How are the situations in problems 11 & 12 different from the ones in problems 1 through 10? *In problems 1 through 10 we worked with a whole number and a fraction, but in problems 11 & 12 both of the numbers in the problem are fractions.* What are the numbers in the problem and what do they represent? What do you need to find? What does the answer tell you? Are the problems grouping or sharing problems? *These are grouping problems because we know the whole amount (length of ribbon) and the size of the share (amount of ribbon it takes to make a badge ribbon or bow), but we don't know how many groups (badge ribbons, bows) can be made.* What does the answer tell you? Show me a diagram for problem 11a and explain how it answers the question.

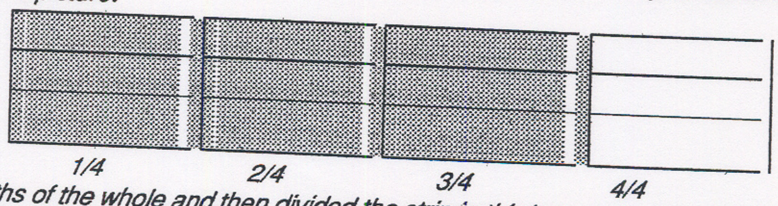


I drew a fraction strip to show 1/2. Then I drew another strip to show 1/6. This lets me see how many 1/6s are in 1/2. The picture shows that 1/2 divided by *1/6 equals 3. So I can make 3 badge ribbons from 1/2 yard of ribbon.* Does anyone have another way to think about this one? *I got a common denominator. So, I wrote 1/2 as 3/6. Then I had to find how many 1/6 are in 3/6. This is the division problem 3/6* divided by *1/6 which is 3.*

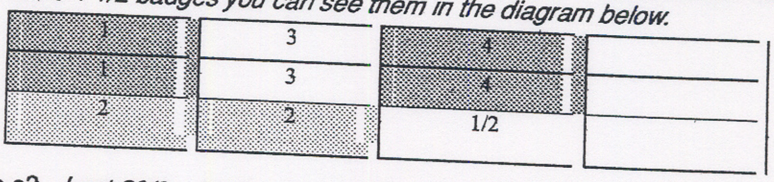
What is different about problem 12? *You have a non-unit fraction amount of ribbon.* Will the same strategies work? *Yes, you can still ask the question, "How many 1/6 are in 3/4?" You can still draw a diagram. And you can still do common denominators.*

Examples:

I used common denominators to write the 3/4 as 9/12 and the 1/6 as 2/12. So I just had to divide 9 by 2 and get 4 1/2 badges. I drew a picture.



I drew the three fourths of the whole and then divided the strip in thirds to make twelfths. But it takes two twelfths to make a sixth. So I have 4 1/2 badges you can see them in the diagram below.



What is the answer to c? *I got 30/8 = 3 6/8 = 3 3/4 badges.*
If there is difficulty dealing with the remainder, go through the common denominator approach and draw a picture to help them see.

What is the answer to problem d and how did you find it? *We need to find the number of 1/6 in 8/3. It is easy if we find a common denominator. 8/3 is equivalent to 16/6 so we can make 16 badges.*

Divide and Conquer!

Activity 3, cont.

Rasheed and Jade have a summer job at a kiosk called Ribbon Remnants. This is a place where you can get small amounts of ribbon very inexpensively from end-of-bolt pieces of ribbon. In each situation that follows, use the chart to organize your reasoning and find your solution.

11. Rasheed takes a customer order to cut ribbons for conference badges. It takes 1/6 of a yard to make a ribbon for a badge. How many badge ribbons can he make from the given remnants of ribbon? For each answer that has a remainder - some ribbon left over - tell what fractional part of another badge ribbon you could make with the amount left over.

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. $\frac{1}{2}$ yard?					
b. $\frac{3}{4}$ yard?					
c. $\frac{5}{8}$ yard?					
d. $2\frac{2}{3}$ yard?					

Divide and Conquer!

Divide and Conquer!

Activity 3, cont.

12. Jade is working on an order for bows for the conference workers to wear so that they can be easily recognized. It takes 2/3 yard of ribbon to make one bow. How many bows can Jade make from each of the following remnants:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. 4/5 yards?					
b. 8/9 yards?					
c. 1 3/4 yards?					
d. 2 1/3 yards?					

TEXTEAMS Rethinking Middle School Mathematics: Numerical Reasoning Activity-158
Summer 2001

Reason and Communicate:

- How does problem 12 differ from problem 11? *Here we need $\frac{2}{3}$ yard for each bow. This is a non-unit fraction. So we will be dividing a non-unit fraction in problem 2. In problem 11 we divided by a unit fraction, $\frac{1}{6}$.*

- In problem *a*, we are making bows with $\frac{4}{5}$ yard. What is the question we are trying to answer? *How many $\frac{2}{3}$ are in $\frac{4}{5}$?*

- Are these grouping or sharing problems? *They are all grouping problems because you are given a quantity to work with and how much each bow takes. You have to find the number of bows, which is like finding the number of groups.*

- Tell me how you found the answer. *I tried common denominators. I wrote both of the fractions as 15ths. This gave me $\frac{10}{15}$ as the amount of ribbon each bow needed and $\frac{12}{15}$ as the amount of ribbon I have. So I have 12 parts and it takes 10 parts to make a bow. I can make 1 bow and have $\frac{2}{15}$ left over for another bow. This means I can make $1\frac{1}{5}$ bows.*

- Did anyone do it differently? *I tried multiplying by the denominator and dividing by the numerator. This gave me $(\frac{4}{5} \times 3) \div 2 = \frac{12}{5} \div 2$. This is the same as $\frac{12}{10}$, which is $1\frac{2}{10}$ or $1\frac{1}{5}$ of a bow.*

Ask similar questions for the remaining problems and ask for drawings where needed to help make sense of the problems.

Here are the answers to the remaining parts:

b. $\frac{24}{18} = \frac{4}{3} = 1\frac{1}{3}$ bows
c. $\frac{21}{8} = 2\frac{5}{8}$ bows
d. $\frac{21}{6}$ or $\frac{7}{2} = 3\frac{1}{2}$ bows

Divide and Conquer!

Activity 3, cont.

12. Jade is working on an order for bows for the conference workers to wear so that they can be easily recognized. It takes 2/3 yard of ribbon to make one bow. How many bows can Jade make from each of the following remnants:

Situation	Think About It (in your own words)	Picture It	Write It In Symbols	Process It (What do you actually do?)	Solve It
a. $\frac{4}{5}$ yards?					
b. $\frac{8}{9}$ yards?					
c. $1\frac{3}{4}$ yards?					
d. $2\frac{1}{3}$ yards?					

Divide and Conquer!

Divide and Conquer!

Activity 3, cont.

13. Based on your work in problems 1 - 12, what general procedure makes sense for dividing with fractions? Remember that a set of steps to do a computation is called an *algorithm*. So this should be your algorithm for dividing fractions.

14. Test your algorithm on these division calculations:

a. $\frac{2}{5} \div 2$

b. $6 \div \frac{2}{3}$

c. $\frac{3}{4} \div \frac{2}{3}$

d. $1\frac{3}{4} \div \frac{1}{2}$

e. $7 \div 3$

f. $\frac{2}{3} \div 1\frac{1}{4}$

15. Write a story problem that can be solved by the division $1\frac{3}{4} \div \frac{1}{2}$. Explain why the calculation matches the story.

Answers:

13. Here are some possibilities:

· Find a common denominator for the given dividend and divisor (the given quantities.) Then divide the numerator of the divisor into the numerator of dividend.

· Multiply the dividend by the quotient of the divisor and divide the result by the numerator of the dividend.

· Multiply the dividend by the reciprocal of the divisor.

14a. [illegible] or $\frac{1}{5}$

b. 9

c. $\frac{9}{8}$ or $1\frac{1}{8}$

d. [illegible] or $\frac{7}{2}$ or 3 1/2

e. 2 1/3

f. [illegible]

15. Answers will vary. Here is a possibility:

Josh is cooking for his nephews and wants to make as many recipes of chili as he can from 1 3/4 pounds of dried beans that he has on hand. Each recipe takes 1/2 cup of dried beans. How many recipes can he make? *Here you have to find 1 3/4 ÷ 1/2 or how many 1/2 are in 1 3/4 to answer the question. There are 3 1/2, so he can make 3 full batches and half of another.*

Divide and Conquer!

Activity 3, cont.

13. Based on your work in problems 1 - 12, what general procedure makes sense for dividing with fractions? Remember that a set of steps to do a computation is called an *algorithm*. So this should be your algorithm for dividing fractions.

14. Test your algorithm on these division calculations:

a $\frac{2}{5} \div 2$

b. $6 \div \frac{2}{3}$

c. $\frac{3}{4} \div \frac{2}{3}$

d. $1\frac{3}{4} \div \frac{1}{2}$

e. $7 \div 3$

f. $\frac{2}{3} \div 1\frac{1}{4}$

15. Write a story problem that can be solved by the division $1\frac{3}{4} \div \frac{1}{2}$. Explain why the calculation matches the story.