How Odd . . .

A-E Strand(s): Probability/Statistics

Topic/Expectation

PS.A.1 Simple probability

d. Know and use the relationship between probability and odds.

PS.B.1 Compound probability

b. Use probability to interpret odds and risks and recognize common misconceptions.

Other Topic/Expectation(s)

PS.A.1 Simple probability

a. Represent probabilities using ratios and percents.

Core Algebra II EOC Content

EOC: R1. Permutations, combinations, and probability

a. Determine the number of ways events can occur using permutations, combinations, and other systematic counting methods.

Rationale

Students (and adults) often confuse probability and odds, and misuses of these terms are common in the media. This task allows a discussion of the difference between and the relationship between these two concepts using interesting contexts found in the media.

Instructional Task

Over the course of a week, students were asked to find statements about odds in magazines, newspapers, or on TV and bring them to class. Here's what some students found:

- Odds of injury from mowing the lawn: 3,623 to 1
- Odds of drowning in a bathtub: 685,000 to 1
- Odds of dating a supermodel: 88,000 to 1
- Odds of becoming a pro athlete: 22,000 to 1
- Odds of getting a royal flush in poker on first five cards dealt: 649,739 to 1

1. Based on what you know about odds, what is wrong with the above statements? Why? How could the statements be changed so that they are correct?

2. According to the list of statements, the odds of getting a royal flush on the first five cards dealt from a standard deck of 52 cards are mistakenly written as 649,739 to 1. [Note: A standard deck of cards used for poker, bridge and other games consists of 4 suits (clubs, diamonds, hearts, spades), each of which includes the ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king.]

a. How many different five-card hands can be dealt from a regular deck?

b. A royal flush consists of five cards—the 10, jack, queen, king, and ace—all of the same suit. How many different ways are there to make a royal flush from a standard deck of cards?

c. What is the probability of getting a royal flush on the first five cards dealt?

d. What are the odds of getting a royal flush on the first five cards dealt?

e. What mistakes were made when the odds of getting a royal flush on the first five cards dealt were reported as 649,739 to 1?

3. Some students found that the chance (probability) of an American home having at least one container of ice cream in the freezer is reportedly 9 in 10.

   a. Assuming this is true, what is the probability that a randomly selected American home has at least one container of ice cream in the freezer?

   b. What are the odds that an American home selected at random will have at least one container of ice cream in the freezer?

   c. How are the probability and odds related in the ice cream example?

4. If you know the odds of an event, how can you find the probability of that event?

5. If you know the probability of an event, how can you find the odds of that event?

Discussion/Further Questions/Extensions

The chance of an event occurring is measured using either probability or odds. The concept of odds is confusing to many. This is not surprising, since we find that many misconceptions about odds are reported in the media as fact.

In truth, if all outcomes are equally likely, the probability of an event, P(E), is the part-to-whole ratio for the number of outcomes.

P(E)=(number of occurrences of the event)/(total number of outcomes)

On the other hand, the odds of an event, Odds(E), are described by the part-to-part ratio.

Odds (E) = (number of occurrences) : (number of non-occurrences)

Example: The *probability* of rolling a six on a die is 1/6 (one out of six), but the *odds* of rolling a six are 1:5 (one to five).

Often, the media intend to demonstrate the unlikelihood of an event occurring by describing the odds of an event as a large number to one (in other words, the number of non-occurrences to the

number of occurrences). However, this ratio actually represents the odds *against* the event, a less common expression that is acceptable only if presented accurately.

Sample Solutions

1. Based on what you know about odds, what is wrong with the above statements? Why? How could the statements be changed so that they are correct?

   The examples don't make sense because the odds of an event should be represented by the ratio of the number of occurrences to the number of non-occurrences. In these statements, then, it seems that the number of occurrences is high for rare events such as drowning in the bathtub. The odds of drowning in a bathtub are more likely 1 to 685,000 (rather than 685,000 to 1, as presented).

2. According to the list of statements, the odds of getting a royal flush on the first five cards dealt from a standard deck of 52 cards are mistakenly written as 649,739 to 1. [Note: A standard deck of cards used for poker, bridge and other games consists of 4 suits (clubs, diamonds, hearts, spades), each of which includes the ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king.]

   a. How many different five-card hands can be dealt from a regular deck?

      The number of different five-card hands can be found by determining the combination of 52 choose 5 since the cards cannot be repeated and the order they are dealt doesn't matter.

      $$C(52,5) = \frac{52!}{5!\cdot 47!} = \frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 2{,}598{,}960$$

   b. A royal flush consists of five cards—the 10, jack, queen, king, and ace—all of the same suit. How many different ways are there to make a royal flush from a standard deck of cards?

      There are four ways since there are four suits.

   c. What is the probability of getting a royal flush on the first five cards dealt?

      $$P(E) = \frac{\#\ of\ \ occurences\ of\ \ the\ event}{total\ \ \#\ \ of\ \ outcomes} = \frac{4}{2{,}598{,}960} = \frac{1}{649{,}740}$$

   d. What are the odds of getting a royal flush on the first five cards dealt?

      The odds of getting a royal flush are the number of occurrences to the number of non-occurrences; that is, 4 to 2,598,956 or 1 to 649,739.

   e. What mistakes were made when the odds of getting a royal flush on the first five cards dealt were reported as 649,739 to 1?

      The person who reported this used the ratio of non-occurrences to occurrences. However, the odds of getting a royal flush on the first five cards dealt is 1 to 649,739.

3. Some students found that the chance (probability) of an American home having at least one container of ice cream in the freezer is 9 in 10.

   a. Assuming this is true, what is the probability that a randomly selected American home has at least one container of ice cream in the freezer?

      The probability that a randomly selected American home has at least one container of ice cream is 9/10.

   b. What are the odds that an American home selected at random will have at least one container of ice cream in the freezer?

      The odds that a randomly selected American home has at least one container of ice cream are 9 to 1.

   c. How are the probability and odds related in the ice cream example?

      The probability and odds in the ice cream example are related in that they both use the ratio of occurrences (9). The probability is a part-to-whole ratio describing the number of occurrences (9) to the total number of possible occurrences (10). The odds are a part-to-part ratio comparing the number of occurrences (9) to the number of non-occurrences (1).

4. If you know the odds of an event, how can you find the probability of that event?

   If you know the odds of an event are a to b, the probability of that event is $\frac{a}{a+b}$.

5. If you know the probability of an event, how can you find the odds of that event?

   If you know the probability of an event is k/n, the odds of that event are k to (n – k).