Is Your Score Normal?

A-E Strand(s): Probability/Statistics

Topic/Expectation

    PS.C.1 Probability Distributions

        b.   Know and use the chief characteristics of the normal distribution.

        c.   Calculate and use the mean and standard deviation to describe the characteristics of a distribution.

Core Algebra II EOC Content

    EOC: R1. Probability Distributions

        b. Identify the principal characteristics of the normal distribution and use them to estimate probabilities.

Rationale

This task involves the characteristics of the mean and standard deviation of a normal distribution. It provides students with a relevant context (PSAT scores) as they examine a normal distribution. The task calls for students to use a normal distribution to determine the probability of individual results.

Instructional Task

In 2005, over 700,000 sophomores took a test called the PSAT. Typically, a larger sample size renders data that fall into a normal distribution curve. Since a large number of students took the test, it is reasonable to assume the scores were normally distributed. The mean score on the math portion of the PSAT in 2005 was 44.1 with a standard deviation of 11.1.[1]

1. Explain why the mean score and standard deviation on the PSAT may differ from year to year.

2. Can the mean score on the PSAT be the same for two years but the standard deviations different? Explain.

3. In 2005, the mean math score was 44.1 and the standard deviation was 11.1. Assuming the scores are normally distributed, draw a sketch of the distribution of the scores, labeling the mean and standard deviation.

4. If the scores are normally distributed, what percentage of scores should fall within two standard deviations of the mean of the scores?

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[1] Source for data on 2005 PSAT scores: College Board. "PSAT/NMSQT Percentiles and Mean Scores." www.collegeboard.com/prod_downloads/counselors/psat/PSATscores.pdf. (Date retrieved: February 7, 2007.)

5. Give an interval describing the range of math scores that are within two standard deviations of the 2005 mean of 44.1.

Discussion/Guiding Questions/Extensions

Students could find the mean and standard deviation of PSAT scores for other years. They could find the mean and standard deviation for a variety of schools' (large and small) scores for one or more years.

What impact does school size have on the distribution of scores? In other words, if a small school had only 15 test takers, would you assume that the scores' distribution would be normal?

This task may call for a discussion of the assumptions underlying the normal distribution and the limitations of such assumptions. Furthermore, students need to understand possible differences between observed patterns from one sample to another.

Sample Solutions

1. Explain why the mean score and standard deviation on the PSAT may differ from year to year.

   While scores on the exams may be normally distributed over time, scores for particular years may have unique centers (means) and spreads (standard deviations). In fact, such variations from year to year are expected.

2. Can the mean score on the PSAT be the same for two years but the standard deviations different? Explain.

   Yes, two normal distributions based on actual data may have the same mean but different standard deviations. Mean is a measure of center, whereas standard deviation is a measure of the spread of the data. For example, if two people took the test and both scored 150, their mean would be 150, but the standard deviation of their scores would be 0. The next year, two people could score 140 and 160. Their mean is still 150, but the standard deviation is 10.

3. In 2005, the mean math score was 44.1 and the standard deviation was 11.1. Assuming the scores are normally distributed, draw a sketch of the distribution of the scores, labeling the mean and standard deviation.
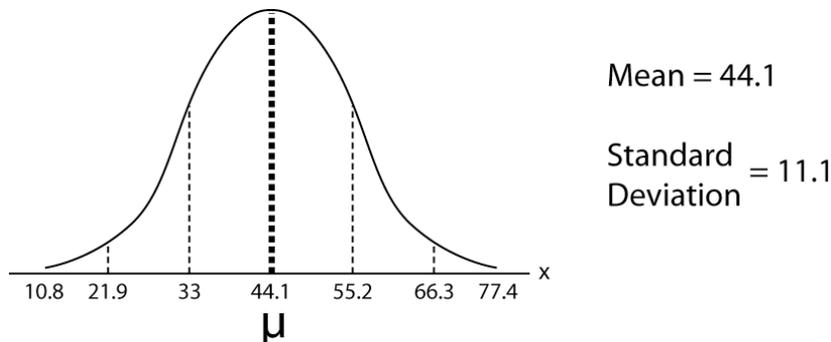
4. If the scores are normally distributed, what percentage of scores should fall within two standard deviations of the mean of the scores?

    In any normal distribution, approximately 34.1% of the data falls within one standard deviation above the mean, and approximately 13.6% of the data falls between one and two standard deviations above the mean. This means that 47.7% of the data lies within two standard deviations above the mean. Similarly, 47.7% of the data lies within two standard deviations below the mean. Therefore, approximately 95% of the data falls within two standard deviations of the mean.

5. Give an interval describing the range of math scores that are within two standard deviations of the 2005 mean of 44.1.

    The scores within two standard deviations of the mean are between 21.9 and 66.3.