

## **Chapter 1:**

*Number, Operation, and  
Quantitative Reasoning*



## Fun Park Party grade 6

OVERARCHING  
FUN PARK

Dawn and seven of her friends are going to Fun Park for her birthday party. She has purchased three Fun Park coupon books with 24 coupons in each book.

<b>Fun Park Prices</b>	
Coupon book with 24 coupons	\$18.00
Individual coupon	\$ 1.00
<b>Games, Rides, Food, and Beverages</b>	
Miniature golf game	3 coupons
Go-cart ride	4 coupons
Video game	1 coupon
Laser tag game	6 coupons
Soft drink	1 coupon
Popcorn	1 coupon
Hotdog	2 coupons
Nachos	2 coupons

Birthday parties are limited to one activity for the group. Any choice of food or drink can be made.

1. If Dawn chooses miniature golf and no food or drink, how many games of miniature golf can each of them play if they all play the same number of games? Explain your answer.
2. If Dawn chooses miniature golf, 1 soft drink for each person, and no food, how many games of miniature golf can each of them play if they all play the same number of games? How did you determine this?
3. If Dawn chooses miniature golf, 1 soft drink for each person, and 1 hot dog for each person, how many games of golf can each of them play if they all play the same number of games? Explain your reasoning.

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates

(6.4) Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, perimeter, area, etc.

(6.5) Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. The student is expected to formulate an equation from a problem situation.

### Scaffolding Questions

- How many coupons will it take for everyone at the party to play one game of miniature golf?
- How many games of golf can be played by everyone at the party with one book of coupons? Two books of coupons? Three books of coupons?
- What is the relationship between the number of coupons and the number of games of golf?
- How can you determine how many coupons will be left for miniature golf after everyone has a soft drink?
- How can you find the number of coupons that will be left for miniature golf after everyone has a soft drink and hot dog?

### Sample Solutions

1. Dawn has a total of 24 coupons in each book and she has 3 books of coupons ( $24 \times 3 = 72$ ). Therefore, she has a total of 72 coupons. A table may be created to show the number of coupons.

Number of books	Number of coupons
1	24
2	48
3	72

There are 72 coupons and 1 miniature golf game takes 3 coupons.

$$72 \text{ coupons} \times \frac{1 \text{ game}}{3 \text{ coupons}} = 24 \text{ games}$$

$$\frac{24 \text{ games}}{8 \text{ people}} = \frac{3 \text{ games}}{1 \text{ person}}$$

Therefore, the total number of games of miniature golf each of the people at the party can play is 3 games.

2. One game of miniature golf takes 3 coupons. One game of miniature golf for everyone at the party takes 24 coupons.

$$3 \frac{\text{coupons}}{\text{game}} \times 1 \frac{\text{game}}{\text{person}} \times 8 \text{ people} = 24 \text{ coupons}$$

One soft drink takes 1 coupon; therefore, 1 soft drink for every person at the party takes 8 coupons.

$$1 \frac{\text{coupon}}{\text{drink}} \times 1 \frac{\text{drink}}{\text{person}} \times 8 \text{ people} = 8 \text{ coupons}$$

Total coupons needed for 1 golf game and 1 soft drink for everyone at the party is 32.

$$24 \text{ coupons for golf} + 8 \text{ coupons for drinks} = 32 \text{ total coupons}$$

Two games of miniature golf for everyone at the party take 48 coupons.

$$3 \frac{\text{coupons}}{\text{game}} \times 2 \frac{\text{games}}{\text{person}} \times 8 \text{ people} = 48 \text{ coupons}$$

One soft drink takes 1 coupon; therefore, 1 soft drink for every person at the party takes 8 coupons.

$$1 \frac{\text{coupon}}{\text{drink}} \times 1 \frac{\text{drink}}{\text{person}} \times 8 \text{ people} = 8 \text{ coupons}$$

Total coupons needed for 2 golf games and 1 soft drink for everyone at the party is 56.

$$48 \text{ coupons for golf} + 8 \text{ coupons for drinks} = 56 \text{ total coupons}$$

Three games of miniature golf for everyone at the party take 72 coupons.

$$3 \frac{\text{coupons}}{\text{game}} \times 3 \frac{\text{games}}{\text{person}} \times 8 \text{ people} = 72 \text{ coupons}$$

There would not be enough coupons for 3 games of golf and a soft drink. If they each get a soft drink, they could play only 2 games of miniature golf each.

The following table can be used to organize and display the different options.

Number of games per person	Total coupons needed for golf	Total coupons needed for drinks	Total coupons needed for golf and drinks
1	24	8	32
2	48	8	56
3	72	8	80

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology

or techniques such as mental math, estimation, and number sense to solve problems

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

3. One game of miniature golf for everyone at the party takes 24 coupons

$$3 \frac{\text{coupons}}{\text{game}} \times 1 \frac{\text{game}}{\text{person}} \times 8 \text{ people} = 24 \text{ coupons}$$

One soft drink takes 1 coupon; therefore, 1 soft drink for every person at the party takes 8 coupons.

$$1 \frac{\text{coupon}}{\text{drink}} \times 1 \frac{\text{drink}}{\text{person}} \times 8 \text{ people} = 8 \text{ coupons}$$

One hot dog for each person takes 2 coupons. One hot dog for everyone at the party takes 16 coupons.

$$2 \frac{\text{coupons}}{\text{hot dog}} \times 2 \frac{\text{hot dogs}}{\text{person}} \times 8 \text{ people} = 16 \text{ coupons}$$

Total coupons needed for 1 golf game, 1 soft drink, and 1 hot dog for everyone at the party is 48.

$$24 \text{ coupons for golf} + 8 \text{ coupons for soft drinks} + 16 \text{ coupons for hot dogs} = 48 \text{ total coupons}$$

Two games of miniature golf for everyone at the party takes 48 coupons.

$$3 \frac{\text{coupons}}{\text{game}} \times 2 \frac{\text{games}}{\text{person}} \times 8 \text{ people} = 48 \text{ coupons}$$

One soft drink takes 1 coupon; therefore, 1 soft drink for every person at the party takes 8 coupons.

$$1 \frac{\text{coupon}}{\text{drink}} \times 8 \text{ drinks} = 8 \text{ coupons}$$

One hot dog for each person takes 2 coupons. One hot dog for everyone at the party takes 16 coupons.

$$2 \frac{\text{coupons}}{\text{hot dog}} \times 1 \frac{\text{hot dog}}{\text{person}} \times 8 \text{ people} = 16 \text{ coupons}$$

Total coupons needed for 2 golf games, 1 soft drink, and 1 hot dog for everyone at the party is 72.

$$48 \text{ coupons for golf} + 8 \text{ coupons for soft drinks} + 16 \text{ coupons for hot dogs} = 72 \text{ total coupons}$$

A table can be created to organize the number of coupons needed for each option.

Number of golf games per person	Total coupons needed for golf	Total coupons needed for soft drinks	Total coupons needed for hotdogs	Total golf coupons + total soft drink coupons + total hotdog coupons
1	24	8	16	48
2	48	8	16	72

### Extension Questions

- If each person at Dawn’s party is limited to 1 game or ride and Dawn has 3 coupon books, which games or rides are possible choices and how many times can each person at the party participate in the game or ride? Explain how to find all the possible choices.

*She has 72 coupons. Possible choices are miniature golf game, go-cart ride, video game, and laser tag game. Miniature golf was considered in the problem.*

*Go-cart rides require 4 coupons. Each person could have 2 rides.*

$$72 \text{ coupons} \times \frac{1 \text{ game}}{4 \text{ coupons}} = 18 \text{ games}$$

$$\frac{18 \text{ games}}{8 \text{ people}} = \frac{2.25 \text{ games}}{1 \text{ person}}$$

*Video games take 1 coupon. If she chooses video games, each person can play 9 games.*

$$72 \text{ coupons} \times \frac{1 \text{ game}}{1 \text{ coupon}} = 72 \text{ games}$$

$$\frac{72 \text{ games}}{8 \text{ people}} = \frac{9 \text{ games}}{1 \text{ person}}$$

Laser tag requires 6 coupons. If she chooses laser tag, each person can play 1 game.

$$72 \text{ coupons} \times \frac{1 \text{ game}}{6 \text{ coupons}} = 12 \text{ games}$$

$$\frac{12 \text{ games}}{8 \text{ people}} = \frac{1.5 \text{ games}}{1 \text{ person}}$$

- Would it cost less for Dawn to purchase coupon books or individual coupons for each possible choice of games or rides for her party? What is the best decision for each of the possible choices? Justify your answers.

Three games of miniature golf for each person would be a total of 72 coupons. Since individual coupons cost \$1 each, the cost of 72 coupons is \$72. One book of coupons sells for \$18 and contains 24 coupons.

$$72 \text{ coupons} \times \frac{1 \text{ book}}{24 \text{ coupons}} \times \frac{\$18}{1 \text{ book}} = \$54$$

It would be cheaper to buy 3 coupon books.

Two go-cart rides for each person would be a total of 64 coupons: \$54 for 3 coupon books is cheaper than \$64 for individual coupon books.

$$8 \text{ people} \times \frac{2 \text{ rides}}{1 \text{ person}} \times \frac{4 \text{ coupons}}{1 \text{ ride}} \times \frac{\$1}{1 \text{ coupon}} = \$64$$

For 9 video games for each person, it would be much cheaper to buy 3 coupon books for the 72 coupons needed instead of spending \$72 for individual coupons.

$$8 \text{ people} \times \frac{9 \text{ video games}}{1 \text{ person}} \times \frac{1 \text{ coupon}}{1 \text{ video game}} \times \frac{\$1}{1 \text{ coupon}} = \$72$$

For 1 game of laser tag for each person, the total number of coupons needed would be 48.

$$8 \text{ people} \times \frac{1 \text{ laser tag game}}{1 \text{ person}} \times \frac{6 \text{ coupons}}{1 \text{ laser tag game}} \times \frac{\$1}{1 \text{ coupon}} = \$48$$

Two coupon books would cost 2(\$18) or \$36, and 48 individual coupons would cost \$48. It would be cheaper to buy 2 coupon books.

## Student work sample

This student's work shows an understanding of the relationships between coupons, books, and coupons needed for activities.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Describes mathematical relationships
- Evaluates the reasonableness or significance of the solution in the context of the problem
- Demonstrates an understanding of mathematical concepts, processes, and skills
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them
- States a clear and accurate solution using correct units

- ① 8 friends & 24 coupons in a book  
 3 people = 3 coupons each for mini-golf

$$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$$

24 coupons needed for one game

they can only play one game each.  
 per book.

She has 3 books so they can play  
 three games each.

# of books	# of coupons	# of games
1	24	1
2	48	2
3	72	3

- ② 72 coupons & 4 coupons for mini-golf and drink  
 - 3 coupons <sup>just</sup> for drinks

64 left

1 round = 24 coupons for mini-golf

64

24 ← 1 round each

~~40~~

24 ← 1 more round each

~~16~~

12

12

12

12

12

12

12

12

12

12

12

12

16 coupons left

5 ← needed for 2 rounds of mini-golf and  
 one soda each

72 coupons - 8 drinks coupons = 64 left

$\begin{array}{r} 54 \\ 24 \\ \hline 16 \\ \hline 48 \end{array}$  coupons - 16 hot dog coupons = 48 left

48 coupons - 24 coupons = 24 left  
(7 round each of golf)

With 24 left you can play one more round of golf

Each person can get 1 drink, 1 hot dog, and 2 rounds of golf with the coupons.



## Fun Park Party grade 7

Dawn is taking several of her friends to Fun Park for her birthday party. She has decided that each person at her party will play 1 game of laser tag and 3 video games and will ride the go-carts 2 times.

<b>Fun Park Prices</b>	
Coupon book with 24 coupons	\$18.00
Individual coupon	\$ 1.00
<b>Games, Rides, Food, and Beverages</b>	
Miniature golf game	3 coupons
Go-cart ride	4 coupons
Video game	1 coupon
Laser tag game	6 coupons
Soft drink	1 coupon
Popcorn	1 coupon
Hotdog	2 coupons
Nachos	2 coupons

Thirty coupons were collected at the video arcade from people at Dawn's party.

1. How many friends were at Dawn's party? Explain your answer.
2. How many total coupons were collected at the laser tag stand and the go-cart rides? How did you determine this?
3. How should Dawn purchase the coupons for her party? Should she purchase only coupon books or a combination of coupon books and individual coupons? Justify your reasoning.

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(F) select and use appropriate operations to solve problems and justify the selections

(G) determine the reasonableness of a solution to a problem

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in

### Scaffolding Questions

- How many coupons does it take to play one game in the video arcade? Three games?
- How can you find the number of people at Dawn's party?
- How can you determine the number of coupons needed for everyone at the party to ride the go-carts 2 times? To play laser tag 1 time?
- How many total coupons will it take for everyone at the party to play 3 video games and 1 game of laser tag and to ride the go-carts 2 times? Explain.
- What is the least number of coupon books Dawn should purchase for her party?
- What is the greatest number of coupon books Dawn should purchase? Will that give her any coupons left over?
- What are some purchase options for the combination of coupon books and individual coupons?
- How can you determine if you have explored all possible options?
- How can you decide which purchase option is best for Dawn?

### Sample Solutions

1. The video arcade attendant collected 30 coupons from people at Dawn's party. Each video game takes 1 coupon; therefore, there were 30 games played.

$$30 \text{ coupons} \times \frac{1 \text{ game}}{1 \text{ coupon}} = 30 \text{ games}$$

Dawn bought enough coupons for each person at her party to play 3 video games.

$$30 \text{ games} \times \frac{1 \text{ person}}{3 \text{ games}} = 10 \text{ people}$$

There were 10 people at Dawn's party (Dawn and 9 friends).

2. There were 10 people at Dawn's party. Dawn purchased 1 game of laser tag per person. Each laser tag game takes 6 coupons; therefore, there were 60 coupons for laser tag.

$$10 \text{ people} \times \frac{1 \text{ game}}{1 \text{ person}} \times \frac{6 \text{ coupons}}{1 \text{ game}} = 60 \text{ coupons}$$

There were 60 coupons collected at the laser tag stand.

There were 10 people at Dawn's party. Dawn purchased 2 go-cart rides per person. Each go-cart ride takes 4 coupons; therefore, there were 80 coupons for laser tag.

$$10 \text{ people} \times \frac{2 \text{ rides}}{1 \text{ person}} \times \frac{4 \text{ coupons}}{1 \text{ ride}} = 80 \text{ coupons}$$

There were 80 coupons collected at the go-cart ride.

There were 60 coupons collected at the laser tag stand.  
There were 80 coupons collected at the go-cart ride.  
Therefore, there were a total of 140 coupons collected at the laser tag stand and the go-cart ride.

$$60 \text{ coupons at the laser tag stand} + 80 \text{ coupons at the go-cart ride} = 140 \text{ total coupons}$$

3. Dawn needed 30 coupons for video games, 60 coupons for laser tag, and 80 coupons for the go-carts. She had to purchase a total of 170 coupons for her party.

$$30 \text{ video coupons} + 60 \text{ laser tag coupons} + 80 \text{ go-cart coupons} = 170 \text{ total coupons}$$

Each coupon book contains 24 coupons; therefore, Dawn needs to purchase 8 coupon books. There are not enough coupons in 7 books.

$$170 \text{ coupons} \times \frac{1 \text{ book}}{24 \text{ coupons}} = 7.08 \text{ books}$$

However, if she does that, she will have 22 coupons left in one book.

$$8(24) = 192 \text{ coupons}$$

$$192 - 170 = 22$$

and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

If she purchases only 7 coupon books, she will only have 168 coupons.

Number of coupon books	Total number of coupons	Total cost of books
1	24	\$18
2	48	\$36
3	72	\$54
4	96	\$72
5	120	\$90
6	144	\$108
7	168	\$126
8	192	\$144

Eight coupon books cost \$144. Seven coupon books cost \$126. She can purchase 2 more coupons for \$2. The cost of 7 coupon books and 2 individual coupons is \$128. She should purchase 7 coupon books and 2 individual coupons to get the best price.

### Extension Questions

- What percentage of the total coupons purchased were for the video games? For the go-cart rides? For the laser tag games? Explain.

*Dawn purchased a total of 170 coupons. She purchased 30 coupons for video games (about 18%), 60 coupons for laser tag (about 35%), and 80 coupons for the go-carts (about 47%).*

$$\frac{30}{170} \approx 0.176 \approx 18\%$$

$$\frac{60}{170} \approx 0.353 \approx 35\%$$

$$\frac{80}{170} \approx 0.471 \approx 47\%$$

- Suppose a total of 88 coupons had been collected by both the video arcade and go-cart attendants. How can you find the number of people in Dawn's party (Dawn and friends)? Assume each person played the same number of each type of game.

*Combine the number of coupons needed for both activities.*

$$3 \text{ for video} + 8 \text{ for go-carts} = 11 \text{ coupons}$$

*If 88 coupons were collected then there were only 8 people at the party.*

$$88 \text{ coupons} \times \frac{1 \text{ person}}{11 \text{ coupons}} = 8 \text{ people}$$

## Fun Park Saturday grade 8

Dawn and her three sisters, Delia, Lisa, and Leslie, have made plans to go to Fun Park on Saturday. They have decided that each of them will choose what games and rides they will participate in. Everyone will purchase 2 soft drinks, 1 hot dog, and a bag of popcorn because they will be there for at least 6 hours.

<b>Fun Park Prices</b>	
Coupon book with 24 coupons	\$18.00
Individual coupon	\$ 1.00
<b>Games, Rides, Food, and Beverages</b>	
Miniature golf game	3 coupons
Go-cart ride	4 coupons
Video game	1 coupon
Laser tag game	6 coupons
Soft drink	1 coupon
Popcorn	1 coupon
Hotdog	2 coupons
Nachos	2 coupons

Dawn and Lisa have decided they will each play 2 games of miniature golf, ride the go-carts 3 times, play 5 video games each, and play 2 games of laser tag. Delia has decided to play 3 games of miniature golf, ride the go-carts 2 times, play 2 video games, and play 3 games of laser tag. Leslie has decided to play 2 games of miniature golf, ride the go-carts 4 times, play 8 video games, and play 2 games of laser tag.

Each of the girls is going to purchase her own coupons. Dawn and Lisa have decided to purchase 2 coupon books each. Delia has decided to purchase only individual coupons. Leslie has decided to purchase 1 coupon book and individual coupons.

1. What is the cost of the coupons for each of the girls? Explain your answer.
2. Is there a better coupon purchase choice for any of the girls? Justify your answer.

## Materials

Graphing calculator

## Connections to Middle School TEKS:

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height,  $a = 1.4h$

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to

## Teacher Notes

### Scaffolding Questions

- How many coupons will each girl need for the day?
- What is the cost of one coupon book? One individual coupon?
- How can you find the total cost of the coupon books and individual coupons for Dawn? Lisa? Delia? Leslie?
- How could you determine the lowest cost for the number of coupons each girl is purchasing?

### Sample Solutions

1. Each girl must determine the total coupons needed for her rides and games. Everyone needs the same number of tickets for soft drinks, popcorn, and hot dogs.

	Drink, popcorn, and hot dog coupons needed	Golf game coupons needed	Go-cart ride coupons needed	Video game coupons needed	Laser tag game coupons needed	Total coupons needed
Dawn	$1(2)+1+2=5$	$3(2)=6$	$4(3)=12$	$1(5)=5$	$6(2)=12$	40
Lisa	$1(2)+1+2=5$	$3(2)=6$	$4(3)=12$	$1(5)=5$	$6(2)=12$	40
Delia	$1(2)+1+2=5$	$3(3)=9$	$4(2)=8$	$1(2)=2$	$6(3)=18$	42
Leslie	$1(2)+1+2=5$	$3(2)=6$	$4(4)=16$	$1(8)=8$	$6(2)=12$	47

Find the total cost of the coupon choices each girl has made.

The total cost  $c$  equals the number of books  $b$  multiplied by the cost of each book (\$18) for Dawn and Lisa.

$$c = 18b$$

$$c = 18(2)$$

$$c = 36$$

The total cost of the coupon purchase choice for Dawn and Lisa is \$36.

For Delia, the total cost  $c$  equals the cost of each individual coupon (\$1) multiplied by the number of individual coupons  $i$ .

$$\begin{aligned}c &= 1i \\c &= 1(42) \\c &= 42\end{aligned}$$

The total cost for Delia's individual coupons is \$42.

For Leslie, the total cost equals the cost of the 1 book she is purchasing plus the cost of individual coupons multiplied by the number of individual coupons she is purchasing. Leslie needs 47 coupons. One book takes care of 24 coupons.

$$47 - 24 = 23$$

So, Leslie is going to purchase 1 book and 23 additional coupons.

$$\begin{aligned}c &= (\$18 \times 1) + (\$1 \times 23) \\c &= \$18 + \$23 \\c &= \$41\end{aligned}$$

The total cost for Leslie's coupons is \$41.

2. The table shows that Dawn and Lisa needed 40 coupons. If they buy 2 coupon books with 24 coupons in each book they will have  $2(24)$  or 48 coupons. They will each have  $48 - 40$  or 8 coupons left at the end of the day. They could each purchase 1 book to take care of 24 coupons and buy an additional 16 individual coupons. The following equation can be solved to find the cost  $c$  of 1 coupon book and 16 individual coupons.

$$\begin{aligned}c &= (\$18 \times 1) + (\$1 \times 16) \\c &= \$18 + \$16 \\c &= \$34\end{aligned}$$

Buying 2 books costs them  $\$18(2)$  or \$36 each. Buying 1 book and 16 tickets would cost them \$34 each. The latter option would be a better choice, resulting in a savings of \$2 each.

If she buys individual coupons, Delia will not have any coupons left at the end of the day. She needs 42

everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

## Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

coupons. She could have purchased 1 coupon book to take care of 24 coupons. She would need  $42 - 24$ , or an additional 18 individual coupons.

$$\begin{aligned}c &= (\$18 \times 1) + (\$1 \times 18) \\c &= \$18 + \$18 \\c &= \$36\end{aligned}$$

Delia's original choice to purchase only individual coupons cost her \$42. Buying one book and 18 additional coupons would only cost her \$36. The better choice is the second option.

Leslie will not have any coupons left at the end of the day. She needed 47 coupons. She could have purchased 2 coupon books that would have given her 48 coupons with one coupon left over at the end of the day.

$$\begin{aligned}c &= 2(\$18) \\c &= \$36\end{aligned}$$

Leslie's original choice to purchase one coupon book and 23 additional coupons cost her \$41. Buying two books would only cost \$36 and would leave her with one coupon left at the end of the day. The second option is the better choice.

### Extension Questions

- Would it be a good idea for some of the girls to consider purchasing coupon books together to save money? Which girls should purchase together? How many books and individual tickets should they purchase? Explain your answers.

*Leslie could purchase with Delia. They would need 42 plus 47, or 89 coupons. Three coupon books would give  $3(24)$ , or 72 coupons. They would each get 36 coupons and need 17 more coupons. Delia would need to buy 6 more coupons for \$6 and Leslie would need to buy 11 more coupons for \$11.*

*The cost of 3 coupon books is \$18 (3) or \$54. Dividing this cost by 2, each girl would pay \$27.*

*Delia's cost:  $\$27 + \$6 = \$33$ .*

*Leslie's cost:  $\$27 + \$11 = \$38$ .*

*Delia is saving  $\$42 - \$33$ , or  $\$9$ .*

*Leslie is saving  $\$41 - \$38$ , or  $\$3$ .*

*Even though Leslie is paying more than Delia, she is still saving  $\$3$  and Delia is saving  $\$9$ .*

*Dawn and Leslie together need 80 coupons. Three coupon books would give 72 coupons. Dawn could purchase with Lisa and split the cost equally for 3 coupon books and 8 individual coupons. Three coupon books cost  $\$54$ . The individual coupons would cost  $\$8$ . The total cost would be  $\$54$  plus  $\$8$ , or  $\$62$ . That would cost them  $\$31$  each instead of  $\$36$  each.*

- If Dawn has only  $\$38$  to spend, what are some of the possible activities and food choices she could make? What would be the best coupon purchase for each of the possible activity combinations?

*We have shown that the cost of 2 coupon books is  $\$36$ . Delia has enough money to buy 2 coupon books plus 2 more coupons. Two coupon books will give 48 coupons plus 2 more coupons. She could buy 50 coupons. The cost of one coupon book with 24 coupons is  $\$18$ . Following is a chart of possible combinations Dawn could choose.*

Food and drink coupons	Golf games/ coupons	Go-cart rides/ coupons	Video game/ coupons	Laser tag/ coupons	Total coupons needed	Best coupon purchase
6	2/6	2/8	2/2	2/12	34	1 book and 10 coupons
6	3/9	2/8	7/7	3/18	8	2 books
6	3/9	3/12	2/2	2/12	41	1 book and 17 coupons
6	2/6	3/12	3/3	3/18	45	2 books
6	4/12	3/12	2/2	3/18	50	2 books and 2 coupons

*The last column shows the best coupon purchase for the combinations. For example:*

*41 coupons purchased as one book (24 coupons) plus 17 coupons would cost \$18 + \$17, or \$35. That is cheaper than purchasing 2 books.*

*45 coupons purchased as one book (24 coupons) plus 21 coupons would cost \$18 + \$21 or \$39. It would be cheaper to buy 2 coupon books at \$36.*

## Homecoming Chili

### grade 6

Alicia's favorite chili recipe calls for 3 pounds of ground beef. The recipe serves 8 people. Alicia bought a package of ground beef that weighs 11.5 pounds to make a large batch of chili for the annual Homecoming Chili Dinner at the local high school.

What is the best estimate of the number of people her chili will serve if she follows the recipe? Explain your reasoning.

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates

(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

### Scaffolding Questions

- If one recipe serves 8 people, how many people will a double recipe serve? A triple recipe? A quadruple recipe?
- How can you determine the amount of ground beef needed for a double recipe? A triple recipe? A quadruple recipe?
- How many pounds of ground beef does it take for each serving? How can you determine this?
- How can you find the number of servings for 11.5 pounds of ground beef?

### Sample Solutions

Alicia has 11.5 pounds of ground beef. One recipe needs 3 pounds of ground beef. Two recipes need 6 pounds of ground beef. Three recipes need 9 pounds of ground beef. Alicia doesn't have enough ground beef to make 4 complete recipes because she would need a total of 12 pounds of ground beef and she has only 11.5 pounds.

Number of recipes	Process	Pounds of beef
1	$1 \times 3$	3
2	$2 \times 3$	6
3	$3 \times 3$	9
4	$4 \times 3$	12

Three pounds of ground beef makes 1 recipe and serves 8 people. Six pounds of ground beef makes 2 recipes and serves 16 people. Nine pounds of ground beef makes 3 recipes and serves 24 people. Twelve pounds of ground beef makes 4 recipes and serves 32 people.

Number of recipes	Process	Number of servings
1	$1 \times 8$	8
2	$2 \times 8$	16
3	$3 \times 8$	24
4	$4 \times 8$	32

A recipe with 3 pounds of ground beef will serve 8 people. A little less than 4 whole recipes with 11.5 pounds of meat will serve about 30 people.

Another way to look at it is to use rates.

$$11.5 \text{ pounds} \times \frac{1 \text{ recipe}}{3 \text{ pounds}} = 3.8 \text{ recipes}$$

$$3.8 \text{ recipes} \times \frac{8 \text{ servings}}{1 \text{ recipe}} = 30.4 \text{ servings}$$

### Extension Questions

- The school principal calls Alicia to let her know that they have sold tickets for 20 more people than they projected. They will need another 20 servings of chili. Alicia agrees to make more chili for the additional 20 servings. How much more ground beef will she need to buy?

*The recipe says that 3 pounds serves 8 people. She needs 20 more servings.*

$$\frac{3 \text{ pounds}}{8 \text{ servings}} \times 20 \text{ servings} = 7.5 \text{ pounds}$$

*She will need 7.5 more pounds of hamburger.*

- Alicia decides to make enough chili for 75 servings instead of the original 30 servings she planned to make. She wants to make sure there is enough chili if more people who have not bought tickets show up the night of the Homecoming Chili Dinner. How much more ground beef will she need to buy?

*It takes 3 pounds to serve 8 people. Alicia needs to buy enough ground beef to serve an additional 45 people.*

$$\frac{3 \text{ pounds}}{8 \text{ servings}} \times 45 \text{ servings} \approx 16.87 \text{ pounds}$$

*She has 11.5 pounds of ground beef, so she needs to buy an additional 16.87 pounds of ground beef to serve 75 people.*

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.



## Spring Sensations grade 6

The first performance of the Maxwell Middle School Spring Sensations will be next Friday in the new school auditorium. The performance is sold out. The auditorium has 840 seats, and each section in the auditorium seats 60 people. The Maxwell Student Council members have volunteered to usher for the performance. There are 24 members in the Student Council. The Student Council will invite other students to usher so that there will be at least two ushers in each section.

1. How many ushers will be needed other than the 24 Student Council members? Explain your reasoning.
2. About how many people will each Student Council member seat? How did you determine this?

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates

(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

### Scaffolding Questions

- If all of the 840 seats are occupied, how many sections are full?
- If 2 ushers are needed for each section, how can you find the total number of ushers needed?
- If 24 Student Council members will usher, how many more ushers are needed so there will be at least 2 ushers in each section?
- If there are 60 seats in each section, how can you determine the number of people each of the ushers will seat?

### Sample Solutions

1. There are 840 seats in the auditorium and 60 seats in each section. Divide the total number of seats by the number of seats in each section to find the number of sections in the auditorium.

$$840 \div 60 = 14$$

There are 14 sections.

Another solution strategy would be to build a table and look for a pattern.

Number of sections	Process	Number of seats
1	60(1)	60
2	60(2)	120
3	60(3)	180
4	60(4)	240
5	60(5)	300
10	60(10)	600
15	60(15)	900

Fifteen sections are 60 seats too many, so there are 14 sections in the auditorium.

$$60(14) = 840$$

There are 14 sections in the auditorium. At least 2 ushers are needed for each section. Multiply the number of sections by the number of ushers for each section to find the total number of ushers needed.

$$14 \text{ sections} \times \frac{2 \text{ ushers}}{1 \text{ section}} = 28 \text{ ushers}$$

Another strategy would be to build a table and look for a pattern.

Number of sections	Process	Number of ushers
1	2(1)	2
2	2(2)	4
3	2(3)	6
14	2(14)	28

There are 28 ushers needed. The Student Council has only 24 members; therefore, they need at least 4 more students to help usher if they are to have at least 2 ushers in each section.

- Each section seats 60 people. There are at least 2 ushers in each section. Divide the number of seats by the number of ushers to find about how many people each usher will seat.

$$60 \div 2 = 30$$

If the Student Council finds more students to help usher, then the approximate number of people each usher will seat will be less than 30. If there are 3 ushers in each section, they will seat about 20 people each. If there are 4 ushers in each section, they will seat about 15 people each. If there are 5 ushers in each section, they will seat about 12 people each.

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

number of seats	number of ushers	number of people each usher seats
60	2	30
60	3	20
60	4	15
60	5	12

### Extension Questions

- If the same students usher for 7 performances, about how many people will each of them have seated after the seventh performance?

*If there are 2 ushers in each section and they seat 30 people each performance, they will seat about 210 people each after the seventh performance. If there are 3 ushers and they seat 20 people each performance, they will seat about 140 each. If there are 4 ushers and they seat 15 people each performance, they will seat about 105 each. If there are 5 ushers who seat 12 people each performance, they will seat about 84 each after the seventh performance.*

- There is a virus going around the school. Only 20 ushers show up for the last performance on Saturday night. How many sections would not have two ushers?

*They need 28 ushers to place 2 ushers at each station. They will have  $28 - 20$ , or 8 sections without ushers.*

- Describe some possible ways the ushers could be assigned and about how many people each usher would seat.

*If there are 14 sections and one person assigned to each section, there would be 6 people left. There could be 2 ushers in 6 sections and they would seat about 30 people each, and the ushers by themselves in 8 sections would seat about 60 people each. If they decide to just share all the sections, then each usher would seat approximately  $840$  divided by  $20$ , or 40 people each section.*

## Bargain Shopping grade 7

The regular price of a rack of swimsuits in the junior department of a clothing store is \$54. The store advertises an end-of-season sale at 40% off the regular price of all swimsuits. Two weeks later the store advertises a summer clearance sale at an additional  $\frac{1}{5}$  off the end-of-season sale price of all swimsuits. Shannon and Mary are on the swim team and swim all year long. They see the clearance sale advertisements and decide this is a great time to shop for bargain swimsuits. Shannon figures that the swimsuits are now 60% off the regular price. Mary disagrees because she figures that the total discount is actually less than 60%.

1. What is the cost of the swimsuits during the end-of-season sale? Justify your reasoning using a model.
2. What is the cost of the swimsuits during the additional  $\frac{1}{5}$ -off sale? How can you show this with a model?
3. Who has figured the discount correctly, Shannon or Mary? Explain your answer using a model.

## Teacher Notes

### Materials

Graphing calculator

Chart paper

Markers

### Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(A) represent multiplication and division situations involving fractions and decimals with concrete models, pictures, words, and numbers

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) estimate and find solutions to application problems involving percent

(7.13) Underlying processes and mathematical tools. The student applies Grade

### Scaffolding Questions

- How can you model this problem with a percent bar? What price would the whole percent bar represent?
- What percent benchmarks could you use for this model?
- How does your percent bar show the discount of the swimsuits at the end-of-season sale? The sale price?
- How can you find the cost of the swimsuits at the end-of-season sale if you know what the amount of the discount is?
- How can you use a model to find the price during the additional  $\frac{1}{5}$ -off sale? What price does the whole model represent?
- How can you use a percent bar as a model to decide if the final cost during the additional  $\frac{1}{5}$ -off sale is equal to or less than 60%? What is the total amount represented by the percent bar in this model?

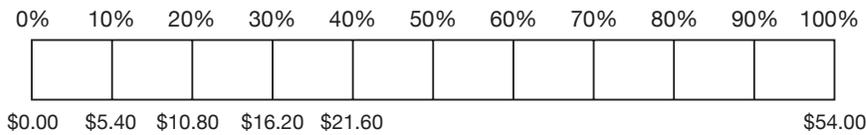
### Sample Solutions

1. Make a percent bar to model the cost of the swimsuits during the end-of-season sale. The whole bar will represent the original cost of the swimsuits (\$54). Benchmarks of 10% and multiples of 10% can be used to find the amount of discount in dollars for 40% off the regular price.

$$10\% \text{ of } \$54 = \$5.40$$

$$4 \times 10\% \text{ or } 40\% \text{ discount} = 4 \times \$5.40 \text{ or } \$21.60$$

As the percentage increases, the dollar amount of the discount also increases proportionally.

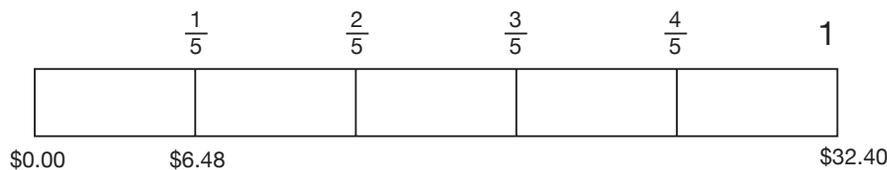


The discount of 40% is \$21.60. Subtract \$21.60 from the original price of \$54.00 to find the sale price during the end-of-season sale.

$$\$54.00 - \$21.60 = \$32.40$$

The price is now \$32.40.

2. Make a fraction bar to model the cost of the swimsuits during the  $\frac{1}{5}$ -off sale. The whole bar represents the cost of the swimsuits after the end-of-season sale, which is the cost before the additional  $\frac{1}{5}$ -off sale. This bar is then divided into 5 equal parts and  $\frac{1}{5}$ -of \$32.40 or \$6.48 is computed.



The additional discount of  $\frac{1}{5}$  off the end-of-season sale price is \$6.48. Subtract \$6.48 from the end-of-season sale price of \$32.40 to find the sale price during the additional  $\frac{1}{5}$ -off sale.

$$\$32.40 - \$6.48 = \$25.92$$

The price after both discounts is \$25.92.

7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

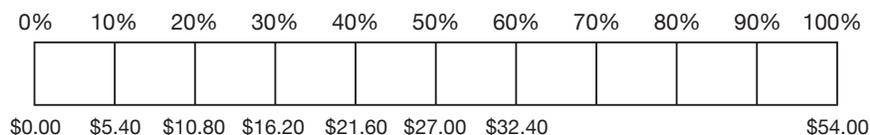
Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

3. Make a percent bar to model the total discount to decide whether Shannon or Mary figured the discount correctly. The whole bar will represent the original cost of the swimsuits. Using 10% benchmarks, 60% of \$54 can be determined.

$$10\% \text{ discount} = \$5.40$$

$$6 \times 10\% \text{ or } 60\% \text{ discount} = \$32.40$$

Another way to find the 60% discount would be to add the 10% discount to the 50% discount ( $\$5.40 + \$27.00 = \$32.40$ ).



Sixty percent off the original price is \$32.40. Subtract \$32.40 from the original price of \$54.00 to find the price with 60% off the original price.

$$\$54.00 - \$32.40 = \$21.60$$

Shannon is not correct because she figures that the swimsuits were 60% off the regular price, which would have been a price of \$21.60. The sale price with both discounts is \$25.92.

Subtract the final sale price of \$25.92 from the original price of \$54.00 to find the total discount amount.

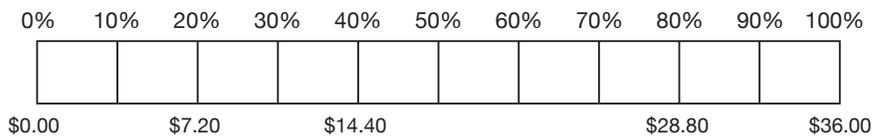
$$\$54.00 - \$25.92 = \$28.08$$

This is less than \$32.40, which corresponds to 60% on the percent bar. Mary is correct because she figures that the total discount is less than 60%.

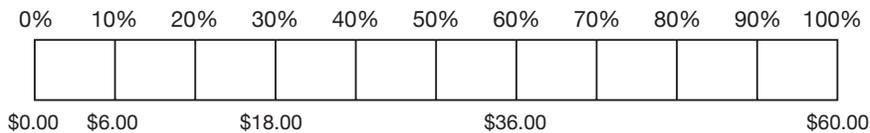
### Extension Questions

- How do you determine the price that was on the price tag at the end-of-season sale if you know the price paid for the swimsuit after all discounts was \$28.80?

*Since the sale was 40% off, followed by an additional  $\frac{1}{5}$  off, working backward can provide the price that was on the price tag. If the swimsuit was discounted  $\frac{1}{5}$  or 20%, then that means  $\frac{4}{5}$  or 80% of that price would be paid. Using benchmarks, find half of 80% (40%), then find half of 40% (20%). Using the model below, determine the money amount at each of these percentages. Since  $80\% + 20\% = 100\%$ , then  $\$28.80 + 7.20 = \$36$ . The swimsuit cost \$36 after the 40% discount.*



*This means that \$36 is 60% of the price tag. Use another model to find 100% of the price on the price tag.*

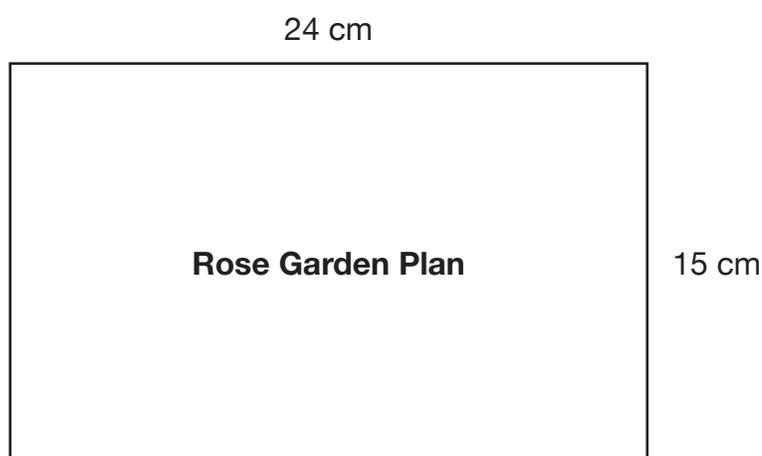


*Find the amount of money associated with some benchmark percents. Since  $60\% + 30\% + 10\% = 100\%$ , then  $\$36 + \$18 + \$6 = \$60$ , which is the price on the price tag before all discounts.*



## Rose Garden Plan grade 7

The town's Heritage Society has decided to plant a rose garden next to the historic Train Depot they have just restored. A landscape architect is drawing a plan for a rectangular rose garden on centimeter grid paper. He makes the following scale drawing, where length is 24 cm and width is 15 cm, to represent the actual rose garden. Three centimeters on the grid paper represents 7 meters in the actual rose garden.



1. Explain how to find the actual length of the rose garden when it is completed.
2. Explain how to find the actual width of the rose garden when it is completed.

## Materials

Graphing calculator

Centimeter grid paper

## Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(G) determine the reasonableness of a solution to a problem

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

## Teacher Notes

### Scaffolding Questions

- How many centimeters in 1 meter?
- How can you change centimeters to meters?
- How does the rectangle in the scale drawing compare to the rectangle in the actual rose garden?
- How do you change the size of a figure without changing its shape?
- What is the ratio of the number of meters in the actual rose garden to the number of centimeters in the scale drawing?
- How can you use the scale in the drawing to find the actual dimensions of the rose garden?

### Sample Solutions

1. The relationship between the measure on the grid paper and the actual measure is

$$\frac{3 \text{ cm on scale drawing}}{7 \text{ meters in the rose garden}}$$

Since the length on the scale drawing is 24, we must find a scale factor that will result in 24 cm.

$$\frac{3}{7} \times \frac{8}{8} = \frac{24}{56}$$

The ratio  $\frac{24}{56}$  is equivalent to  $\frac{3}{7}$  and represents 24 cm in the scale drawing to 56 m in the actual rose garden. Therefore, the length of the actual rose garden will be 56 meters.

Possible solution using the equation method:

$$\frac{3 \text{ cm}}{7 \text{ m}} = \frac{24 \text{ cm}}{x}$$

Find the scale factor from 3 cm to 24 cm and multiply 7 m by this same scale factor of 8 to get 56 m.

2. The solution methods used in problem 1 may be used to find the actual width; however, here are some other possible methods for finding the width of the actual rose garden.

Possible solution using a table and equation:

Scale measurement	Process	Actual measurement
3 cm	$\frac{7}{3}$ (3)	7 m
6 cm	$\frac{7}{3}$ (6)	14 m
9 cm	$\frac{7}{3}$ (9)	21 m
x	$\frac{7}{3}$ (x)	y

The ratio of  $\frac{y}{x}$  in the table above is  $\frac{7}{3}$  and represents a constant ratio or constant of proportionality,

$\frac{7 \text{ meters in the rose garden}}{3 \text{ cm on scale drawing}}$ . The rule  $y = \frac{7}{3}x$  can be written from the process column in the table above where y represents the number of meters in the actual measurement and x, the number of centimeters in the scale drawing. This rule means that the number of meters in the actual rose garden is  $\frac{7 \text{ meters in the rose garden}}{3 \text{ cm on scale drawing}}$  times the number of centimeters in the scale drawing. To find the number of meters in the width of the rose garden, multiply  $\frac{7}{3}$  by the number of centimeters in the width of the scale drawing.

$$y = \frac{7}{3}x$$

$$y = \frac{7}{3}(15)$$

$$y = 35 \text{ meters}$$

There are 35 meters in the width of the actual rose garden represented by 15 centimeters in the scale drawing.

Possible solution using a proportion and solving with properties of equality:

$$\frac{3}{7} = \frac{15}{s}, \frac{7}{3} = \frac{s}{15}$$

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

Multiply both sides of the equation by 15 to get the following solution:

$$15 \times \frac{7}{3} = 15 \times \frac{s}{15}$$

$$\frac{105}{3} = s$$

$$35 \text{ m} = s \text{ or } s = 35 \text{ meters}$$

### Extension Questions

- The Heritage Society decides they will put both a rose garden and a vegetable garden in the same amount of space as the original rose garden. What are several options for the design of the rose garden/vegetable garden combination? The new scale drawing uses a scale of 2 centimeters on the grid paper to represent 5 meters in the garden. What will the dimensions of the new scale drawing be?

*The problem involves “undoing” what was done in problems 1 and 2 where the dimensions of the scale drawing were given and the dimensions of the actual rose garden had to be found. In this problem, the dimensions of the actual rose garden are known from problems 1 and 2 ( $l = 56 \text{ m}$ ,  $w = 35 \text{ m}$ ), and the dimensions of the scale drawing must be found.*

*Another difference involves the scale used: 2 cm on the grid represents 5 m in the actual garden. The equation  $\frac{2 \text{ cm}}{5 \text{ m}} = \frac{x}{35 \text{ m}}$  can be written and solved using the scale factor method. Scale up 5 m to 35 m using a scale factor of 7. Multiply 2 cm by the scale factor 7 to get 14 cm for the width in the new scale drawing.*

*The equation  $\frac{2 \text{ cm}}{5 \text{ m}} = \frac{x}{56 \text{ m}}$  is solved in a similar way. Scale up 5 m to 56 m using a scale factor of 11.2. Multiply 2 cm by 11.2 to get 22.4 cm for the length in the new scale drawing. The dimensions of the new scale drawing using 2 centimeters to represent 5 meters in the actual garden will be length = 22.4 cm and width = 14 cm.*

*This area can then be divided into the rose garden and the vegetable garden in a variety of ways.*

- Suppose the dimensions of the scale drawing in the original problem are doubled. How would this affect the scale so that the actual size of the garden does not change? Explain.

*The actual size of the garden was found to be 35 meters by 56 meters in the original problem. When the dimensions of the scale drawing are doubled, they become 30 cm by 48 cm. The question is “What scale is used in the drawing (30 cm by 48 cm) to represent 35 meters by 56 meters in the actual garden?”*

*The ratio of centimeters in the width of the scale drawing to corresponding meters*

*in the width of the actual garden can be written as 30 cm : 35 m or 6 cm : 7 m. The ratio of centimeters in the length of the scale drawing to corresponding meters in the length of the actual garden can also be written as 48 cm : 56 m or 6 cm : 7 m. This common ratio 6 cm : 7 m shows the scale that was used in the scale drawing with  $l = 48$  cm and  $w = 30$  to represent the dimensions of the actual garden of 35 meters by 56 meters.*



## Talk, Talk, Talk

### grade 8

Mrs. Kim decided to buy her son, Jason, a cellular phone so that he can easily communicate with his parents when he is away from home. Mrs. Kim found two companies that offer special rates for students. Talk Cheap cellular phone service has no monthly basic fee but charges \$0.55 a minute. Talk Easy cellular phone service charges a basic monthly fee of \$35 plus \$0.15 for each minute used. Both companies do not round the time to the nearest minute like many of their competitors do; they charge only for the exact amount of time used. Build a table, make a graph, and write a rule to represent the cost of cellular service for both companies.

1. If price is the only factor, which plan is better?
2. Which company should Mrs. Kim choose if Jason never uses more than 30 minutes of cellular phone time each month?
3. If Jason knows the cost of each plan for 30 minutes, can he double this cost to find the cost for 60 minutes? Explain your answer.

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(A) compare and contrast proportional and non-proportional relationships

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected

### Scaffolding Questions

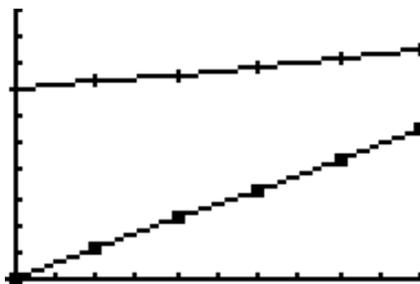
- Look at your table. How much would each company charge for 10 minutes? 20 minutes? 30 minutes? 25 minutes? 50 minutes?
- Look at your graph. Should the points be connected? Why?
- Looking at your graph, can you describe a rule you might use to determine *when* each plan is best?
- Look at your graph. Why does one line include the point of origin but the other does not?
- Why is it reasonable for the graph of the cost for service with both companies to be linear?
- What rule can you write to find the cost for any number of minutes for Talk Easy cellular phone service? For Talk Cheap cellular phone service?
- How can you express your rule in words?
- How could you decide the cost for service from each company for 200 minutes?

### Sample Solutions

Students might begin making the table by picking “round” numbers of minutes and finding the corresponding costs for the two companies.

# of Minutes	0	10	20	30	40	50	60
Talk Cheap	\$0.00	\$5.50	\$11.00	\$16.50	\$22.00	\$27.50	\$33.00
Talk Easy	\$35.00	\$36.50	\$38.00	\$39.50	\$41.00	\$42.50	\$44.00

Using a graphing calculator, students might plot the points and determine the rule for the lines corresponding to the cellular plans.



Students should write a rule to represent the cost  $y$  in dollars in terms of the number of minutes  $x$ .

The cost for Talk Easy equals \$35 plus 15 cents per minute times the number of minutes:  $y = 35.00 + 0.15x$ .

The cost for Talk Cheap is 55 cents per minute times the number of minutes:  $y = 0.55x$ .

1. Students could answer this question by graphing the rules and examining the graph to determine that Talk Cheap is the cheaper plan, but after a certain amount of time Talk Easy becomes the cheaper plan. Students can use the trace function on the graphs to find the point of intersection of the lines that represents the cost of each plan, or they may use the table functions to determine when Talk Easy becomes more economical.

```

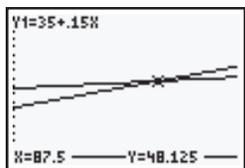
Plot1 Plot2 Plot3
Y1=35+.15X
Y2=.55X
Y3=
Y4=
Y5=
Y6=
Y7=
  
```

X	Y <sub>1</sub>	Y <sub>2</sub>
85.5	47.825	47.025
86	47.9	47.3
86.5	47.975	47.575
87	48.05	47.85
87.5	48.125	48.125
88	48.2	48.4
88.5	48.275	48.675

X=87.5

```

WINDOW
Xmin=57
Xmax=104
Xscl=0
Ymin=-3
Ymax=90
Yscl=5
Xres=
  
```



From the table we can see that the cost is the same at 87.5 minutes. The charge for  $Y_1$ , Talk Easy, is greater than the cost for  $Y_2$ , Talk Cheap, when the number of minutes is less than 87.5. The charge for the  $Y_1$ , Talk Easy, is less than the cost for  $Y_2$ , Talk Cheap, when the number of minutes is greater than 87.5.

to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

The graph of  $Y_1$ , Talk Easy, is above the graph of  $Y_2$ , Talk Cheap, for minutes less than 87.5. The graph of  $Y_1$ , Talk Easy, is below the graph of  $Y_2$ , Talk Cheap, for minutes greater than 87.5.

Using one of these methods students will conclude that Talk Cheap is a more economical plan if the phone is used for less than 87.5 minutes; otherwise, Talk Easy is the better plan.

2. Mrs. Kim should choose Talk Cheap if Jason never uses more than 30 minutes of talk time. This can be determined by using the table or graph shown above.
3. The rule for the Talk Cheap plan is  $y = 0.55x$  and the graph of the rule passes through the origin. Therefore, the cost of Talk Cheap plan is proportional to the time used, so Jason can figure the cost of 60 minutes by doubling the cost of 30 minutes of talk time for this plan. However, because there is a basic monthly fee, Talk Easy charges are not proportional to the time used. That is, the graph of the rule,  $y = 35.00 + 0.15x$ , does not pass through the origin, so this rule does not represent a proportional relationship. Jason cannot figure the cost of 60 minutes by doubling the cost of 30 minutes of talk time for this plan.

$$y = 35.00 + 0.15(30) = \$39.50$$

$$y = 35.00 + 0.15(60) = \$44$$

### Extension Questions

- How would the graphs be affected if Talk Easy increased or decreased its basic fee, or if Talk Cheap began charging a fee?

*The point where the graph of the line intersects the y-axis would be changed to the new starting fee amount.*

- How would the graph of Talk Cheap be affected if the company increased or decreased its cost per minute?

*The steepness (slope) of the graph would change.*

### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

## Half-Life Happening grade 8

Scientists use a pattern to calculate what they call “half-life.” In physics, half-life is the time required for one-half of a radioactive material to decay. During the next half-life, half of the remaining radioactive material decays. The pattern continually repeats. As the amount of remaining radioactive material approaches zero, there is a point where scientists consider it immeasurable.

The half-life of Lead-214 is 27 minutes. This means that every 27 minutes, half of the radioactive materials in Lead-214 has decayed.

The original amount is 1 gram. Below is a table that shows the amount after a given number of minutes.

Number of half-life	Time lapsed	Measure of the number of grams remaining		
		Fractional form	Decimal form	Exponential form
0	0 minutes	$\frac{1}{1}$	1.0	$2^0$
1	27 minutes	$\frac{1}{2}$	0.5	$2^{-1}$
	54 minutes			
				$2^{-3}$
4			0.0625	
5				

1. Complete the table above.
2. What is the decimal form of the fractional part of Lead-214 remaining after 81 minutes?

3. What is the exponential form of the fractional part of Lead-214 remaining after 108 minutes?
4. Using scientific notation, give the part of Lead-214 that is remaining after 2 hours and 15 minutes of decay.
5. If you know the time lapse at the fifth half-life, can you double this amount to find the time lapse at the tenth half-life? Explain.
6. If you know the amount remaining of Lead-214 at the fifth half-life, can you double this amount to find the amount remaining of Lead-214 at the tenth half-life? Explain.
7. What is the amount of decay at the fifth half-life? Can you double this amount to find the amount of decay of Lead-214 for tenth half-life? Explain.

## Teacher Notes

### Materials

Graphing calculator

### Connections to Middle School TEKS

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships

(D) express numbers in scientific notation, including negative exponents, in appropriate problem situations using a calculator

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(B) add, subtract, multiply, and divide rational numbers in problem situations

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

### Scaffolding Questions

- How many minutes are in a half-life interval?
- What fraction of the remaining radioactive materials decays during each half-life?
- What process do you use to convert a fraction to decimal form? What do you do to find the fractional part remaining after 81 minutes?
- What process do you use to write a fraction in exponential form? What do you do to find the exponential form remaining after 108 minutes?
- How can you find the number of half-life after 2 hours and 15 minutes of decay?

### Sample Solutions

1. To complete the table each new amount is found by multiplying the previous amount by one-half. The number of minutes increases by 27 minutes per half-life.

Number of half-life	Time lapsed	Measure of the number of grams remaining		
		Fractional form	Decimal form	Exponential form
0	0 minutes	$\frac{1}{1}$	1.0	$2^0$
1	27 minutes	$\frac{1}{2}$	0.5	$2^{-1}$
2	54 minutes	$\frac{1}{4}$	0.25	$2^{-2}$
3	81 minutes	$\frac{1}{8}$	0.125	$2^{-3}$
4	108 minutes	$\frac{1}{16}$	0.0625	$2^{-4}$
5	135 minutes	$\frac{1}{32}$	0.03125	$2^{-5}$

2. Students should use the table they created to answer this question. The decimal form of the fractional part of Lead-214 remaining after 81 minutes is 0.125 grams.
3. The exponential form of the fractional part of Lead-214 remaining after 108 minutes is  $2^{-4}$  grams.
4. Students must convert 2 hours and 15 minutes to minutes. Two hours is  $2(60)$ , or 120 minutes; 120 minutes plus 15 minutes is 135 minutes. From the table, the decimal form of measures of amount remaining is 0.3125. Change this to scientific notation.

0.3125 in scientific notation is  $3.125 \times 10^{-2}$

5. Yes, the time lapse is growing at a constant rate of 27 minutes per half-life. The time lapse for the fifth half-life is  $5(27)$ , or 135 minutes, so the time lapse of the tenth half-life will be  $10(27)$ , or 270 minutes.
6. No, the measures of amount remaining are not growing at a constant rate. In fact, they are decreasing exponentially. For example, the change from the first half-life to the second half-life is  $0.25 - 0.5$  or a decrease of 0.25. The change from the second half-life to the third half-life is  $0.125 - 0.5$ , or a decrease of 0.125.
7. For fifth half-life, the amount remaining is  $\frac{1}{32}$ ; therefore, the amount of decay is  $\frac{31}{32}$ . No, the amount of decay is not growing at a constant rate. The amount remaining for the tenth half-life is  $\frac{1}{1024}$ ; therefore, the amount of decay is  $\frac{1023}{1024}$ .

(A) compare and contrast proportional and non-proportional relationships

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### Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

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Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

### Extension Questions

- Can you write an equation to represent the decay of Lead-214 ( $y$ ) in terms of the number of half-life ( $x$ )? Explain your equation.

$$y = \left(\frac{1}{2}\right)^x$$

*Every half-life the amount of Lead-214 decreases by  $\frac{1}{2}$ , so an exponent can be used to represent the half-life number, and that will give us the amount of decay for that half-life number. For example, for half-life 5,*

$$y = \left(\frac{1}{2}\right)^5 \text{ so } y = 0.03125$$

