Middle School Mathematics Assessments

Proportional Reasoning

The Charles A. Dana Center
at The University of Texas at Austin

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The Charles A. Dana Center at The University of Texas at Austin works to support education leaders and policymakers in strengthening Texas education. As a research unit of UT Austin's College of Natural Sciences, the Dana Center maintains a special emphasis on mathematics and science education. We offer professional development institutes and produce research-based mathematics and science resources for educators to use in helping all students achieve academic success. For more information, visit the Dana Center website at www.utdanacenter.org.

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TEKS and TAKS Resources

The mathematics Texas Essential Knowledge and Skills (TEKS) were developed by the state of Texas to clarify what all students should know and be able to do in mathematics in kindergarten through grade 12. Districts are required to provide instruction that is aligned with the mathematics TEKS, which were adopted by the State Board of Education in 1997 and implemented statewide in 1998. The mathematics TEKS also form the objectives and student expectations for the mathematics portion of the Texas Assessment of Knowledge and Skills (TAKS) for grades 3 through 10 and for the Grade 11 Exit Level assessment.

The mathematics TEKS can be downloaded in printable format, free of charge, from the Texas Education Agency website (www.tea.state.tx.us/teks). Bound versions of the mathematics and science TEKS are available for a fee from the Charles A. Dana Center at The University of Texas at Austin (www.utdanacenter.org).

Resources for implementing the mathematics TEKS, including professional development opportunities, are available through the Texas Education Agency and the Charles A. Dana Center, formerly the state-designated Mathematics Center for Educator Development. Online resources can be found in the Mathematics TEKS Toolkit at www.mathtekstoolkit.org.

Additional products and services that may be of interest are available from the Dana Center at www.utdanacenter.org. These include the following:

- TEKS, TAAS, and TAKS: What's Tested at Grades 3–8? charts
- Mathematics Abridged TEKS charts
- Mathematics TEKS "Big Picture" posters
- *Mathematics Standards in the Classroom; Resources for Grades 3–5*
- *Mathematics Standards in the Classroom; Resources for Grades 6–8*
- *Algebra I Assessments* and the corresponding professional development
- *Geometry Assessments* and the corresponding professional development
- *Algebra II Assessments* and the corresponding professional development
- TEXTEAMS professional development mathematics institutes
- TEKS for Leaders professional development modules for principals and other administrators

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Introduction

The Dana Center developed *Middle School Mathematics Assessments: Proportional Reasoning* as a resource for teachers to use to provide ongoing assessment integrated with middle school mathematics instruction.

Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) states: "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students."[1] Further, NCTM (1995) identified the following six standards to guide classroom assessment:[2]

Standard 1: Assessment should reflect the mathematics that all students need to know and be able to do.

Standard 2: Assessment should enhance mathematics learning.

Standard 3: Assessment should promote equity.

Standard 4: Assessment should be an open process.

Standard 5: Assessment should promote valid inferences about mathematics learning.

Standard 6: Assessment should be a coherent process.

Implementing these assessment standards may require significant changes in how teachers view and use assessment in the mathematics classroom. Teachers should assess frequently to monitor individual performance and guide instruction.

What is *Middle School Mathematics Assessments: Proportional Reasoning*?

Middle School Mathematics Assessments: Proportional Reasoning contains problems that reflect what all students need to know and be able to do in sixth-, seventh-, and eighth-grade mathematics. The resource focuses on exploring proportionality through the content and process strands. These assessments may be formative, summative, or ongoing. The problems focus on students' conceptual understanding as well as their procedural knowledge. The tasks require more than right or wrong answers; they focus on how students are thinking about a situation.

Why focus on proportional reasoning?

The "big idea" of proportionality, including proportional relationships and proportional reasoning, is central to an understanding of middle and high school mathematics, and

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[1] National Council of Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

[2] National Council of Teachers of Mathematics. 1995. *Assessment Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

provides a critical foundation for formal algebra study. The concept of proportionality operates as a single thread running through and connecting each of the content and process strands of middle school mathematics.

Proportionality involves recognizing quantities that are related by multiplication. Numbers, tables, graphs, words, and equations are used to think about the quantities and their relationships. Fluency with proportionality develops through problem solving and reasoning in many areas, including ratio and proportion, percent, similarity, scaling, linear equations, slope, and probability.

What is the purpose of *Middle School Mathematics Assessments: Proportional Reasoning*?

The purpose of these assessments is to make clear to teachers, students, and parents what is being taught and learned about proportionality throughout middle school mathematics. Teachers should use evidence of student insight, student misconceptions, and student problem-solving strategies to guide their instruction. Teachers may also use the questions included with the assessments to guide learning and to assess student understanding. The use of these assessments should help teachers enhance student learning and provide them with a source of evidence on which they may base their instructional decisions.

What is the format of *Middle School Mathematics Assessments: Proportional Reasoning*?

This book contains 51 problems divided by chapter according to their TEKS strand. The problems address how proportionality can be used to show understanding of the TEKS.

The problems have been divided into six categories:

Number, Operation, and Quantitative Reasoning
Patterns, Relationships, and Algebraic Thinking
Geometry and Spatial Reasoning
Measurement
Probability and Statistics
Underlying Processes and Mathematical Tools

Each problem

- includes a mathematics task,
- is aligned with the Grades 6, 7, and 8 mathematics Texas Essential Knowledge and Skills (TEKS) student expectations,
- is aligned with the Texas Assessment of Knowledge and Skills (TAKS) objectives,
- includes “scaffolding” questions that the teacher may use to help the student to analyze the problem,

- provides a sample solution,* and
- includes extension questions to bring out additional mathematical concepts in a summative discussion of solutions to the problem.

*The sample solution is only one way that a problem may be approached and is not necessarily the "best" solution. For many of the problems there are other approaches that will also provide a correct analysis of the problem. The authors have attempted to illustrate a variety of methods in the different problem solutions. Several of the problems include samples of anonymous student work.

Following this introduction are alignments of all the problems to the TEKS and to the grade-level TAKS objectives.

What is the solution guide?

The solution guide is a problem-solving checklist that may be used to understand what is necessary for a complete problem solution. When assigning the problem, the teacher will give the students the solution guide and will indicate which of the criteria should be considered in the problem analysis. In most problems all of the criteria are important, but initially the teacher may want to focus on only two or three criteria. On the page before a student work sample in this book, comments on some of the criteria that are evident from the student's solution are given. The professional development experience described below will help the teacher use this tool in the classroom and will also help guide the teacher to use other assessment evaluation tools.

TEXTEAMS Practice-Based Professional Development—*Middle School Mathematics Assessments: Proportional Reasoning*

The Dana Center has developed a three-day TEXTEAMS institute in which participants experience selected assessments, examine the assessments for alignment with the TEKS and TAKS, analyze student work to evaluate student understanding, consider methods for evaluating student work, view a video of students working on the assessments, develop strategies for classroom implementation, and consider how the assessments support the TAKS. Teachers should contact their local school district or regional service center to determine when this institute is offered.

Middle School Mathematics Assessment Solution Guide

Name of Student:

Name of Problem:

The teacher will mark the criteria to be considered in the solution of this particular problem	Criteria	Check if solution satisfies this criteria
	Describes mathematical relationships	
	Recognizes and applies proportional relationships	
	Develops and carries out a plan for solving a problem that includes *understand the problem, select a strategy, solve the problem, and check*	
	Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations	
	Evaluates the reasonableness or significance of the solution in the context of the problem	
	Demonstrates an understanding of mathematical concepts, processes, and skills	
	Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them	
	Communicates clear, detailed, and organized solution strategy	
	Uses appropriate terminology, notation, and tools	
	States a clear and accurate solution using correct units	

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 1							
6	Fun Park Party	6.2C	6.4A, 6.5				6.11A, B, C, D
7	Fun Park Party	7.2F, G	7.3B				7.13A, B, C, D 7.14A
8	Fun Park Saturday	8.2D	8.5A				8.14A, B, C, D 8.15A
6	Homecoming Chili	6.2C, D					6.11A, B, C, D 6.12A
6	Spring Sensations	6.2C, D					6.11A, B, C, D
7	Bargain Shopping	7.2A, B	7.3A				7.13A, B, C, D 7.14A
7	Rose Garden Plan	7.2D, G					7.13A, B, C, D
8	Talk, Talk, Talk	8.1B	8.3A, B 8.4				8.14A, B, C, D 8.15A
8	Half-Life Happening	8.1B, D 8.2B	8.3A				8.14A, B, C, D 8.15A

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 2							
6	At Home in Space		6.3A, B, C, 6.4A, B				
7	Solar Cells for Science		7.4A, B 7.5A				
8	City in Space		8.3A 8.4 8.5A,B				
6	Community Clean-Up		6.3A, C				
6	Towering Pizzas		6.3A, B, C 6.4A, B 6.5				
7	South Texas Natives		7.3A, B 7.4C				
7	Working Smarter		7.4A, C 7.5B				
8	Fast Food Workout		8.3B 8.4 8.5A, B				
8	Global Warming: Texas-Size		8.3A 8.4 8.5A, B				

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 3							
6	By the Sea		6.3A, C	6.7			6.12A
7	By the Sea			7.6 7.7			
8	By the Sea			8.6A, B 8.7			8.9A
6	Sweet Trip to the Candy Shop			6.7			6.12A, B
6	The Round Table		6.3A, C	6.6C			
7	Sorting Rectangles	7.2D	7.3B	7.6D			
7	Mighty Mascot		7.3B	7.6D 7.9			
8	Javier Builds a Model		8.3B 8.5A	8.7B 8.8C			8.14A, C, D 8.15A
8	Student Council President	8.2A	8.3B 8.5A	8.6A	8.9B		

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 4							
6	Lights at the Marleys' and Farleys'	6.2B	6.3B 6.4A		6.8A, B, D		6.11A, C 6.12A
7	Gardens at the Marleys' and Farleys'	7.2D	7.3A, B	7.6D	7.9		7.13A, B, D 7.14A
8	Storage Boxes at the Marleys' and Farleys'	8.2D	8.3A, B	8.7B	8.8B, C 8.9B 8.10B		8.14A, B, D 8.15A
6	Extravaganza		6.3A, B, C 6.4 A		6.8D		
6	Matchmaker				6.8D	6.10D	6.12A 6.13B
7	Bug Juice	7.2B, D	7.3B		7.9		
7	Photographic Memories		7.3B 7.4B	7.6D 7.7A 7.8C	7.9	7.11B	7.13A, C, D 7.14A
8	In the Rafters	8.2D	8.3B	8.6A 8.7B	8.9A,B		
8	Liberty Enlightening the World	8.2D	8.3B	8.7B	8.8C 8.9B 8.10B		

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 5							
6	Perplexing Polygons		6.3C			6.9A, B	
7	Perplexing Polygons		7.3B			7.10A, B	
8	Perplexing Polygons		8.3B			8.11A, B	
6	Science Quiz					6.10B, D	
7	Big Money Prizes		7.3B			7.12A, B	
8	Five Friends					8.12A, B	

Mathematics TEKS Alignment

	Problem	Number, operation, and quantitative reasoning	Patterns, relationships, and algebraic thinking	Geometry and spatial reasoning	Measurement	Probability and statistics	Underlying processes and mathematical tools
Chapter 6							
6	What's in Your Wallet?	6.1A 6.2C, D	6.3A, B, C			6.10B, D	6.11A, B, C, D, 6.12A, B 6.13B
7	What's in Your Wallet?	7.2D	7.3B 7.4B	7.7A			7.13A 7.14A 7.15B
8	What's in Your Wallet?	8.1B 8.2C, D	8.3B 8.4 8.5A				8.14A, C, D 8.15A 8.16B
6	Gone Fishin'	6.2C	6.3A, B, C			6.10A, C, D	6.11A, C, D 6.12A, B
6	Secret Recipe		6.3A, B		6.8A, D		6.11B, D 6.12A 6.13A, B
7	Rx	7.2B, D, G	7.3B 7.5B				7.13A, B 7.14A
7	It's a Weighty Matter	7.2B, D	7.3B 7.4A	7.7A			7.13A, D 7.14A 7.15B
8	Java Joe	8.1B 8.2A	8.3B			8.12C 8.13B	8.14D 8.15A
8	How Green Is Green?	8.1B 8.2A, B, D	8.3B 8.4 8.5A		8.8C		8.14A, B, C, D 8.15A 8.16B

Mathematics Grade 6 TAKS Alignment

This chart shows the problems that have been aligned to the Grade 6 Texas Assessment of Knowledge and Skills (TAKS).

Problem Name	Grade 6 TAKS Objective					
	1	2	3	4	5	6
At Home in Space		X				
By the Sea			X			
Community Clean-Up		X				
Extravaganza		X		X		
Fun Park Party	X					X
Gone Fishin'						X
Homecoming Chili	X					X
Lights at the Marleys' and Farleys'		X		X	X	
Matchmaker	X	X		X		
Perplexing Polygons					X	
The Round Table			X			
Science Quiz					X	
Secret Recipe						X
Spring Sensations	X					X
Sweet Trip to the Candy Shop			X			
Towering Pizzas	X	X	X		X	
What's in Your Wallet?						X

Mathematics Grade 7 TAKS Alignment

This chart shows the problems that have been aligned to the Grade 7 Texas Assessment of Knowledge and Skills (TAKS).

Problem Name	Grade 7 TAKS Objective					
	1	2	3	4	5	6
Bargain Shopping	X					X
Big Money Prize					X	
Bug Juice	X	X		X		
By the Sea			X			
Fun Park Party	X	X				X
Gardens at the Marleys' and Farleys'		X	X	X		
It's a Weighty Matter						X
Mighty Mascot			X			
Perplexing Polygons					X	
Photographic Memories		X	X	X	X	
Rose Garden Plan	X					X
Rx						X
Solar Cells for Science		X				
Sorting Rectangles			X			
South Texas Natives		X				
What's in Your Wallet?						X
Working Smarter		X				

Mathematics Grade 8 TAKS Alignment

This chart shows the problems that have been aligned to the Grade 8 Texas Assessment of Knowledge and Skills (TAKS).

Problem Name	Grade 8 TAKS Objective					
	1	2	3	4	5	6
By the Sea			X			
City in Space		X				
Fast Food Workout		X				
Five Friends					X	
Fun Park Saturday	X	X				X
Global Warming: Texas-Size		X				
Half-Life Happening	X	X				X
How Green is Green?						X
In the Rafters		X	X	X		
Java Joe						X
Javier Builds a Model			X			
Liberty Enlightening the World	X	X	X	X		
Perplexing Polygons					X	
Storage Boxes at the Marleys' and Farleys'	X	X	X	X		
Student Council President			X			
Talk, Talk, Talk	X	X				X
What's in Your Wallet?						X