Chapter 2:
Patterns, Relationships, and Algebraic Thinking

At Home in Space grade 6

The International Space Station, ISS, is a state-of-the-art laboratory in space. It is a human experiment where we can learn to live and work “off planet.” The knowledge gained will help humans adjust to living in space in preparation for expeditions to Mars and beyond. The space station must be large enough to accommodate the more than 30 experiments onboard and provide living space for 6 astronauts. The space station is in the shape of a rectangular solid.

1. The table below shows volumes of an 18-square-foot area with different heights. Study the table and answer the following questions: What patterns can you find? Evaluate the patterns for a proportional relationship. Support your answer. Create a rule to describe how one number affects another.

Volume of a solid with a base area of 18 ft^2

Area in ft^2	Height in ft	Volume in ft^3
18	1	18
18	2	36
18	3	54
18	4	72
18	5	90
18	6	108
18	7	126
18	8	144

2. According to NASA, the floor space in the average American house is about 1,800 square feet. The ceiling is usually 8 feet high. How many cubic feet are in an average American house?
3. If the ISS has 43,000 cubic feet of pressurized volume, how many average American houses would fit in it?

Teacher Notes

Materials

Calculator

Cubes

Rectangular boxes

Grid paper

Connections to TEKS

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations

(B) represent ratios and percents with concrete models, fractions, and decimals

(C) use ratios to make predictions in proportional situations

(6.4) Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, perimeter, area, etc.

Scaffolding Questions

- Can you describe a strategy to find volume?
- How are units of measurement used to express area? Volume?
- Describe how you could show the floor area with cubes.
- What ratios can be found using the data in the table?
- How does area relate to volume? Volume to area? Volume to height?
- How is the living space at home like or different from the living space in the ISS?
- What solid figure best describes a simple house?
- What is the floor area of the average American house?
- Can you describe a method for finding floor area?
- How can you use cubes to model a rectangular house?
- Does a simpler problem created by dividing the area and volume by 100 model the same pattern?
- Is there a relationship between the height and the number of cubes?
- Which item from the table is represented by the variable?

Sample Solutions

1. Answers will vary. Here are some patterns the students may find.
   - The area remains the same or constant.
   - The volume is always the area times the height.

- If the height is 2, then the area is $\frac{1}{2}$ of the volume. If the height is 4, then the area is $\frac{1}{4}$ the volume.
- Some ratios noticed:

  The ratio of the volume to the height:

$$\frac{72\text{ ft}^3\text{ of volume}}{4\text{ ft of height}} = \frac{18\text{ ft}^3\text{ of volume}}{1\text{ ft of height}}$$

$$\frac{54\text{ ft}^3\text{ of volume}}{3\text{ ft of height}} = \frac{18\text{ ft}^3\text{ of volume}}{1\text{ ft of height}}$$

$$\frac{36\text{ ft}^3\text{ of volume}}{3\text{ ft of height}} = \frac{18\text{ ft}^3\text{ of volume}}{1\text{ ft of height}}$$

  Notice that this ratio is the same for all given values.

  These patterns show that the volume is proportional to the height because the ratios are equal to each other. When the height increases by 1, the volume increases by 18.

  The ratio of the volume to the area:

$$\frac{72\text{ ft}^3\text{ of volume}}{18\text{ ft}^2\text{ of area}} = \frac{4\text{ ft}^3\text{ of volume}}{1\text{ ft}^2\text{ of area}}$$

$$\frac{54\text{ ft}^3\text{ of volume}}{18\text{ ft}^2\text{ of area}} = \frac{3\text{ ft}^3\text{ of volume}}{1\text{ ft}^2\text{ of area}}$$

$$\frac{36\text{ ft}^3\text{ of volume}}{18\text{ ft}^2\text{ of area}} = \frac{2\text{ ft}^3\text{ of volume}}{1\text{ ft}^2\text{ of area}}$$

  This ratio is not constant.
- To create a rule students may use cubes to show the area. As the height increases by 1, the student thinks of "1" as another layer that is congruent to the layer below. The student may also see from the table that the area times the height is equal to the volume of the prism.

  Let height = *h* and volume = *v*. The area is 18, so the rule for this relationship is $18h = v$.

2. Students might also use the rule they wrote to show the relationship in the table. The area of the base times the height equals the volume.

$$1{,}800\text{ ft}^2(8\text{ ft}) = 14{,}400\text{ ft}^3$$

(B) generate formulas to represent relationships involving perimeter, area, volume of a rectangular prism, etc., from a table of data

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

3. The calculator may assist with this solution. Some students may use repeated addition such as 14,400 + 14,400 + 14,400 = 43,200, or some may divide 43,000 by 14,400 to find that the answer is approximately 3 houses.

Extension Questions

- According to NASA, the average American house has a 100 amp service from the electric company. The following conversion is also true:

  100 amps x 110 volts = 11,000 watts or 11 kilowatts

  If the International Space Station uses approximately 110 kilowatts, about how many houses could it power?

  The student can set up a relationship between kilowatts used per house and the kilowatts used on the space station to find the number of houses that would use the same amount of kilowatts.

$$\frac{11\text{ kw of power}}{1\text{ house}} = \frac{11\text{ kw of power x }10}{1\text{ house x }10} = \frac{110\text{ kw of power}}{10\text{ houses}}$$

  The same relationship can be shown using the following table:

Houses	1	2	3	4	5	6	7	8	9	10
Kilowatts	11	22	33	44	55	66	77	88	99	110

Resources used in this section

NASA Human Spaceflight, spaceflight.nasa.gov

NASA Fun Stuff! Station Overview, www.hq.nasa.gov/osf/funstuff/stationoverview/sld019.htm and www.hq.nasa.gov/osf/funstuff/stationoverview/sld021.htm

Solar Cells for Science
grade 7

The United States and 15 other nations created the International Space Station, ISS, as a world-class laboratory in space. The ISS provides a gravity-free environment for about 30 experiments. Solar energy is used to run the research and maintain the ISS. This power is collected from the sun with photovoltaic modules made up of two rectangular panels called *arrays*.

As space research increases, the demands for power increase. In the next construction phase, astronauts plan to install two more sets of solar arrays on the space station. The photovoltaic modules will then have a total surface area of 1632 square meters, which is twice the present amount. The modules will contain a total of 65,000 solar power cells, which is 32,500 solar power cells more than the present amount.

Photovoltaic modules

Number of array panels	2	4	6	8	10	12	14
Solar cells	32,500	65,000			162,500		
Area in m^2		1,632	2,448			4,896	

1. Complete the table above. Describe any patterns you see in the sequences.

2. Make a conjecture about the proportionality of the number of solar cells to the number of array panels. Graph the number of solar cells compared to the number of array panels. Does the graph support your conjecture? Why or why not?

3. Make a conjecture about the proportionality of the number of solar cells to the area of the array panels. Graph the number of solar cells compared to the area of the array panels. Does the graph support your conjecture? Why or why not?

Teacher Notes

Materials

Graph paper

Graphing calculator

Connections to TEKS

(7.4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(A) generate formulas involving conversions, perimeter, area, circumference, volume, and scaling

(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling

(7.5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to:

(A) use concrete models to solve equations and use symbols to record the actions

Scaffolding Questions

- How can you describe a strategy that will work to find the missing information?
- Describe how to find the number of solar cells for 6 panels.
- How can you use the information in the table and the information given in the directions to find the area of 2 array panels?
- What ratios may be found using the given information?
- What patterns or sequences does the table reveal?
- What patterns can translate into a rule for the relationship between the number of array panels and the number of solar cells, or the area?
- What are the attributes of a proportional relationship?
- What questions analyze data for a proportion?
- What kind of relationship does the graphed data show?
- Does the line pass through the origin? How do you know?

Sample Solutions

1. To complete the table, students may notice the pattern in the row for the number of array panels. Each term is a multiple of two. The number of solar cells is increasing by 32,500. The number of solar cells sequence is then

   32,500 65,000 97,500 130,000 162,500 etc.

   The problem states that the surface area for 4 panels is twice the surface area for 2 panels. Since 1,632 divided by 2 is 816, the first table value in the area row is 816.

   $$1,632 + 816 = 2,448$$

Each new value is 816 more than the previous value.

816 1,632 2,448 3,264 4,080 etc.

Another approach may be to set up a ratio between the area and the number of array panels.

$$\frac{1632\ m^2}{4\ panels} = \frac{408\ m^2}{1\ panel}$$

With the ratio in lowest terms, the student can find the other areas by multiplying the area, 408 m^2, by the number of array panels.

Find the ratio between array panels and solar cells:

$$\frac{4\ \text{array panels}}{65{,}000\ \text{solar cells}} = \frac{1\ \text{array panel}}{16{,}250\ \text{solar cells}}$$

With the ratio in lowest terms, the student can find the other amounts of solar cells by multiplying the solar cells,16,250, by the number of array panels.

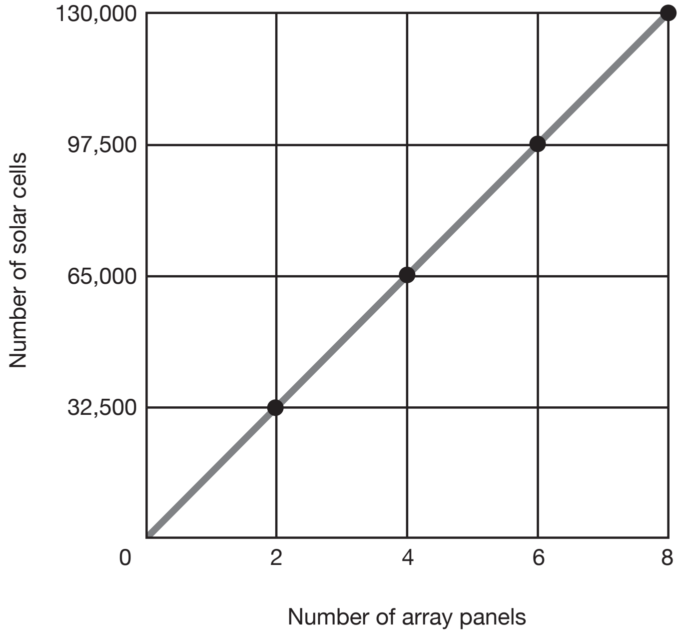
Photovoltaic modules

# of array panels	2	4	6	8	10	12	14
Solar cells	32,500	65,000	97,500	130,000	162,500	195,000	227,500
Area in m^2	816	1632	2448	3264	4080	4896	5712

2. Students may conjecture that the number of solar cells increases as the number of panels increases. The number of solar cells increases at the constant rate of 32,500 for every 2 panels, or 16,250 cells per 1 panel. If the relationship is proportional, the number of panels will increase by the constant rate. For example, if the number of panels doubles, then the number of solar cells doubles. The scatterplot of the data lies on a straight line through the origin. When a proportional relationship is graphed, the line goes through the origin.

Connections to TAKS

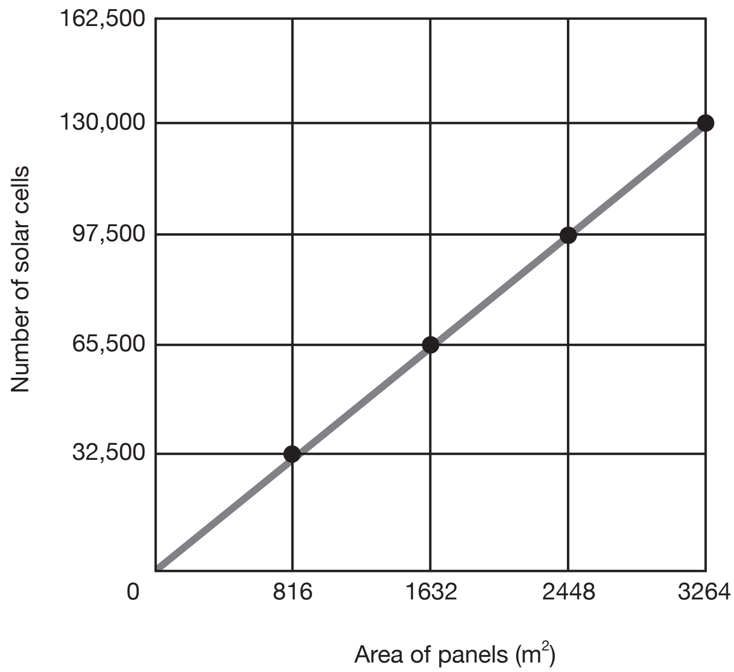
Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.



3. The ratio of the number of solar panels to the area is

$$\frac{32{,}500 \text{ solar cells}}{816 \text{ m}^2} = \frac{65{,}000 \text{ solar cells}}{1632 \text{ m}^2} = \frac{97{,}500 \text{ solar cells}}{2448 \text{ m}^2} \approx \frac{39.83 \text{ solar cells}}{1 \text{ m}^2}$$

The scatterplot of the data lies on a line through the origin.



Extension Questions

- The NASA engineers design a new solar array panel. The photovoltaic modules now have $\frac{1}{4}$ the total surface area they had before and $\frac{1}{4}$ the number of solar cells, but they still collect the same amount of power. Complete a new table showing the numbers of solar cells and area in m^2.

  The original surface area for 2 panels was 816 m^2 and had 32,500 solar cells, so 1 panel would have an area of 408 m^2 for every 16,250 solar cells. If the new panel uses only $\frac{1}{4}$ the area and $\frac{1}{4}$ the solar cells, then the new panel would have an area of 102 m^2 and have 4,062.5 solar cells.

Resources used in this section

NASA Human Spaceflight, spaceflight.nasa.gov

City in Space
grade 8

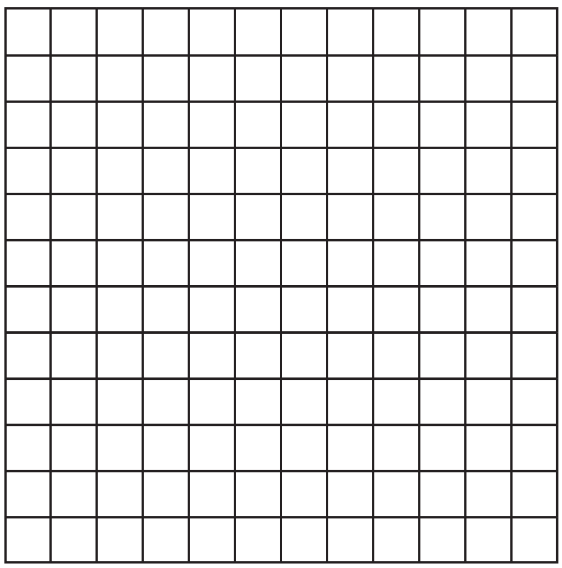
The International Space Station, ISS, is more than just a laboratory in space. It represents a “city in space,” a partnership of 16 nations learning and working in a gravity-free environment. The United States and Russia installed photovoltaic modules to store solar power for the station’s electricity. The U.S.’s photovoltaic modules are made of two rectangular panels. Each panel measures 34 meters long and 12 meters wide. Two panels provide 23 kilowatts of electricity per day.

Photovoltaic modules

# of array panels	2	4	6	8	10
Area in m^2	816	1,632	2,448	3,264	4,080
Kilowatts	23	46	69	92	115

Use the information from the table above to answer the following questions.

1. Describe the patterns you see in the table above.
2. Explain the relationship between the number of panels and the number of kilowatts. Graph the relationship. Is this a proportional relationship? How do you know? Create a rule to describe this relationship.



3. Predict the number of kilowatts if there had been 16 array panels.
4. Show how to determine the number of array panels if there are 276 kilowatts being produced.
5. Describe the relationship between the area and the number of kilowatts. Write a rule that describes the relationship. Create a graph of the relationship. How can you tell from the rule and the graph whether this is a proportional relationship?
6. Due to a workload increase, the ISS engineers need to increase the electrical power by 50%. If the ISS has 8 panels now, how much solar power in kilowatts is needed to meet the workload increase?

Materials

Grid paper

Graphing calculator

Connections to TEKS

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(A) compare and contrast proportional and non-proportional relationships

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

Teacher Notes

Scaffolding Questions

- How do the ratios in the columns compare with each other?
- What are the proportional relationships?
- What type of graph best illustrates a proportional relationship?
- Does the line on the graph go through the origin? Explain how you know.
- Is the data discrete or continuous? Why?
- How does the area change as the number of panels changes?
- How do the kilowatts change as the number of panels changes?
- How does the ratio of a solar panel to kilowatts compare with the ratio of an area to kilowatts?
- Are any ratios equal to each other?
- What conjectures can you make from the table's data?

Sample Solutions

1. The number of panels increased by 2 because it takes 2 panels to make each module. Two panels have an area of 816 m^2, so the area increases by 816 m^2 for every 2 panels. Two panels provide 23 kw of power; for every 2 panels, the number of kilowatts increases by 23.

$$\frac{\text{23 kilowatts}}{\text{2 array panels}} = \frac{\text{11.5 kilowatts}}{\text{1 array panel}}$$

$$\frac{\text{46 kilowatts}}{\text{4 array panels}} = \frac{\text{11.5 kilowatts}}{\text{1 array panel}}$$

$$\frac{\text{69 kilowatts}}{\text{6 array panels}} = \frac{\text{11.5 kilowatts}}{\text{1 array panel}}$$

$$\frac{\text{92 kilowatts}}{\text{8 array panels}} = \frac{\text{11.5 kilowatts}}{\text{1 array panel}}$$

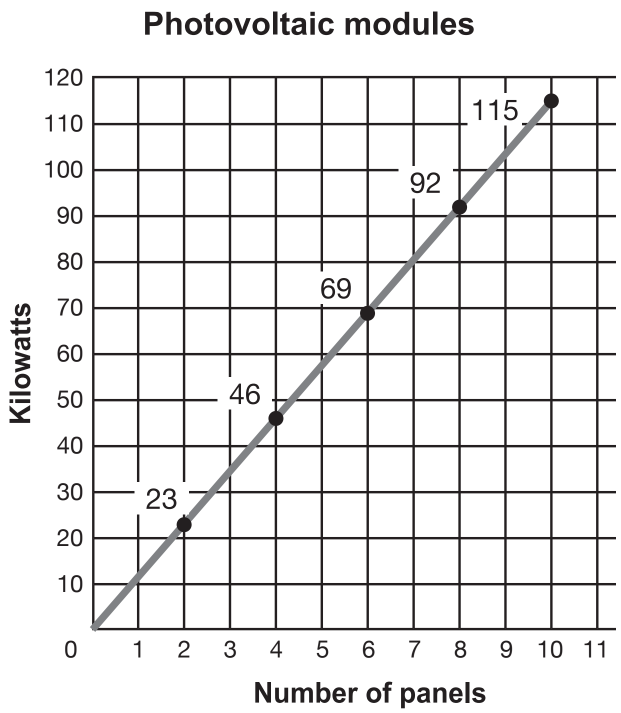
$$\frac{\text{115 kilowatts}}{\text{10 array panels}} = \frac{\text{11.5 kilowatts}}{\text{1 array panel}}$$

2. The ratio of the number of kilowatts to the number of array panels is constant.

The rate of change is 11.5 kilowatts per panel.

The rule that describes this relationship is $k = 11.5n$, where n represents the number of array panels and k represents the number of kilowatts.

The relationship is proportional because it has a rule of the form $y = kx$, where k is the constant of proportionality. The graph is a straight line and the line goes through the origin. A non-proportional relationship can also be a straight line, but the line will not go through the origin.



3. The rule may be used to find the number of kilowatts when the number of array panels is 16.

$$k = 11.5n$$

$$k = 11.5\ (16) = 184 \text{ kilowatts.}$$

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(B) use an algebraic expression to find any term in a sequence

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

4. The number of kilowatts is 276.

A table may be used to determine the value of *n* when *k* =276.

Number of array panels	Number of kilowatt hours
1	11.5
2	23
3	34.5
...	...
23	264.5
24	276
25	287.5
...	...

The number of array panels is 24.

5. The student may show the ratios as unit rates.

$$\frac{816 \text{ m}^2}{23 \text{ kw}} \approx \frac{35.5 \text{ m}^2}{1 \text{ kw}}$$
$$\frac{1,632 \text{ m}^2}{46 \text{ kw}} \approx \frac{35.5 \text{ m}^2}{1 \text{ kw}}$$
$$\frac{2,448 \text{ m}^2}{69 \text{ kw}} \approx \frac{35.5 \text{ m}^2}{1 \text{ kw}}$$
$$\frac{3,264 \text{ m}^2}{92 \text{ kw}} \approx \frac{35.5 \text{ m}^2}{1 \text{ kw}}$$
$$\frac{4,080 \text{ m}^2}{115 \text{ kw}} \approx \frac{35.5 \text{ m}^2}{1 \text{ kw}}$$

The ratio of area to kilowatts is not affected by the increase in solar panels. The area increases in proportion to the number of kilowatts collected by the panels. If the area doubles, the number of kilowatts doubles. When the area triples, the number of kilowatts also triples.

6. To increase by 50% means that the amount is 100% plus 50%. Multiply by 150% or 1.5.

$$\frac{92 \text{ kilowatts}}{8 \text{ array panels}} = \frac{92 \text{ kilowatts } \times 1.5}{8 \text{ array panels } \times 1.5} = \frac{138 \text{ kilowatts}}{12 \text{ array panels}}$$

There would be 12 panels for 138 kilowatts.

Extension Questions

- Every 45 minutes the modules store energy from the daylight side of their orbit and again from the dark side of their orbit. How many power storage cycles take place in one day?

  Every 45 minutes a cycle takes place can be written as

$$\frac{1 \text{ cycle}}{45 \text{ minutes}}$$

  Since there are 60 minutes in 1 hour and 24 hours in 1 day, rates may be multiplied to convert to cycles per day.

$$\frac{1 \text{ cycle}}{45 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \text{ day}} = \frac{32 \text{ cycles}}{1 \text{ day}}$$

Resources used in this section

NASA Human Spaceflight, spaceflight.nasa.gov

Community Clean-Up
grade 6

The state has an all-volunteer beach cleanup program to clean beaches and educate citizens about the problem of beach trash. Volunteers pick up trash along the coast twice a year. In the last 14 years, 273,000 volunteers have removed 450,000 kilograms of trash from the state's beaches. As a service project, the math club members decide to educate and recruit students for this year's beach clean-up.

1. Create and label several ratios to study relationships among the beach cleanup facts. Discuss your findings.

2. If a volunteer picks up the same ratio of trash every year, how much total trash will he or she pick up in ten years?

3. Determine the total trash to be removed this year if there are 50 volunteers. Predict the total trash in kilograms to be collected in the next five years and in the next 15 years. Graph the results.

4. If the 50 volunteers picked up double the amount of trash for the 15-year period, how much trash would be picked up? Graph this situation and compare to the graph in problem 3.

5. How could the math club use any of this information to recruit student volunteers for this year's beach clean-up?

Materials

Calculator

Poster board

Chart paper or construction paper

Markers or crayons

Connections to TEKS

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations

(C) use ratios to make predictions in proportional situations

Teacher Notes

Scaffolding Questions

- What determines how you set up a ratio?
- Must it be in lowest terms?
- What is the relationship of volunteers to years?
- How do metric tons of trash compare with years?
- What is the ratio of the amount of trash picked up per volunteer?
- Are the relationships proportional? Why or why not?
- What steps can you take to predict trash pick-up in other years?
- How will a chart or graph help?

Sample Solutions

1. Ratios may vary.

   The ratio of volunteers to years shows that 19,500 volunteers are needed each year to pick up trash if the amount of trash stays the same each year.

$$\frac{273{,}000 \text{ volunteers} \div 14}{14 \text{ years} \div 14} = \frac{19{,}500 \text{ volunteers}}{1 \text{ year}}$$

   The ratio of trash pick-up to years shows that 32,143 kg of trash is the average amount of trash picked up in one year.

$$\frac{450{,}000 \text{ kg} \div 14}{14 \text{ years} \div 14} \approx \frac{32{,}143 \text{ kg}}{1 \text{ year}}$$

   The ratio of amount of trash picked up in a year to volunteers shows that each volunteer picked up approximately 1.65 kg of trash a year.

$$\frac{32{,}143 \text{ kg of trash} \div 19{,}500}{19{,}500 \text{ volunteers} \div 19{,}500} \approx \frac{1.65 \text{ kg of trash}}{1 \text{ volunteer}}$$

2. The amount of trash picked up in 14 years seems like a very large amount. However, the ratios in the answer to problem 1 show that each volunteer picked up about 1.65 kg of trash per year.

   The table below shows the relationship of years to the amount of trash picked up by each volunteer. The total amount a volunteer would pick up in 5 years is 8.25 kg, so for 10 years it would be twice as much. In 10 years a volunteer would pick up 16.5 kg of trash.

Years	Kilograms of trash
1	1.65
2	3.3
3	4.95
4	6.6
5	8.25
6	9.9
7	11.55
8	13.2
9	14.85
10	16.5

3. If 50 volunteers pick up trash for 1 year using the ratio of 1 volunteer to 1.65 kg of trash per year, then 50 volunteers can pick up 50 x 1.65 kg = 82.5 kg of trash per year. A table may be used to predict what amount of trash will be picked up in 5 and 15 years.

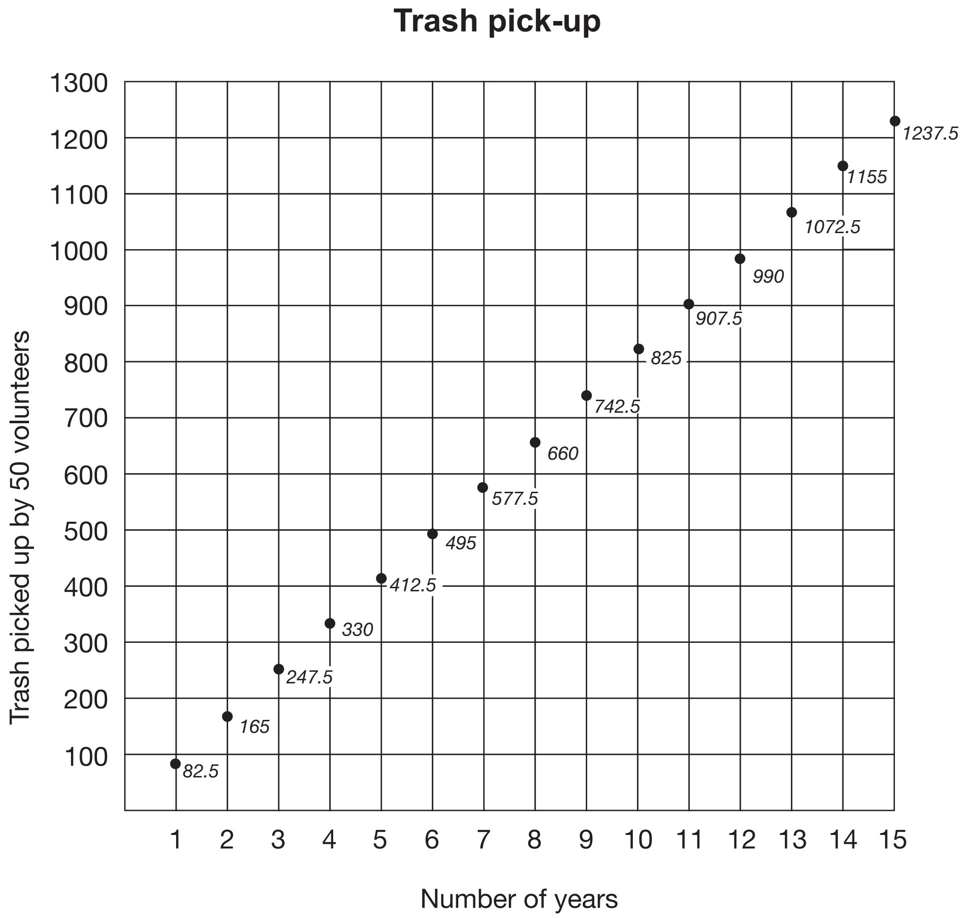
Years	Amount of trash in kg picked up by 50 volunteers
1	82.5
2	165
3	247.5
4	330
5	412.5
10	825
15	1237.5

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Each year the amount of trash increased by 82.5 kg. In 5 years 50 volunteers will collect 412.5 kg of total trash. In 15 years, they will collect 3 times as much, which will total 1,237.5 kg of trash.

Here is a possible graph:



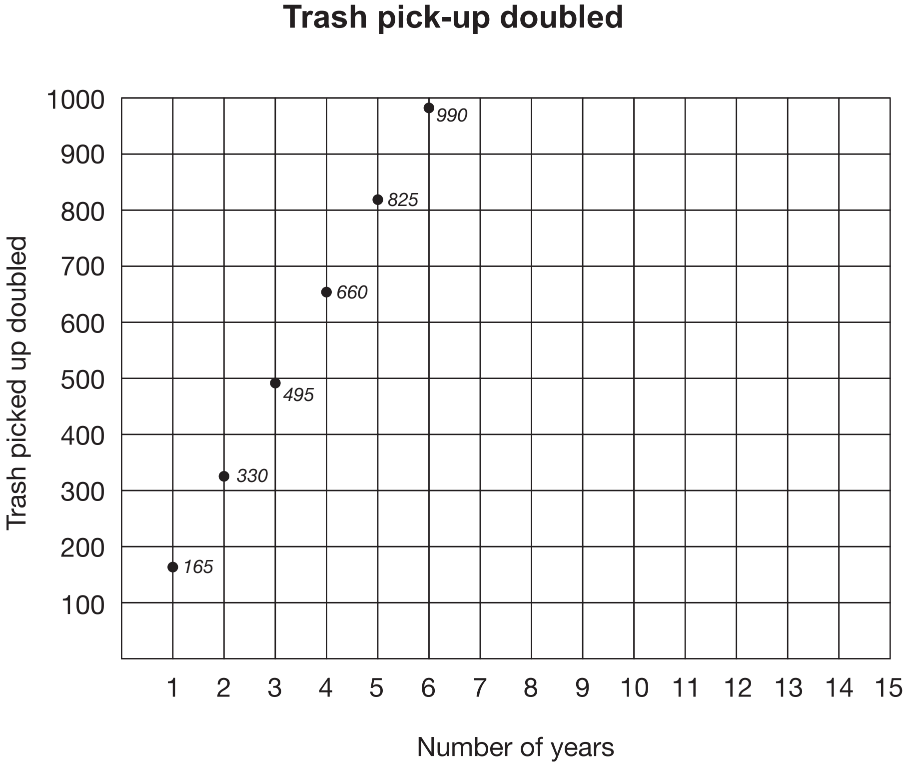
4. If 50 volunteers pick up double the amount of trash, the ratio of volunteers to trash will be 1 volunteer to 3.3 kg of trash per year.

$$\frac{3.3\text{ kg}}{1\text{ volunteer}} \times 50\text{ volunteers} = 165\text{ kg per year}$$

To find out how much trash will be picked up in 15 years multiply 165 times the number of years.

$$\frac{165\text{ kg}}{1\text{ year}} \times 15\text{ years} = 2,475\text{ kg}$$

Here is a possible graph:



The graph of this situation would show a line that is steeper and increases faster.

5. Students will use a variety of methods for recruiting volunteers. Accept all ideas that use correct mathematical facts with different styles of persuasion. It is important to stress the mathematical relationships in this activity. For example, students may want to make posters or a sample flyer using tables or graphs. Artistic or visual learners may create a montage by attaching plastic wrappers and other trash to a poster to illustrate the point of the data. Slogans or catchy phrasing might be used.

Extension Questions

- If the classroom trashcan holds approximately 2 kg of trash, how many trashcans of trash would equal the beach trash picked up in one year?

  One trashcan holds about the same amount of trash as one volunteer collects in a year; therefore, it would take 19,500 trashcans to hold the trash pick-up in one year.

- In one month in 2002, 2,600 volunteers removed 45,000 kg of trash from Galveston Island. If the island has 32 miles of coastline, about how much trash was picked up per mile?

$$\frac{45{,}000 \text{ kg} \div 32}{32 \text{ miles} \div 32} \approx \frac{1{,}406 \text{ kg}}{1 \text{ mile}}$$

About 1,406 kg of trash per mile was picked up.

Resources used in this section

Texas Adopt-A-Beach Program, www.glo.state.tx.us.

International Coastal Cleanup, www.coastalcleanup.org/top10.cfm

Texas Natural Resource Conservation Commission, www.tnrcc.state.tx.us

Student Work Sample

This student's work shows the use of various forms of one to simplify the ratios.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Describes mathematical relationships
- Recognizes and applies proportional relationships
- Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations
- Evaluates the reasonableness or significance of the solution in the context of the problem
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them
- States a clear and accurate solution using correct units

Community Clean-Up

1 (A) Ratios: years : volunteers

$$\frac{14 \text{ years}}{273{,}000 \text{ volunteers}} \div \boxed{\frac{14}{14}} = \frac{1 \text{ year}}{19{,}500}$$

(B) volunteers : Kilo

$$\frac{273{,}\not{000} \text{ vol.}}{450{,}\not{000} \text{ Kilo}} \div \boxed{\frac{273}{273}} = \frac{1 \text{ volunteer}}{1.65 \text{ Kilo of trash}}$$ about

(C) Kilo : years

$$\frac{450{,}000 \text{ kilo}}{14 \text{ years}} \div \boxed{\frac{14}{14}} = \frac{32{,}142}{1 \text{ year}}$$ (about)

(A) You need 19,500 volunteers in a year to pick up trash for each of the 14 years

(B) 1 volunteer must pick up 1.65 kilo of trash each year for the 14 years.

(C) In 1 year about 32,142 kilo of trash has to be picked up.

(2) $\frac{1 \text{ vol}}{1.65 \text{ Kilo}}$ in 1 year then $\frac{1 \text{ vol.}}{16.5 \text{ kilo}}$ in 10 years

1.65 kilo each yr. x 10 yr. = 16.5 kilo

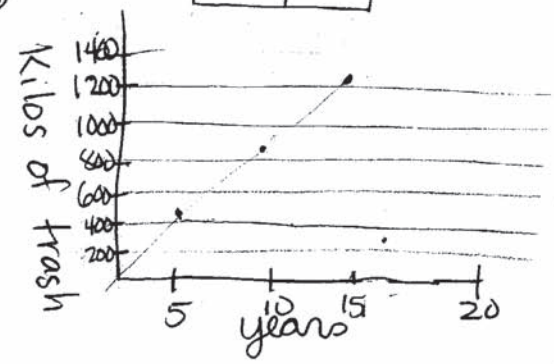
3. 1 vol picks up 1.65 each year ⟩ mult x 50
50 vol pick up 82.5 a year.

50 people pick up 82.5 in one year
x 5
412.5 in 5 year

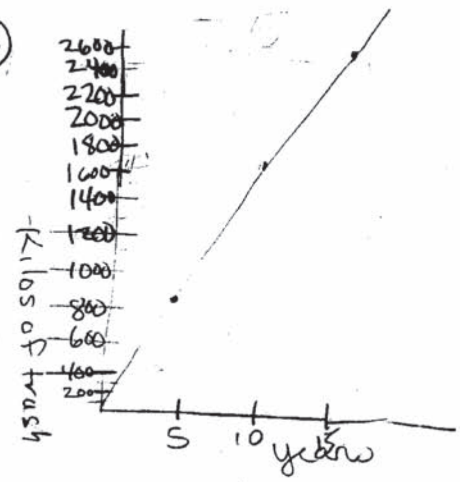
# of yrs	trash
5	412.5
10	825.0
15	1237.5

15 year = 1237.5 kilos

③



④



"Trash 50 volunteers will pick up"

4. Just double the amount 1237.5 x 2 =
2475 Kilos of trash

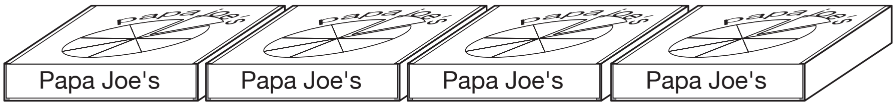
5 = 825.
10 = 1650
15 = 2475 kilo

this graph goes up faster for every x the y has doubled in the pairs (x, y)

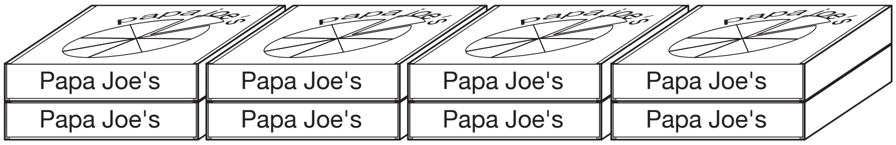
5. I would use the graph in #4 to help convince students to help. The more trash we get people to pick up the the cleaner our beaches will be.

Towering Pizzas
grade 6

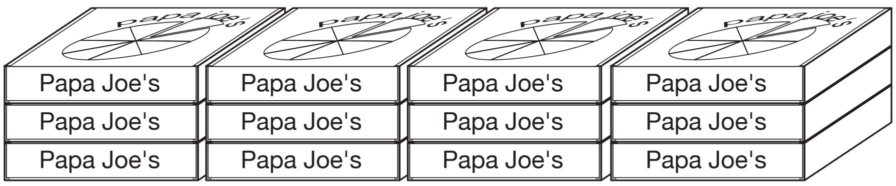
Papa Joe's Pizza Palace prepares pizza boxes by folding them in advance for its deliveries each day. On one shelf, a row of the pizza boxes are arranged as shown below:



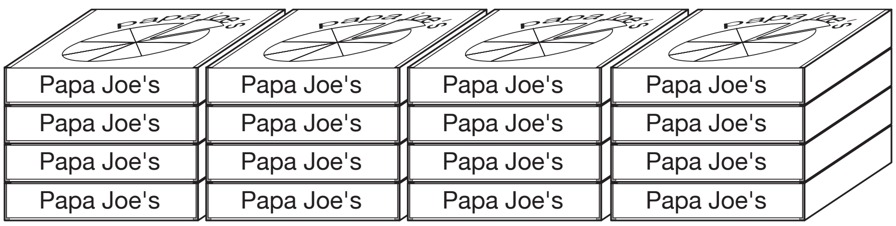
When there are two rows of pizza boxes on a shelf, the arrangement looks like this:



Three rows of pizza boxes stacked on a shelf looks like this:



The next picture shows the arrangement for a stack of four rows of pizza boxes.



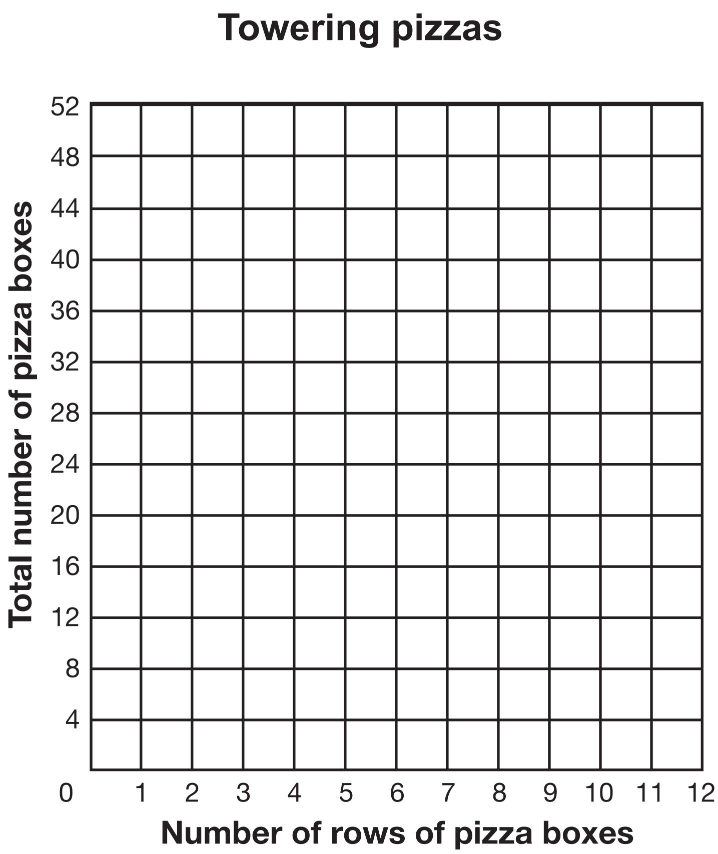
As more pizza boxes are folded, the pattern continues like the boxes in the pictures above.

1. Describe the relationship between the number of rows of pizza boxes and the total number of pizza boxes.

2. Complete the following table of data to represent the pizza box pattern at Papa Joe's Pizza Palace. Use the process column to show how you find the total number of pizza boxes for each row.

Number of rows of pizza boxes	Process	Total number of pizza boxes
1		
2		
3		
4		
6		
10		
12		

3. Complete the following coordinate graph of data to represent the pizza box pattern at Papa Joe's Pizza Palace.



4. Consider the ratio of the total number of pizza boxes to the number of rows. Compare the ratios for 3 rows and 6 rows.

5. Use the ratio of the total number of pizza boxes to the number of rows to answer the following two questions.

   a. How can this ratio be used to find the total number of pizza boxes in 7 rows?

   b. How can this ratio be used to find the number of rows if there are 44 pizza boxes?

6. Write a rule to describe the relationship between the number of pizza boxes and the number of rows.

7. Explain at least two methods for how to find the total number of pizza boxes in 30 rows.

Teacher Notes

Materials

Calculator

Square tiles to represent pizza boxes

Connection to Middle School TEKS

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations

(B) represent ratios and percents with concrete models, fractions, and decimals

(C) use ratios to make predictions in proportional situations

(6.4) Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, perimeter, area, etc.

Scaffolding Questions

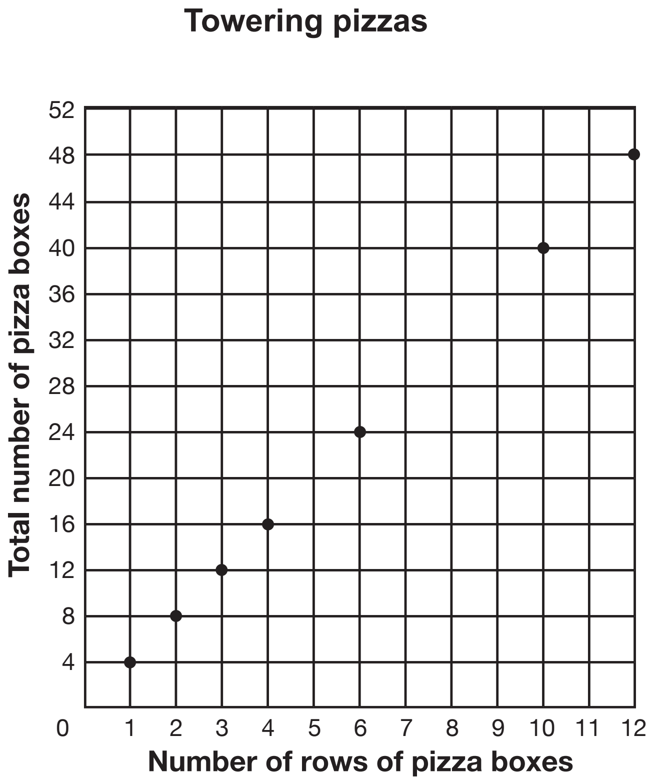
- What is the total number of pizza boxes in the first two rows?
- How does the total number of pizza boxes in the first two rows compare with the number 2?
- What does the ordered pair (3, 12) on the coordinate graph represent about the pizza boxes?
- What form of the number 1 can be used with the ratio $\frac{1}{4}$ to find the number of rows of pizza boxes if the total number of pizza boxes is 12?

Sample Solutions

1. The relationship between the number of rows of pizza boxes and the total number of pizza boxes can be described in a variety of ways.
   - The total number of pizza boxes is 4 times the number of rows of pizza boxes.
   - The number of rows of pizza boxes is $\frac{1}{4}$ of the total number of pizza boxes.
   - The ratio of the total number of pizza boxes to the number of rows of pizza boxes is always 4 to 1.
2. The table can be completed as shown below.

Number of rows of pizza boxes	Process	Total number of pizza boxes
1	1 x 4	4
2	2 x 4	8
3	3 x 4	12
4	4 x 4	16
6	6 x 4	24
10	10 x 4	40
12	12 x 4	48

3. The following coordinate graph shows the pattern of the pizza box data from the table of data in the solution to problem 2.



4. The ratios of the total number of pizza boxes to the number of rows for rows 3 and 6 are equivalent. The ratio for row 3 is

$$\frac{12 \text{ pizza boxes}}{3 \text{ rows}} = \frac{4 \text{ pizza boxes}}{1 \text{ row}}$$

The ratio for row 6 is

$$\frac{24 \text{ pizza boxes}}{6 \text{ rows}} = \frac{4 \text{ pizza boxes}}{1 \text{ row}}$$

5. The ratio of the total number of pizza boxes to the number of rows of pizza boxes is 4 pizza boxes per row.

   a. The ratio 4 pizza boxes per row can be used to find the total number of pizza boxes in 7 rows.

(B) generate formulas to represent relationships involving perimeter, area, volume of a rectangular prism, etc., from a table of data

(6.5) Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. The student is expected to formulate an equation from a problem situation

Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Objective 5: The student will demonstrate an understanding of probability and statistics.

Multiply the ratio by 1 in the form of $\frac{7}{7}$ to multiplicatively increase the number of rows from 1 to 7.

$$\frac{4 \text{ pizza boxes}}{1 \text{ row}} \times \frac{7}{7} = \frac{28 \text{ pizza boxes}}{7 \text{ rows}}$$

So 28 total pizza boxes are needed to have 7 complete rows of pizza boxes.

b. The ratio of 4 pizza boxes per row can be used to find the number of rows of pizza boxes for 44 pizza boxes.

Multiply the ratio by 1 in the form of $\frac{11}{11}$ to multiplicatively increase the total number of pizza boxes from 4 to 44.

$$\frac{4 \text{ pizza boxes}}{1 \text{ row}} \times \frac{11}{11} = \frac{44 \text{ pizza boxes}}{11 \text{ rows}}$$

So 44 total pizza boxes are needed to have 11 complete rows of pizza boxes.

6. The number of boxes of pizza is four times the number of rows of pizza. Let *r* represent the number of rows and let *p* represent the number of pizza boxes.

$$p = 4r$$

7. **Method: table of data**

Using the table of data generated in the solution to problem 2 leads to a variety of methods.

Number of rows of pizza boxes	Process	Total number of pizza boxes
1	1 x 4	4
2	2 x 4	8
3	3 x 4	12
4	4 x 4	16
6	6 x 4	24
10	10 x 4	40
12	12 x 4	48

For example, use the pattern in the table to continue the process until 30 rows.

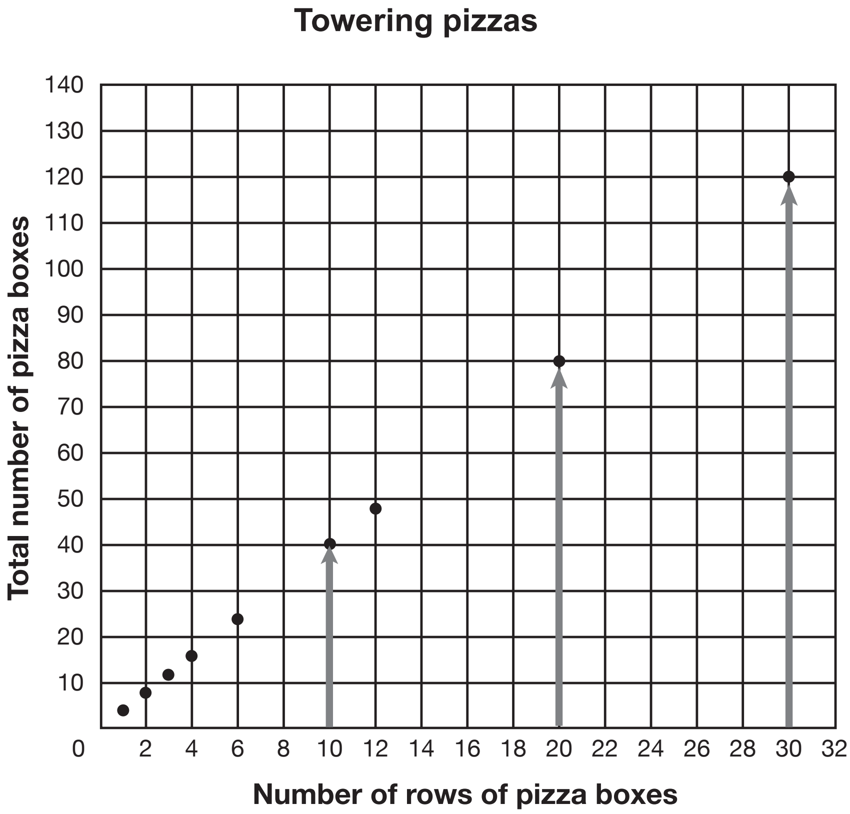
Number of rows of pizza boxes	Process	Total number of pizza boxes
1	1 x 4	4
2	2 x 4	8
3	3 x 4	12
4	4 x 4	16
6	6 x 4	24
10	10 x 4	40
20	2(10 x 4)	80
30	3(10 x 4)	120

Since 30 ÷ 10 = 3, the number of rows of pizza boxes, 10, multiplied by 3 increases the rows of pizza boxes multiplicatively to 30. Therefore, the total number of pizza boxes in 30 rows increased multiplicatively by the same factor is 120, since 40 x 3 = 120.

Method: graphing

The total number of pizza boxes in 30 rows can also be found by extending the graph of data using the same pattern of the data.

Note how the vertical length on the graph doubles when increasing from 10 rows of pizza boxes to 20 rows of pizza boxes (for 80 total pizza boxes) and triples when increasing from 10 rows of pizza boxes to 30 rows of pizza boxes (for 120 total pizza boxes).



Method: equation

Using equation methods, the total number of pizza boxes, *t,* in 30 rows can be obtained by using the rule that describes the relationship between the number of pizza boxes and the number of rows.

The total number of pizza boxes is 4 times the number of rows of pizza boxes.

$$p = 4r$$

$$p = 4(30)$$

$$p = 120$$

There are 120 pizza boxes.

Extension Questions

- How can you explain whether the point (23, 96) represents a point on the coordinate graph for the pattern of the pizza boxes?

  Because the total number of pizza boxes is 4 times the number of rows of pizza boxes, the 96 in the ordered pair should be four times 23. However, 4 times 23 is 92, so the point (23, 96) does not represent a point on the coordinate graph for the pattern of pizza boxes.

  Also, the graph could be extended to 23 rows of pizza boxes on the horizontal axis. Following the pattern of increasing 4 pizza boxes for every row, the total number of pizza boxes would be 92.

- Using symbols, how can you express the relationship between the number of rows of pizza boxes and the total number of pizza boxes?

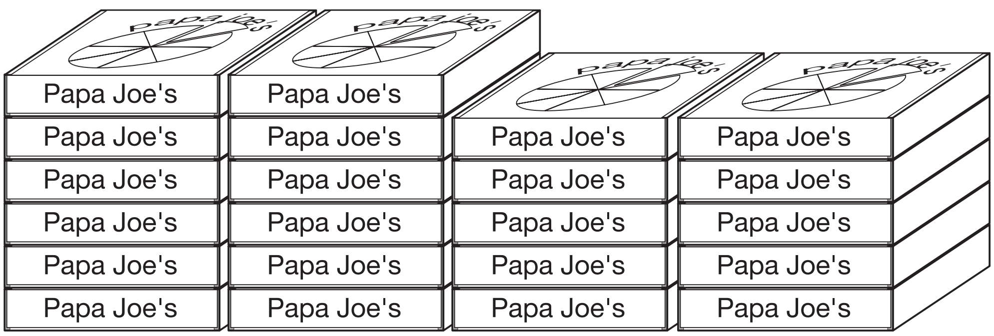
  If the number of rows of pizza boxes is represented by r and the total number of pizza boxes is represented by t, the relationship can be expressed symbolically by $t = 4r$ *or* $\frac{t}{r} = 4$ *or* $\frac{r}{t} = \frac{1}{4}$.

- If Papa Joe's Pizza Palace expands its business and prepares three shelves all the same size with an equal number of rows of pizza boxes on each shelf, how many rows of pizza boxes are on each shelf if the total number of pizza boxes is 288?

  The 288 pizza boxes will be divided among the three shelves equally.

  Since 288 ÷ 3 = 96, there will be 96 pizza boxes on each shelf. Then the 96 pizza boxes will be split equally into rows containing 4 boxes each. Since 96 ÷ 4 = 24, there will be 24 rows of 4 pizza boxes on each of the 3 shelves.

- What equation shows the relationship between the number of rows of pizza boxes and the total number of boxes on the shelf pictured below?



The equation 22 = 5 x 4 + 2 shows that there is a total of 22 pizza boxes in 5 rows of 4 plus 2 more pizza boxes in the top row.

South Texas Natives
grade 7

The South Texas region is home to more plant, butterfly, and animal species than any other biological region in Texas. Due to the introduction of foreign plants and foreign animals, South Texas native plants and animals are in danger of being overrun and even becoming extinct. The South Texas Natives program helps landowners replant native plants in private and public lands. To make seeds available, volunteers collect seeds of these threatened native plants.

With this in mind, the school science club volunteers to collect buffalo grass seeds. As an incentive, the South Texas Natives program pays $20 dollars for each kilogram of buffalo grass seeds collected.

1. Fifteen students help collect seeds for the science club project. How much money did the science club earn if each student collected 2 kilograms? Explain your solution.

2. Describe the relationship between the number of kilograms and the amount of money received. Is the relationship proportional? Explain why or why not.

3. Describe the relationship between the amount of money earned and the number of students. Is the relationship proportional? Explain why or why not.

4. The science club has a goal to raise $1,000 dollars during this school year. What percentage of this year's goal was met by the 15 students with the seed collection project? Explain your reasoning.

5. What percentage of the money did each student collect? Explain your solution.

Teacher Notes

Materials

Calculator (optional)

Connections to TEKS

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) estimate and find solutions to application problems involving percent; and

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

(7.4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(C) describe the relationship between the terms in a sequence and their positions in the sequence

Scaffolding Questions

- If you created a table to help solve the problem, what patterns do you notice in your table?
- What ratios compare the numbers in the data?
- How much money does just one student earn?
- What is the relationship between kilograms and dollars?
- How are ratios, decimals, and percentages the same?
- How much did the club raise collecting seeds?
- How much money does the club need to reach this year's goal?
- What makes a proportion?
- What would the data look like in a graph?

Sample Solutions

1. A table may be created to help solve the problem.

Buffalo grass seed

Students	1	2	3	4	5	5x3=15
Kilograms	2	4	6	8	10	10x3=30
Dollars	40	80	120	160	200	200x3=600

If five students collect 10 kg of seed, they earn $200; therefore, 15 students will earn $600. Another approach is to note from the information in the problem that the ratio of the amount of money to the number of students is

$$\frac{\$20}{1\text{ kilogram}} \times \frac{2\text{ kilograms}}{1\text{ student}} = \frac{\$40}{1\text{ student}}$$

Multiply 1 in the form of $\frac{15}{15}$ to increase the number of students to 15.

$$\frac{\$40}{1\text{ student}} \times \frac{15}{15} = \frac{\$120}{15\text{ students}}$$

2. The ratio of amount of money in dollars to the number of kilograms is as follows: The club earns 20 dollars for every kilogram of seed collected. Yes, this is a proportional relationship. There is a constant of proportionality of $20 per kilogram. If the student collects zero kilograms, the student earns zero dollars. On a line graph, the data would show a straight line that goes through the origin.

   A rule may be created for the amount of money earned. The number of kilograms times $20 equals *y*, the amount of money earned.

$$20n = y$$

3. The ratio of the amount of money earned to the number of students was shown in problem 1.

$$\frac{\$20}{1\text{ kilogram}} \times \frac{2\text{ kilograms}}{1\text{ student}} = \frac{\$40}{1\text{ student}}$$

   The ratio is a constant. Therefore, this is a proportional relationship.

   The relationship may be described using the following rule: *y* = 40*s,* where *y* is the amount of money earned and *s* is the number of students.

4. Students in class may compare $600 with $1,000 and simplify to $60 for every $100, resulting in 60%. A proportion could be used to show 60%.

$$\frac{\$600}{\$1{,}000} = \frac{x}{100\%}$$

   Students may also see the ratio $600 to $1,000 as a division problem. The quotient 0.60 converts to 60%.

5. Each student collected 2 kilograms, worth $40. The $40 may be compared to $600, the total amount earned by the club.

   Students may set up the ratio $40 to $600 and divide,

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

resulting in approximately 0.067. When the decimal is converted to a percentage, the answer is 6.7%.

$$\frac{\$40}{\$600} = 0.06\overline{6} = 6\frac{2}{3}\%$$

Extension Question

- The science teacher finds a price list for seed collections by the individual seed. If coma shrub seeds are 5 cents per seed, how many seeds must each student collect to earn the same amount as with the buffalo grass?

  The student may solve by dividing $40 by $.05, resulting in 800 seeds. Another strategy may be to think of the ratio of nickels to dollars. If 1 dollar has 20 nickels, then 40 dollars equals 800 nickels. To earn 800 nickels the student has to collect 800 seeds, since the ratio is a nickel to 1 coma seed.

Resources used in this section

South Texas Natives, www.southtexasnatives.org

Working Smarter
grade 7

A simple machine with ropes and pulleys gives a worker a mechanical advantage when doing work. Using this device, even a baby can lift an elephant. The number of strands of rope affects the force needed to lift a weight. In the table below, the force pulling on the rope is a person's body weight.

Mechanical advantage

# of rope strands	1	2	3	4	5	6
Force (body weight lbs.)	1,500	750	500	375	300	250
Weight lifted (lbs.)	1,500	1,500	1,500	1,500	1,500	1,500
Mechanical advantage	1.00	.5	$.33\frac{1}{3}$	.25	.2	$.16\frac{2}{3}$

1. Analyze the table above for patterns. Study the ratios of weight lifted to the force (body weight) and make a conclusion and a rule about this relationship.
2. What is the relationship between the force (body weight) and the lifted weight? Is the relationship proportional? Explain why or why not. Make a rule that describes this relationship.
3. Calculate the missing data in the table below using 150 pounds of body weight.

Mechanical advantage

# of rope strands	1	2	3	4	5	6
Force (body weight lbs.)						
Weight lifted (lbs.)						
Mechanical advantage	1.00	.5	$.33\frac{1}{3}$	.25	.2	$.16\frac{2}{3}$

4. Demonstrate how you can use your mechanical advantage by writing a problem situation requiring the use of the data in your table.

Teacher Notes

Materials

Picture of ropes and pulleys (optional)

Connections to TEKS

(7.4) Patterns, relationships, and algebraic thinking.
The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(A) generate formulas involving conversions, perimeter, area, circumference, volume, and scaling

(C) describe the relationship between the terms in a sequence and their positions in the sequence

(7.5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to:

(B) formulate a possible problem situation when given a simple equation

Scaffolding Questions

- What are the patterns in the table?
- How do the ropes and pulleys help to do work?
- What is mechanical advantage?
- How do the sequences in the table support the use of ropes and pulleys?
- Do all the numbers change in the data? Why or why not?
- What determines the set-up of a ratio?
- What rule develops from each sequence?
- How is mechanical advantage computed?
- What are the different ways to express ratios?
- How are ratios in decimal form converted to common fraction form?
- What relationships exist between mechanical advantage and other numbers in the table?
- What units of measurement are used to measure body weight?
- Does the type of unit used for weight affect the mechanical advantage ratio?

Sample Solutions

1. Patterns found include

- The rope strands increase by one.
- The weight to be lifted stays the same.
- The ratio of weight lifted to rope strands is equal to the force (body weight in lbs).
- The ratio of force (body weight) to weight lifted is equal to the mechanical advantage.

Some sample ratios are

$$\frac{1{,}500 \text{ lbs weight lifted}}{1{,}500 \text{ lbs force (body weight)}} = \frac{1}{1} \text{ or}$$

$$\frac{1{,}500 \text{ lbs weight lifted}}{750 \text{ lbs force (body weight)}} = \frac{2}{1} \text{ or}$$

$$\frac{1{,}500 \text{ lbs weight lifted}}{500 \text{ lbs force (body weight)}} = \frac{3}{1}$$

Conclusion: The values of the simplified ratios are the number of strands of rope.

The rule showing this relationship would be the weight lifted to the force (body weight in lbs) is equal to the number of strands of ropes.

$$\frac{\text{weight lifted}}{\text{force (body weight)}} = \text{number of strands of rope and pulleys needed}$$

2. Some sample ratios are

$$\frac{750 \text{ lbs force (body weight)}}{1{,}500 \text{ lbs weight lifted}} = \frac{1}{2} \text{ or}$$

$$\frac{500 \text{ lbs force (body weight)}}{1{,}500 \text{ lbs weight lifted}} = \frac{1}{3} \text{ or}$$

$$\frac{375 \text{ lbs force (body weight)}}{1{,}500 \text{ lbs weight lifted}} = \frac{1}{4}$$

Conclusion: The ratio force (body weight in lbs) to weight lifted equals the mechanical advantage. The rule that describes the relationship is

$$\frac{\text{amount of force (body weight lbs)}}{\text{weight lifted (lbs)}} = \text{mechanical advantage}$$

This ratio is not constant. The relationship is not a proportional relationship.

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

3.

Mechanical advantage

Rope strands	1	2	3	4	5	6
Force (body weight lbs)	150	150	150	150	150	150
Weight (lbs)	150	300	450	600	760	900
Mechanical advantage	1	.5	$.33\frac{1}{3}$	.25	.2	$.16\frac{2}{3}$

4. A mechanic who weighs 180 pounds needs to lift a motor weighing 300 pounds. If he uses ropes and pulleys, what is the least amount of strands he needs to use to be able to lift the motor?

$$\frac{\text{300 lbs weight to be lifted}}{\text{180 lbs force (body weight)}} \approx \frac{1.67}{1}$$

The mechanic needs to use at least two strands of rope to be able to lift the motor.

Extension Questions

- Ramon and Jamal must lift a block weighing 500 pounds. They have enough rope to set up two strands. Ramon weighs 125 pounds. What does Jamal's weight have to be for the two to perform the task?

$$\frac{\text{500 lbs weight to be lifted}}{\text{2 strands}} = \text{250 lbs force (body weight)}$$

The sum of Jamal's weight and Ramon's weight needs to be equal to or greater than 250 pounds.

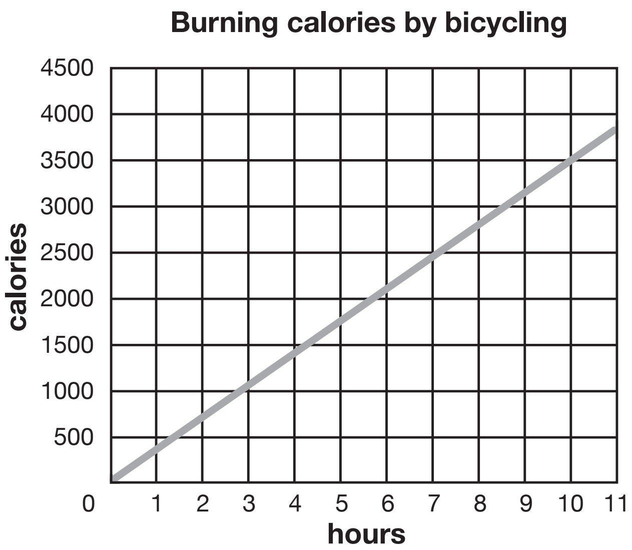
$$250 - 125 = 125$$

Jamal's weight needs to be equal to or greater than 125 pounds.

Fast Food Workout
grade 8

Fast food has become a part of the busy American lifestyle. Experts point out that fast food is often high in calories.

Kala eats fast food often. To maintain her weight, Kala exercises on her bicycle. She knows one hour of bicycling burns many calories. Kala also knows a female should eat about 2,000 calories per day to maintain her weight.



Bicycle exercise

Number of hours	Calories burned
2	700
4	1,400
6	2,100
8	2,800

1. Is the relationship shown above proportional? How do you know? What does the point (10, 3500) mean? Write a rule to show the relationship between the number of hours on a bicycle and the number of calories burned.

2. Kala will go bicycling three days a week for one hour a day. Predict how many calories she will burn in a week. Explain your reasoning.

3. How long does Kala have to ride her bicycle to burn up calories from a hamburger, fries, and a soda? The meal totals 1,166 calories. Explain your reasoning.

4. If Kala eats an average of 2,100 calories per day, how many hours per week does she need to bicycle to maintain her weight? Explain your reasoning.

Teacher Notes

Materials

Graphing calculator

Straight edge

Connections to TEKS

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

Scaffolding Questions

- What points on the graph can be read without estimation?
- Which is the independent variable and which is the dependent variable?
- How does time affect the calories burned?
- How can you evaluate the situation for proportionality?
- How does the line graph support or not support a proportional relationship?
- What is the rate Kala burns up calories by bicycling?
- How does a rule help predict an outcome?

Sample Solutions

1. The relationship between the number of calories burned and the number of hours is proportional because the graph is a line graph that goes through the origin. The rate of change is the same for each hour. The data in the table needs to be studied to find the rate of calories burned in one hour. Any ratio of the number of calories burned to the number of hours may be simplified into 350 calories per hour.

$$\frac{700 \text{ calories}}{2 \text{ hours}} = \frac{350 \text{ calories}}{1 \text{ hour}} = 350 \text{ calories per hour}$$

$$\frac{1400 \text{ calories}}{4 \text{ hours}} = \frac{350 \text{ calories}}{1 \text{ hour}} = 350 \text{ calories per hour}$$

$$\frac{2100 \text{ calories}}{6 \text{ hours}} = \frac{350 \text{ calories}}{1 \text{ hour}} = 350 \text{ calories per hour}$$

$$\frac{2800 \text{ calories}}{8 \text{ hours}} = \frac{350 \text{ calories}}{1 \text{ hour}} = 350 \text{ calories per hour}$$

The point (10, 3500) means 10 hours of bicycling burns 3,500 calories.

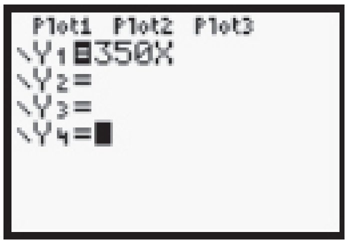
Kala burns 350 calories in one hour. The total number of calories burned is 350 calories per hour times the number of hours. The rule for the total number of calories, *y*, is equal to 350 times the number of hours exercised, *x*.

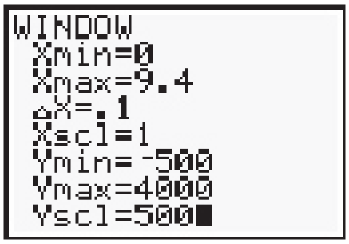
$$y = 350x$$

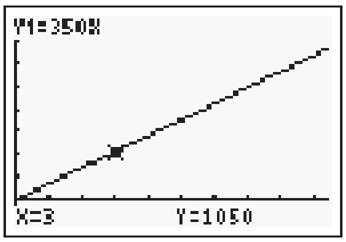
2. The student may use the table to find that the value is halfway between 2 and 4 hours. There are 350 calories burned per hour.

   700 + 350 =1,050 or 1400 – 350 = 1,050

   Another strategy is to enter the rule in a graphing calculator, create a graph, and trace to *x* = 3 hours to get the *y*-value, the calories burned.







The student may substitute 3 hours for *x* in the rule.

$$y = 350(3)$$

$$y = 1{,}050$$

When *x* is 3 hours, then *y* is 1,050 calories.

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(B) use an algebraic expression to find any term in a sequence

Connection to the TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

3. The student may substitute the value of 1,166 in the rule.

$$y = 350x$$
$$1{,}166 = 350x$$
$$1{,}166 \div 350 = x$$
$$x = 3\tfrac{1}{3}\text{ hours}$$

Another strategy might be to use the ratio

$$\frac{350\text{ calories}}{1\text{ hour}} = \frac{700\text{ calories}}{2\text{ hours}} = \frac{1050\text{ calories}}{3\text{ hours}} = \frac{1400\text{ calories}}{4\text{ hours}}$$

Because 1,166 calories is between 1,050 calories, the amount for 3 hours, and 1,400 calories, the amount for 4 hours, Kala must exercise between 3 and 4 hours.

4. If Kala is to maintain her weight, she must exercise two hours per week. A table may help organize the information for the solution.

Calories consumed in one week

								Total
Day	1	2	3	4	5	6	7	7
Calories per day	2,100	2,100	2,100	2,100	2,100	2,100	2,100	14,700
Extra calories	100	100	100	100	100	100	100	700

To find how many hours of bicycling it takes to burn up 700 calories, the student may substitute 700 into the rule.

$$y = 350x$$
$$700 = 350x$$
$$\frac{700}{350} = \frac{350}{350}x$$
$$2 = x$$
$$x = 2\text{ hours}$$

Another strategy is to look at the original table and see that 700 calories is the amount for 2 hours.

Extension Questions

- To lose a pound, Kala must burn up 3,500 calories. If she keeps the same workout schedule, how many days must she exercise to lose one pound?

  The ratio may be used to determine the number of hours.

$$\frac{350 \text{ calories}}{1 \text{ hour}} = \frac{3500 \text{ calories}}{x \text{ hours}}$$

$$\frac{350 \text{ calories} \cdot 10}{1 \text{ hour} \cdot 10} = \frac{3500 \text{ calories}}{x \text{ hours}}$$

$$\frac{3500 \text{ calories}}{10 \text{ hours}} = \frac{3500 \text{ calories}}{x \text{ hours}}$$

$$x = 10 \text{ hours}$$

  It takes ten hours of bicycling to lose one pound If she exercises for one hour per day, Kala must exercise for 10 days.

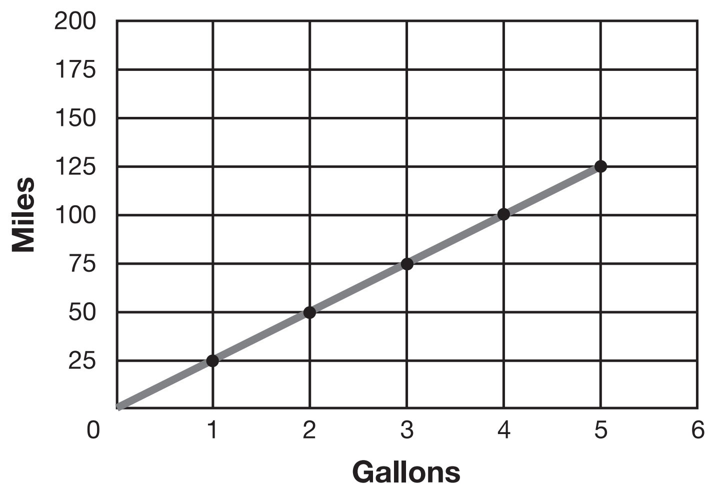
Global Warming: Texas-Size
grade 8

The burning of fossil fuels such as gasoline is one way carbon dioxide, a greenhouse gas, is added to the atmosphere. Carbon dioxide (CO_2) traps the earth's heat and contributes to global warming. Texas leads the nation in emissions of greenhouse gases. With millions of vehicles on the road, carbon dioxide is a Texas-size problem. Each time a vehicle burns one gallon of gasoline, it produces about 20 pounds of carbon dioxide, CO_2.

1. Is this relationship proportional? How do you know? Write a rule to show the relationship between the number of gallons burned and the CO_2 produced. How many pounds of CO_2 are given off by burning 5 gallons of gas? Explain your reasoning.

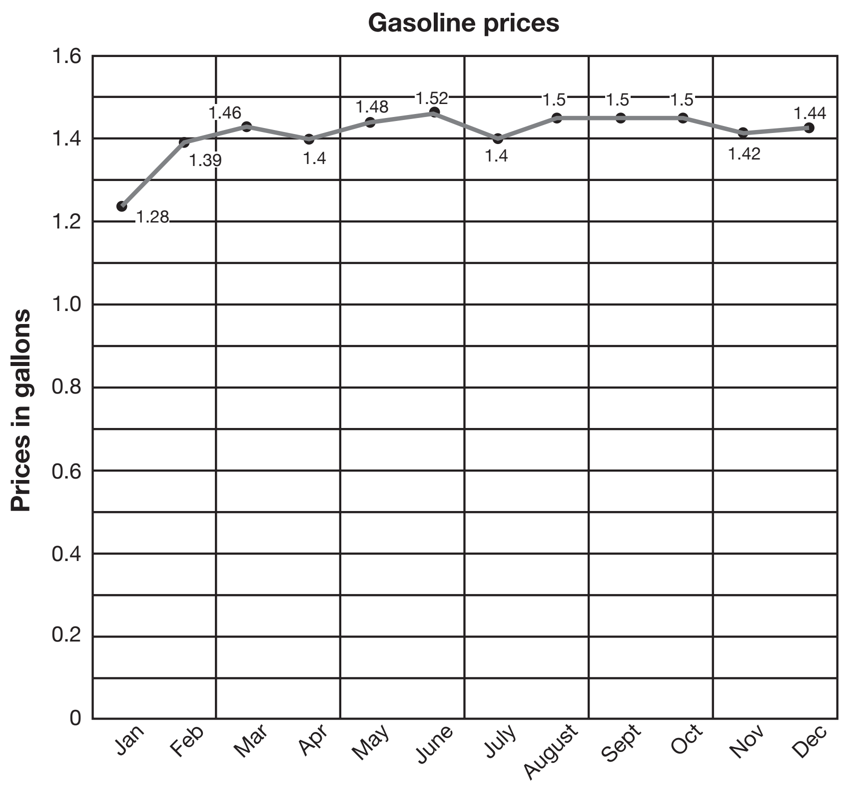
2. The graph below shows the relationship between the number of miles traveled and the number of gallons of gasoline used.

Average mileage per gallon in Texas



In Texas, the average driver travels 12,000 miles per year. What is the carbon dioxide emission in one year? Show your solution steps.

3. Look at gasoline prices for one year; use the information from problem 2 to project the cost of driving a car for one year. Justify your answer.



4. Compare graphs from problem 2 and problem 3. Explain if they show a proportional relationship.

Materials

Graphing calculator

Ruler

Connections to TEKS

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(A) compare and contrast proportional and non-proportional relationships

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make

Teacher Notes

Scaffolding Questions

- What is the ratio of gallons to pounds of carbon dioxide?
- What other ratios are in the table?
- What patterns help create the equation?
- What clues determine the operations to use in the rule?
- What kind of relationship is illustrated on the line graph?
- What rate of change does the graph express?
- How are the independent variable and the dependent variable determined?
- What are some visual clues of proportionality on a graph?
- How can a ruler assist in reading a graph?

Sample Solutions

1. There is a constant rate of change; for every 1 gallon of gasoline burned, 20 pounds of CO_2 are produced. If 0 gallons of gasoline are burned, then no CO_2 is produced. Therefore, there is a proportional relationship between the gallons burned and the pollution produced.

   The number of pounds of CO_2 is equal to 20 pounds per gallon times the number of gallons. A rule for this proportion could be written as $p = 20g$, where g is the gallons and p is the pounds of CO_2.

   The rule may be used to determine the number of pounds when the number of gallons is 5.

$$p = 20g$$

$$p = 20(5) = 100$$

One hundred pounds of CO_2 are produced by burning 5 gallons of gasoline.

The ratio could also be used to answer the question. Multiply by 5 to make the number of gallons be 5 gallons.

$$\frac{20 \text{ pounds}}{1 \text{ gallon}} = \frac{20 \text{ pounds} \cdot 5}{1 \text{ gallon} \cdot 5} = \frac{100 \text{ pounds}}{5 \text{ gallons}}$$

2. The graph is a straight line that passes through the origin. There is a constant rate of change.

25 miles for 1 gallon

50 miles for 2 gallons

75 miles for 3 gallons

100 miles for 4 gallons

125 miles for 5 gallons

The rate is 25 miles per gallon.

Proportions may be used to find the number of gallons for 1,200 miles.

$$\frac{25 \text{ miles}}{1 \text{ gallon}} = \frac{12{,}000 \text{ miles}}{x \text{ gallons}}$$

$$\frac{25 \text{ miles x } 480}{1 \text{ gallon x } 480} = \frac{12{,}000 \text{ miles}}{480 \text{ gallons}}$$

The driver uses 480 gallons in one year.

To find the number of pounds of CO_2, use the rule $p = 20g$ when g is 480 gallons.

$$p = 20(480) = 9{,}600$$

Therefore, 9,600 pounds of CO_2 would be produced in one year.

3. The student finds the average of the cost of gasoline for the year. The sum of the price of gas for 12 months, \$17.29, is divided by 12 months. The mean is about \$1.44 per gallon. The information from problem 2 helps

predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(B) use an algebraic expression to find any term in a sequence

Connections to TAKS

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

with this solution. From problem 2, the student gathers that the yearly total is 480 gallons.

$\frac{\$1.44 \times 480}{1\text{ gallon} \times 480} = \frac{\$691.20}{480\text{ gallons}}$ The driver spends $691.20 to buy 480 gallons of gas.

4. The graph in problem 2 is a proportion because the line goes through the origin (0, 0) and it has a constant rate of change; the ratio is

   25 miles : 1 gallon of gasoline

   The graph in problem 3 is not a straight line and does not go through the origin (0, 0). The comparison of gasoline prices is not proportional. There is not a constant rate of change among the prices of gasoline over the months.

Extension Questions

- A large Douglas fir tree takes in 16,000 pounds of carbon dioxide per year. How many trees can counteract the car emission of 10,000 pounds of carbon dioxide?

$$\frac{16{,}000\text{ pounds carbon dioxide}}{1\text{ tree}} = \frac{10{,}000\text{ pounds carbon dioxide}}{x}$$

$$\frac{16{,}000\text{ pounds carbon dioxide} \times .63}{1\text{ tree} \times .63} \approx \frac{10{,}000}{.63}$$

so it takes approximately 1 tree

One Douglas fir tree can counteract 10,000 pounds of carbon dioxide.

Resources used in this section

Environmental Protection Agency, www.epa.gov/OMSWWW/

Public Citizen, www.citizen.org/pressroom/release.cfm?ID=524