Chapter 3:
*Geometry and
Spatial Reasoning*

By the Sea
grade 6

Tourists spot a family of whales swimming off the coast of California. They first spot the mother whale, Millie. Willie, the father whale, is behind Millie. Billie, the baby whale, trails behind.

1. Use the list of coordinates and instructions below to draw each family member. Connect points with lines as you graph.

		Millie	Willie	Billie
Point	A	(1, 6)	(2, 18)	(.5, 1)
	B	(1, 8)	(2, 22)	(.5, 2)
	C	(2, 9)	(4, 24)	(1, 2.5)
	D	(1, 10)	(2, 26)	(.5, 3)
	E	(1, 12)	(2, 30)	(.5, 4)
	F	(3, 10)	(6, 26)	(1.5, 3)
	G	(4, 10)	(8, 26)	(2, 3)
	H	(6, 15)	(12, 36)	(3, 5.5)
	I	(11, 15)	(22, 36)	(5.5, 5.5)
	J	(13, 12)	(26, 30)	(6.5, 4)
	K	(13, 8)	(26, 22)	(6.5, 2)
	L	(3, 8)	(6, 22)	(1.5, 2)
		(connect *L* to *A*)	(connect *L* to *A*)	(connect *L* to *A*)
		(start over)	(start over)	(start over)
	M	(11, 10)	(22, 26)	(5.5, 3)
	N	(12, 9)	(24, 24)	(6, 2.5)
	O	(13, 9)	(26, 24)	(6.5, 2.5)
		(start over)	(start over)	(start over)
	P	(10, 12)	(20, 30)	(5, 4)

2. Find the body lengths for each family member. Describe how you determined the length.

Whale	Head $\overline{HI}$	Belly $\overline{LK}$	Nose $\overline{JO}$	Tail $\overline{AB}$
Millie				
Willie				
Billie				

3. Compare the body lengths of the three whales from problem 2 using ratios and explain what each ratio means.

4. The length of Millie's mouth is 2.4 units. Explain how to find the length of Willie's mouth.

Materials

Grid paper

Straight edge

Ruler

Scissors

String

Connections to Middle School TEKS

(6.7) Geometry and spatial reasoning. The student uses coordinate geometry to identify location in two dimensions.

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships.

(A) uses ratios to describe proportional situations

(C) uses ratios to make predictions in proportional situations

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models.

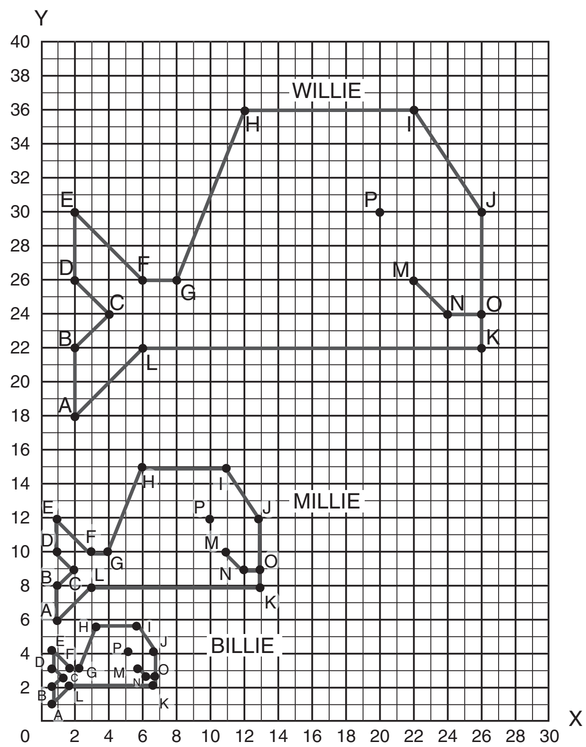
Teacher Notes

Scaffolding Questions

- Which number in the ordered pair represents the *x*-value? The *y*-value?
- What information does the first number in each ordered pair indicate?
- What information does the second number in each ordered pair indicate?
- How would you find the length of a horizontal segment on a grid?
- How would you find the length of a vertical segment on a grid?
- What is a ratio?
- How would you describe the relationship between the body lengths of Millie and Willie? Of Millie and Billie? Of Willie and Billie?

Sample Solutions

1.



Label the axes on the grid. Label the scale used on each axis. Name each figure on the grid: Willie, Millie, and Billie.

2. The number of units for each segment can be found by counting. Another method would be to find the absolute value of the difference between the *y*-values for a vertical segment or the absolute value of the difference between the *x*-values for a horizontal segment as shown below.

Head $\overline{HI}$ for Millie: H (6, 15) I (11, 15)

$HI = |11 - 6|$ or $|6 - 11|$

$HI = 5$

Tail $\overline{AB}$ for Willie: A (2, 18) B (2, 22)

$AB = |22 - 18|$ or $|18 - 22|$

$AB = 4$

Whale	Head $\overline{HI}$	Belly $\overline{LK}$	Nose $\overline{JO}$	Tail $\overline{AB}$
Millie	5	10	3	2
Willie	10	20	6	4
Billie	2.5	5	1.5	1

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

3. The following ratios describe some of the relationships between the designated body lengths for pairs of whales.

Head comparisons

Millie to Willie: $\frac{5}{10} = \frac{1}{2}$ Millie to Billie: $\frac{5}{2.5} = \frac{2}{1}$ Willie to Billie: $\frac{10}{2.5} = \frac{4}{1}$

Belly comparisons

Millie to Willie: $\frac{10}{20} = \frac{1}{2}$ Millie to Billie: $\frac{10}{5} = \frac{2}{1}$ Willie to Billie: $\frac{20}{5} = \frac{4}{1}$

Nose comparisons

Millie to Willie: $\frac{3}{6} = \frac{1}{2}$ Millie to Billie: $\frac{3}{1.5} = \frac{2}{1}$ Willie to Billie: $\frac{6}{1.5} = \frac{4}{1}$

Tail comparisons

Millie to Willie: $\frac{2}{4} = \frac{1}{2}$ Millie to Billie: $\frac{2}{1} = \frac{2}{1}$ Willie to Billie: $\frac{4}{1} = \frac{4}{1}$

The ratios comparing Millie and Willie's body lengths show that Millie has dimensions that are half the corresponding dimensions of Willie. The ratios comparing corresponding body lengths of Millie and Billie show that Millie's dimensions are twice those of Billie. The ratios comparing body lengths of Willie and Billie show that Willie's dimensions are 4 times the corresponding dimensions of Billie.

4. Willie's dimensions are twice the corresponding dimensions of Millie. If Millie's mouth is 2.4 units long, then Willie's mouth is twice as long, or 2 times 2.4 units. Willie's mouth is 4.8 units long.

Extension Questions

- Tourists spot another whale. This whale is Tillie, a cousin to the whale family. Tillie has dimensions that are three times the corresponding dimensions of Millie. How long is Tillie's belly?

  The ratio comparing the length of Tillie's belly and the length of Millie's belly is 3 to 1. If Millie's belly is 10 units long, then Tillie's belly is 3 times as long. Tillie's belly is 30 units long.

- Use a ratio to describe the relationship between the length of Tillie's tail and the length of Billie's tail.

  The length of Millie's tail is 2 units. The ratio comparing the length of Tillie's tail to the length of Millie's tail is 3 to 1. This means that the length of Tillie's tail is three times the length of Millie's tail or 6 units long. Billie's tail is one unit long; therefore, the ratio of Tillie's tail length to Billie's tail length is 6 to 1.

By the Sea
grade 7

Tourists spot a family of whales swimming off the coast of California. They first spot the mother whale, Millie. Willie, the father whale, is close by. Billie, the baby whale, trails behind.

1. Use the list of coordinates and instructions below to draw the whale family on grid paper. Connect points with line segments as you graph.

		Millie	**Willie**	**Billie**
Point	A	(11, 14)	(1, -19)	(-7.5, 4)
	B	(11, 16)	(1, -15)	(-7.5, 5)
	C	(12, 17)	(3, -13)	(-7, 5.5)
	D	(11, 18)	(1, -11)	(-7.5, 6)
	E	(11, 20)	(1, -7)	(-7.5, 7)
	F	(13, 18)	(5, -11)	(-6.5, 6)
	G	(14, 18)	(7, -11)	(-6, 6)
	H	(16, 23)	(11, -1)	(-5, 8.5)
	I	(21, 23)	(21, -1)	(-2.5, 8.5)
	J	(23, 20)	(25, -7)	(-1.5, 7)
	K	(23, 16)	(25, -15)	(-1.5, 5)
	L	(13, 16)	(5, -15)	(-6.5, 5)
		(connect *L* to *A*)	(connect *L* to *A*)	(connect *L* to *A*)
		(start over)	(start over)	(start over)
	M	(21, 18)	(21, -11)	(-2.5, 6)
	N	(22, 17)	(23, -13)	(-2, 5.5)
	O	(23, 17)	(25, -13)	(-1.5, 5.5)
		(start over)	(start over)	(start over)
	P	(20, 20)	(19, -8)	(-3, 7)

2. Find the body lengths for each family member. Describe how you determined the length.

Whale	Head $\overline{HI}$	Belly $\overline{LK}$	Nose $\overline{JO}$	Tail $\overline{AB}$
Millie				
Willie				
Billie				

3. Are the shapes that represent the members of the whale family similar? Justify your answer.

4. Billie wants to swim to his mom, Millie. Translate Billie so that vertex H is moved from coordinates (-5, 8.5) to coordinates (6, 8.5). Explain how to translate Billie to his new position in the coordinate plane. Name Billie's new coordinates and plot the points to relocate Billie. How did this translation affect the original coordinates of the figure?

5. Willie wants to join his family. Translate Willie so that vertex H is moved from coordinates (11, -1) to coordinates (11, 6). Describe how to translate Willie to his new location in the coordinate plane. Name Willie's new coordinates and plot the points to relocate Willie. What effect did this translation have on the original coordinates of the figure?

6. Tourists spot another whale, Tillie. This whale is a cousin to the whale family. Tillie is three times as long as Millie. Explain how to find the length of Tillie's belly.

Teacher Notes

Materials

Calculator

Half-centimeter grid paper

Scissors

Tracing paper

Straight edge

Ruler

Strips of paper about 20 inches long and 1 inch wide

Connections to Middle School TEKS

(7.6) Geometry and spatial reasoning. The student compares and classifies shapes and solids using geometric vocabulary and properties. The student:

(D) uses critical attributes to define similarity

(7.7) Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane. The student:

(A) locates and names points on a coordinate plane using ordered pairs of integers

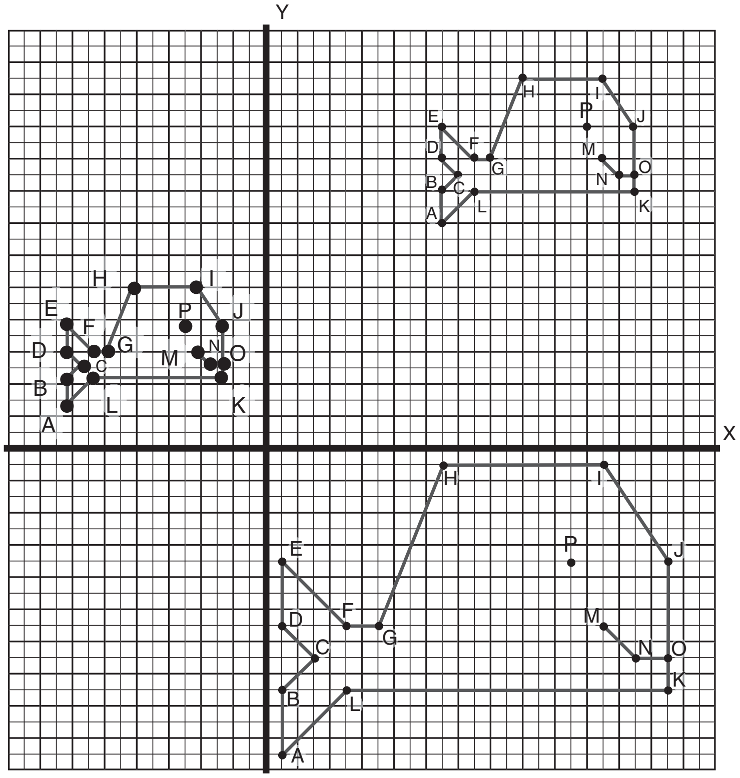
(B) graphs translations on a coordinate plane

Scaffolding Questions

- What makes two figures similar?
- How could you show that two whales are similar?
- How do corresponding angles of the whale family compare? Why?
- How do corresponding sides of the whale family compare?
- How do you locate ordered pairs on a coordinate graph?
- What does it mean to translate a figure?
- Describe how you would translate a figure 3 units to the right in the coordinate plane. How does this translation affect the coordinates (x, y) of the figure?
- How would you translate a figure 2 units up in the coordinate plane? How does this affect the coordinates (x, y) of the figure?
- How does a translation affect the length of the sides of a figure? The angles of a figure?

Sample Solutions

1.



2. The length of selected segments in the whale figures is given in the table below:

Whale	Head $\overline{HI}$	Belly $\overline{LK}$	Nose $\overline{JO}$	Tail $\overline{AB}$
Millie	5	10	3	2
Willie	10	20	6	4
Billie	2.5	5	1.5	1

(6.7) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student:

(B) estimates and finds solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

3. The shapes of all members of the whale family are similar. The dimensions of Willie are twice the corresponding dimensions of Millie and 4 times the corresponding dimensions of Billie, as shown in the table above. The ratio of corresponding sides for any two of the whales in this family can be shown to be equivalent.

For example,

Millie's head length : Willie's head length	Millie's belly length : Willie's belly length
5 : 10	10 : 20
Millie's nose length : Willie's nose length	Millie's tail length : Willie's tail length
3 : 6	2 : 4

The ratios 5 : 10, 10 : 20, 3 : 6, and 2 : 4 are all equivalent to 1 : 2. A comparison of the lengths of other corresponding sides of the whale figures for Millie and Willie will also have a ratio of 1 : 2. This ratio of 1 : 2 means that all the side lengths of Millie are half the corresponding side lengths of Willie, or all the side lengths of Willie are twice the corresponding side lengths of Millie. Therefore, the corresponding sides of the two figures (Willie and Millie) are proportional.

The corresponding interior angles have the same measurement. This can be verified by comparing the measures of the angles using tracing paper. Trace one angle of Willie on tracing paper and place the angle over the corresponding angle of Millie. Continue this process of tracing each angle of Willie's figure and comparing with the corresponding angle in Millie's figure. Angle measurement comparisons can also be made using a protractor.

This process can be repeated for angle comparisons of Willie and Billie.

Willie's head length : Billie's head length	Willie's belly length : Billie's belly length
10 : 2.5	20 : 5
Willie's nose length : Billie's nose length	Willie's tail length : Billie's tail length
6 : 1.5	4 : 1

Each of these ratios is equivalent to 4 : 1. The length of a measure on Willie is four times the length of a corresponding measure on Billie.

The table below shows that the ratio of a length for Millie to a corresponding length for Billie is equivalent to 2 : 1.

Millie's head length : Billie's head length	Millie's belly length : Billie's belly length
5 : 2.5	10 : 5
Millie's nose length : Billie's nose length	Millie's tail length : Billie's tail length
3 : 1.5	2 : 1

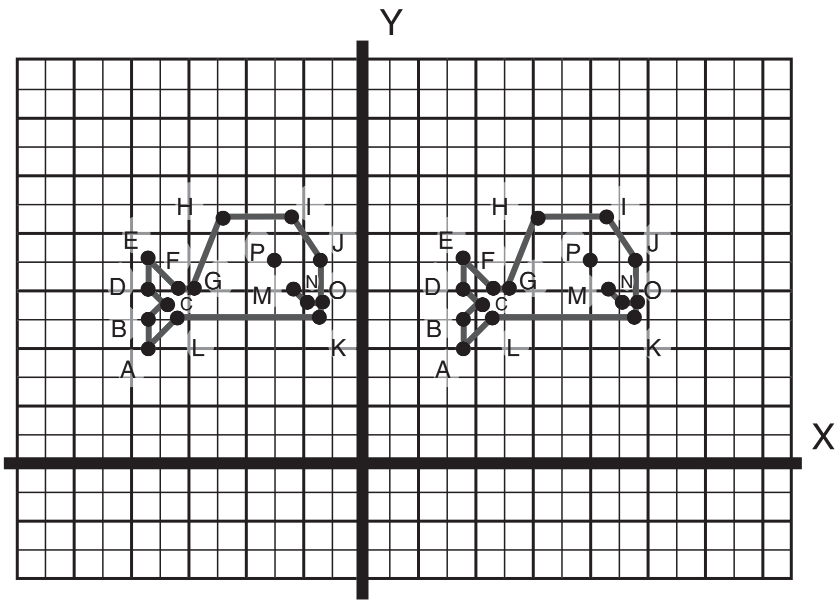
It should also be noted that when all the sides of a given figure are scaled up or down by the same scale factor, the resulting figure is similar to the original. Corresponding angle measurements remain the same, and corresponding sides are in proportion.

4. If the point (-5, 8.5) is translated to the point (6, 8.5), the point has been moved to the right a distance of 6 – (-5), or 11 units. Each vertex will move 11 units to the right. Billie's new coordinates are shown below. In this translation, 11 was added to the *x*-coordinate of the original ordered pairs. For example, point A in the original figure was (-7.5, 4) and became (-7.5 + 11, 4), or (3.5, 4) in the translation. By adding 11 to the *x*-coordinate -7.5, the new *x*-coordinate 3.5 is determined. The *y*-coordinate of the new point remained the same as the corresponding *y*-coordinate of the original point on the figure. A rule for translating Billie's figure 11 units to the right in the coordinate plane can be expressed as follows:

   For every point on Billie's figure with coordinates (x, y), add 11 to the x-coordinate and leave the y-coordinate the same.

$$(x, y) \quad (x + 11, y)$$

		Billie
Point	A	(3.5, 4)
	B	(3.5, 5)
	C	(4, 5.5)
	D	(3.5, 6)
	E	(3.5, 7)
	F	(4.5, 6)
	G	(5, 6)
	H	(6, 8.5)
	I	(8.5, 8.5)
	J	(9.5, 7)
	K	(9.5, 5)
	L	(4.5, 5)
		(connect *L* to *A*)
		(start over)
	M	(8.5, 6)
	N	(9, 5.5)
	O	(9.5, 5.5)
		(start over)
	P	(8, 7)

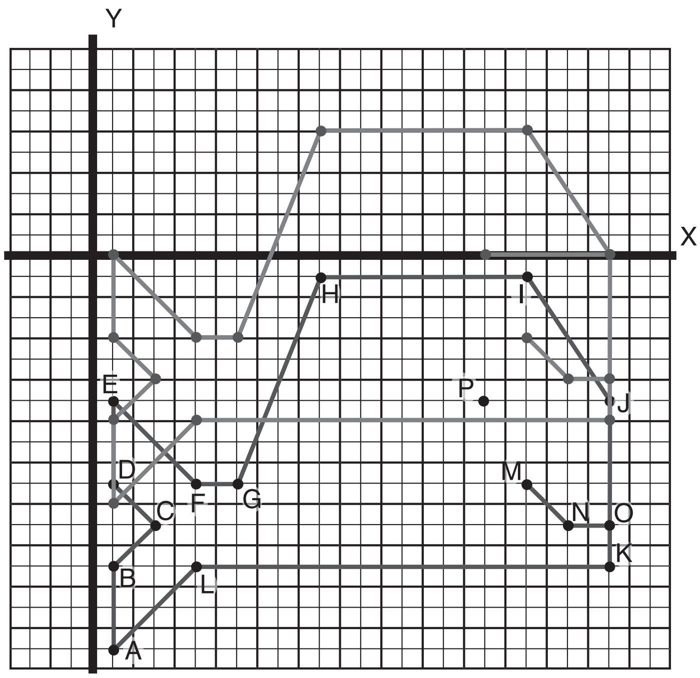


5. If the point (11, -1) is translated to the point (11, 6), the point has been moved up a distance of 6 – (-1), or 7 units. Each vertex will move 7 units up. Willie's new coordinates of the vertices are given in the table below. Each new coordinate is found by adding 7 to the original *y*-coordinate. For example, 7 was added to the *y*-coordinate -13 of point C to get a new *y*-coordinate of -6. This vertical translation did not affect the *x*-coordinate of point C. A rule for translating Willie in the coordinate plane 7 units up can be expressed as follows:

$$(x, y) \longrightarrow (x, y + 7)$$

Each point on Willie's figure with coordinates (x, y) is translated 7 units up by keeping the x-coordinate the same and adding 7 to the y-coordinate.

Point	Willie
A	(1, -12)
B	(1, -8)
C	(3, -6)
D	(1, -4)
E	(1, 0)
F	(5, -4)
G	(7, -4)
H	(11, 6)
I	(21, 6)
J	(25, 0)
K	(25, -8)
L	(5, -8)
	(connect *L* to *A*)
	(start over)
M	(21, -4)
N	(23, -6)
O	(25, -6)
	(start over)
P	(19, -1)



6. Tillie's dimensions are three times the corresponding dimensions of Millie. If Millie's belly is 10 units long, then Tillie's belly is 3 times as long. Tillie's belly is 30 units long.

Extension Questions

- How do the perimeters of the original figures compare?

  Use the edge of a strip of paper (1 inch wide and 20 inches long) to mark off the length of each side of Willie starting at the tail, $\overline{ED}$*, with one end of the strip at point E. Then continue marking off consecutive segments along the edge of the paper in a clockwise direction until all segments that form Willie's outline have been marked off. Label this strip of paper "Willie's Perimeter." Take another strip of paper and repeat this procedure to find the perimeter of Millie. Label this second strip of paper "Millie's Perimeter."*

  Cut off excess paper on both strips so that each strip of paper represents the perimeter for each whale figure. Next, compare the lengths of the two strips of paper. The strip of paper representing Willie's perimeter is twice the length of the strip of paper representing Millie's perimeter. This shows that Willie's perimeter is twice Millie's perimeter. The ratio of Willie's perimeter to Millie's perimeter is 2 : 1, the same as the ratio of any two corresponding sides.

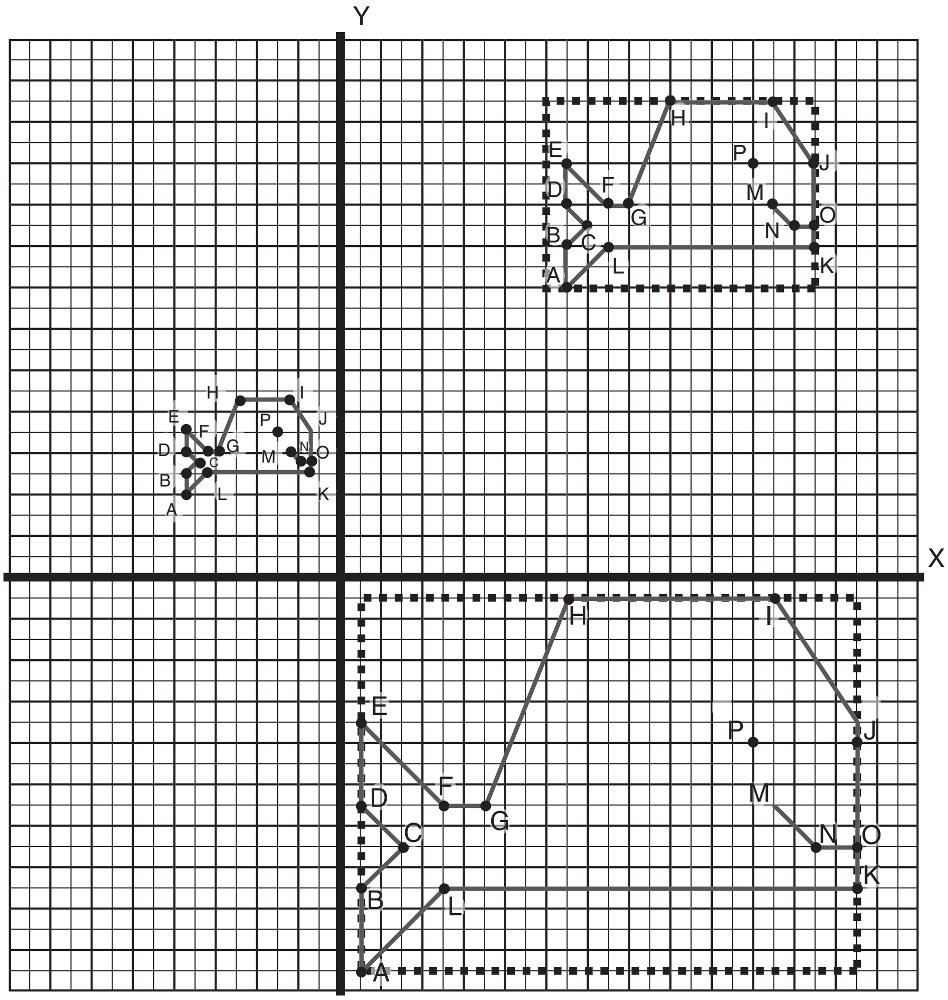
  In a similar way, the perimeter of Willie can be shown to be 4 times the perimeter of Billie; and the perimeter of Billie can be shown to be half the perimeter of Millie. In each situation, the ratio of corresponding perimeters is the same as the ratio of corresponding sides for the similar figures.

- How do the perimeters of the translated figures compare?

  Since a translated figure is congruent to the original figure, side lengths do not change and perimeters remain the same.

- How do the areas of the original figures compare?

  To find the area of Willie and Millie, find the area of the rectangle containing each and subtract the areas of the figures not included in the area of Willie and Millie. For example: Extend $\overline{HI}$*,* $\overline{OJ}$*, and* $\overline{AB}$ *in each of the figures. Draw a horizontal line through point A parallel to line segment HI. The intersections of these lines form a rectangle around each figure.*



Count the number of units to find the length and width of each side of the rectangle containing the figure.

Find the area of each rectangle:

Area of rectangle containing Willie

A = length x width

A = 24 x 18

A = 432 square units

Area of rectangle containing Millie

A = length x width

A = 12 x 9

A = 108 square units

Next, count the squares and partial squares that are not included in the areas of Willie and Millie. Half squares may be combined to estimate a whole square. An estimate of the number of squares not included in the area of Millie is 44 and of Willie is 176. Subtract the area not included from the area of each rectangle above to find the area of each whale figure.

Willie's area: 432 square units – 176 square units = 256 square units

Millie's area: 108 square units – 44 square units = 64 square units

Since 256 square units is 4 times 64 square units, Willie's area is 4 times the area of Millie. In a similar way, it can be shown that the area of Millie is 4 times the area of Billie and the area of Willie is 16 times the area of Billie. In each case, the ratio of the areas of two whale figures is the square of the ratio of two corresponding sides. For example, the ratio of corresponding sides for Willie and Millie is 2 : 1 and the ratio of corresponding areas is 4 : 1. The ratio of corresponding sides for Willie and Billie is 4 : 1 and the ratio of corresponding areas can be shown to be 16 : 1 or $4^2 : 1$.

- How do the areas of the translated figures compare?

  The areas of the translated figures remain the same. This can be verified by tracing the original whale figure on tracing paper and placing it on top of the translated figure to check if the figures will align. The figures are the same size and shape and have the same area.

By the Sea
grade 8

Tourists spot a family of whales swimming off the coast of California. They first spot the mother whale, Millie. Willie, the father whale, is close by. Billie, the baby whale, trails behind.

1. Use the list of coordinates and instructions below to draw Millie on grid paper. Connect points with line segments as you graph. To draw Willie and Billie, use the given rules to find the coordinates of each point. Plot the points to graph Willie and Billie, making sure you connect the points with line segments as you graph.

		Millie	**Willie**	**Billie**
RULE		**(*x*, *y*)**	**(2*x*, 2*y*)**	**($\frac{1}{2}x$, *y*)**
Point	A	(1, 2)		
	B	(1, 4)		
	C	(2, 5)		
	D	(1, 6)		
	E	(1, 8)		
	F	(3, 6)		
	G	(4, 6)		
	H	(6, 11)		
	I	(11, 11)		
	J	(13, 8)		
	K	(13, 4)		
	L	(3, 4)		
		(connect *L* to *A*)	(connect *L* to *A*)	(connect *L* to *A*)
		(start over)	(start over)	(start over)
	M	(11, 6)		
	N	(12, 5)		
	O	(13, 5)		
		(start over)	(start over)	(start over)
	P	(10, 8)		

2. Compare Millie's coordinates with those of Willie and Billie. What is the relationship between the coordinates of Millie and Willie? Of Millie and Billie?

3. Compare and contrast the shapes of the three whales. What do you observe?

4. Billie becomes playful and does flips in the water. Reflect Billie using the *x*-axis as the line of reflection and list the new coordinates. How do these coordinates compare with the original coordinates? Write a rule in words and symbols that reflects a point with coordinates (x, y) about the *x*-axis.

5. Billie continues to play. Now reflect Billie using the *y*-axis as the line of reflection and list the new coordinates. How do these new coordinates compare with the previous coordinates? Write a rule in words and symbols for a reflection of a point with coordinates (x, y) about the *y*-axis.

Materials

Calculator

Half-centimeter grid paper

Tracing paper

Scissors

Ruler

Connections to Middle School TEKS

(8.6) Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. The student is expected to:

(A) generate similar shapes using dilations including enlargements and reductions

(B) graph dilations, reflections, and translations on a coordinate plane

(8.7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

(D) locate and name points on a coordinate plane using ordered pairs of rational numbers

Teacher Notes

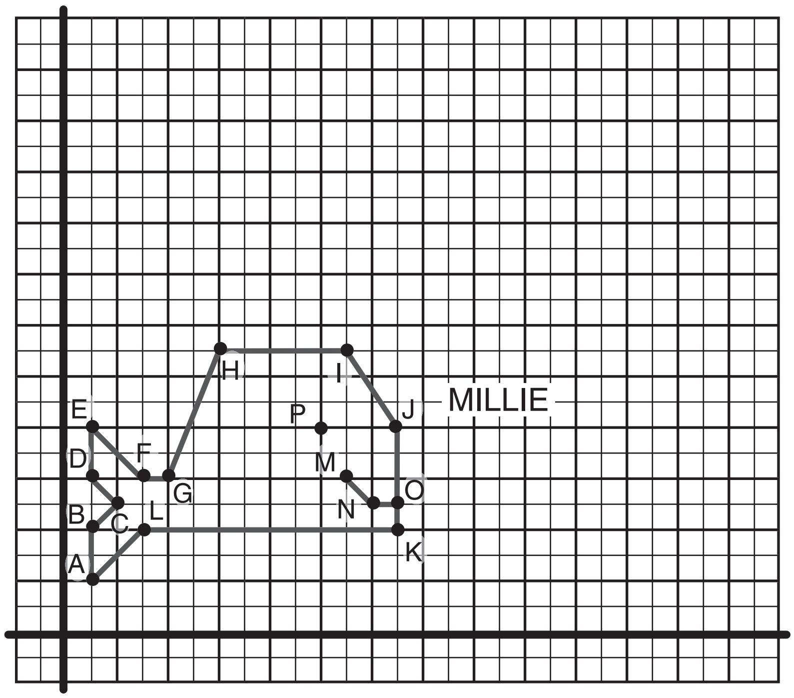
Scaffolding Questions

- How can you use the rule (2*x*, 2*y*) for finding the coordinates for Willie?
- What does the rule for Billie's coordinates tell you to do?
- What does it mean for two figures to be similar?
- How do the side lengths of Millie and Willie compare? How do the corresponding angles of Millie and Willie compare?
- How do the side lengths of Millie and Billie compare? How do the corresponding angles of Millie and Billie compare?
- How do the side lengths of Willie and Billie compare? How do the corresponding angles of Willie and Billie compare?
- How do you reflect a geometric figure about the *x*-axis? The *y*-axis?
- How do the original coordinates and corresponding new coordinates compare in a reflection about the *x*-axis? About the *y*-axis?
- How can you state a rule in words for reflecting a point with coordinates (*x*, *y*) about the *x*-axis? About the *y*-axis?
- How could you write these rules using symbols?

Sample Solutions

1.

	Millie	Willie	Billie
RULE	(x, y)	$(2x, 2y)$	$(\frac{1}{2}x, y)$
Point			
A	(1, 2)	(2, 4)	(.5, 2)
B	(1, 4)	(2, 8)	(.5, 4)
C	(2, 5)	(4, 10)	(1, 5)
D	(1, 6)	(2, 12)	(.5, 6)
E	(1, 8)	(2, 16)	(.5, 8)
F	(3, 6)	(6, 12)	(1.5, 6)
G	(4, 6)	(8, 12)	(2, 6)
H	(6, 11)	(12, 22)	(3, 11)
I	(11, 11)	(22, 22)	(5.5, 11)
J	(13, 8)	(26, 16)	(6.5, 8)
K	(13, 4)	(26, 8)	(6.5, 4)
L	(3, 4)	(6, 8)	(1.5, 4)
	(connect *L* to *A*)	(connect *L* to *A*)	(connect *L* to *A*)
	(start over)	(start over)	(start over)
M	(11, 6)	(22, 12)	(5.5, 6)
N	(12, 5)	(24, 10)	(6, 5)
O	(13, 5)	(26, 10)	(6.5, 5)
	(start over)	(start over)	(start over)
P	(10, 8)	(20, 16)	(5, 8)

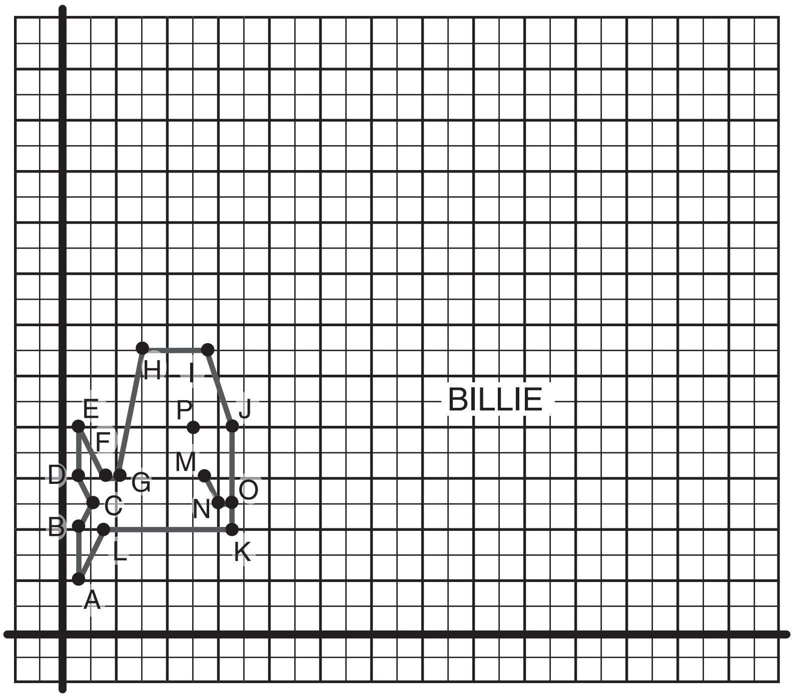


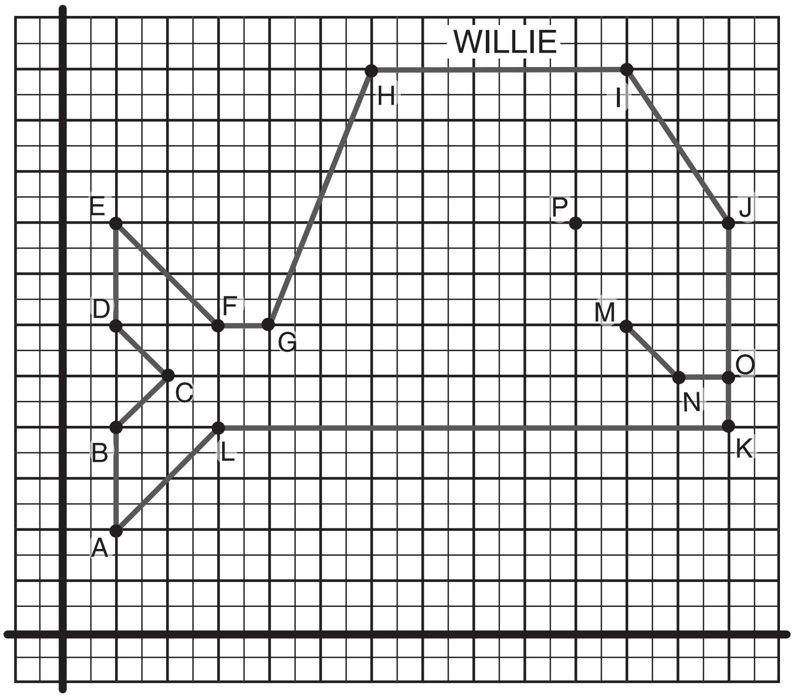
(8.9) Measurement. The student uses indirect measurements to solve problems. The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.





2. Using the completed table in problem 1, students see that the coordinates of the points for Willie are two times the coordinates of corresponding points for Millie. The rule used to create Willie (2*x*, 2*y*) states that each coordinate for Millie (*x*, *y*) is multiplied by 2. For Billie, each *x*-coordinate is half the corresponding *x*-coordinate for Millie. The *y*-coordinate does not change. The rule used to create Billie ($\frac{1}{2}x$, *y*) states that each *x*-coordinate of Millie is multiplied by $\frac{1}{2}$.

3. When comparing the shapes of the whales, measure the lengths of line segments and angles and then examine the relationships of the corresponding measurements to determine if the shapes are similar. To calculate the size of segments that are vertical or horizontal, students can count the number of units to find length. Students can use the Pythagorean Theorem to find the lengths of the slanted segments.

Millie and Willie are similar shapes. Each line segment of Willie is twice the length of the corresponding line segment of Millie, and each corresponding angle is congruent. An example of each type of segment and its corresponding length for Millie and Willie is shown in the table below.

Segment of figure	Millie	Willie
Tail segment AB	Vertical segment 2 units	Vertical segment 4 units
Head segment HI	Horizontal segment 5 units	Horizontal segment 10 units
Forehead segment JI	Slanted segment $2^2 + 3^2 = c^2$ $4 + 9 = c^2$ $13 = c^2$ $\sqrt{13} = c$ $3.6 \approx c$ $c \approx 3.6$ units	Slanted segment $4^2 + 6^2 = c^2$ $16 + 36 = c^2$ $52 = c^2$ $\sqrt{52} = c$ $7.2 \approx c$ $c \approx 7.2$ units

The ratios of corresponding sides of the figures are all 2 units on Willie for every 1 unit on Millie. For example:

$$\frac{\text{length of Willie's tail}}{\text{length of Millie's tail}} = \frac{4 \text{ units}}{2 \text{ units}} = \frac{2 \text{ units}}{1 \text{ unit}}$$

$$\frac{\text{length of Willie's head}}{\text{length of Millie's head}} = \frac{10 \text{ units}}{5 \text{ units}} = \frac{2 \text{ units}}{1 \text{ unit}}$$

$$\frac{\text{length of Willie's forehead}}{\text{length of Millie's forehead}} \approx \frac{7.2 \text{ units}}{3.6 \text{ units}} = \frac{2 \text{ units}}{1 \text{ unit}}$$

The measurements of corresponding angles are equal. This can be verified by using a protractor or tracing paper. For example, $m\angle H = 112°$ for both Willie and Millie; $m\angle A = 45°$ for Willie and Millie; and $m\angle 1 = 124°$ for both whale figures. Those angles having measures greater than 180˚ can be verified using tracing paper. Trace the angle of Millie and align with the corresponding angle of Willie.

Because the sides of Millie have been scaled up using a scale factor of 2 to form Willie's figure and corresponding angles are congruent, Millie and Willie are similar whale figures.

On the other hand, Millie and Billie are not similar whale figures. Each line segment of Billie is not proportional to the corresponding line segment of Millie, and each corresponding angle is not congruent. An example of each type of segment and its corresponding length for Millie and Willie is shown in the table below.

Segment of figure	Millie	Billie
Tail segment AB	Vertical segment 2 units	Vertical segment 2 units
Head segment HI	Horizontal segment 5 units	Horizontal segment 2.5 units
Forehead segment JI	Slanted segment $2^2 + 3^2 = c^2$ $4 + 9 = c^2$ $13 = c^2$ $\sqrt{13} = c$ $3.6 \approx c$ $c \approx 3.6$ units	Slanted segment $2^2 + 1.5^2 = c^2$ $4 + 2.25 = c^2$ $6.25 = c^2$ $\sqrt{6.25} = c$ $2.5 \approx c$ $c \approx 2.5$ units

The ratios of corresponding sides of the figures are not equal. For example:

$$\frac{\text{length of Billie's tail}}{\text{length of Millie's tail}} = \frac{2 \text{ units}}{2 \text{ units}} = \frac{1 \text{ unit}}{1 \text{ unit}}$$

$$\frac{\text{length of Billie's head}}{\text{length of Millie's head}} = \frac{2.5 \text{ units}}{5 \text{ units}} = \frac{0.5 \text{ units}}{1 \text{ unit}}$$

$$\frac{\text{length of Billie's forehead}}{\text{length of Millie's forehead}} \approx \frac{2.5 \text{ units}}{3.6 \text{ units}} = \frac{0.694 \text{ units}}{1 \text{ unit}}$$

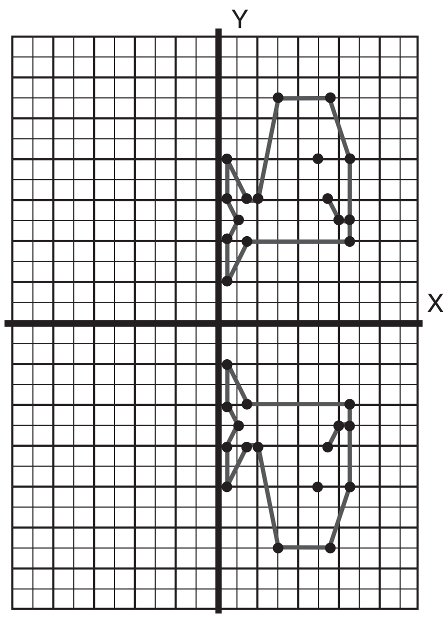
The measures of corresponding angles are not equal. This can be verified by using a protractor or tracing paper. For example, $m\angle H = 112°$ for Millie, while $m\angle H = 105°$ for Billie. Likewise, Willie and Billie are not similar whale figures.

4. The new coordinates for Billie when reflected about the *x*-axis are given in the table below. By comparing these new coordinates to the corresponding coordinates of the original figure, the *x*-coordinates are the same. However, the new *y*-coordinates are the opposite of the corresponding original *y*-coordinates.

Rule in words: A reflection of a point with coordinates (*x*, *y*) about the *x*-axis produces a point with the same *x*-coordinate and a *y*-coordinate that is the opposite of the original *y*-coordinate.

Rule in symbols: $(x, y) \rightarrow (x, -y)$

		Billie
Point	A	(.5, -2)
	B	(.5, -4)
	C	(1, -5)
	D	(.5, -6)
	E	(.5, -8)
	F	(1.5, -6)
	G	(2, -6)
	H	(3, -11)
	I	(5.5, -11)
	J	(6.5, -8)
	K	(6.5, -4)
	L	(1.5, -4)
		(connect *L* to *A*)
		(start over)
	M	(5.5, -6)
	N	(6, -5)
	O	(6.5, -5)
		(start over)
	P	(5, -8)

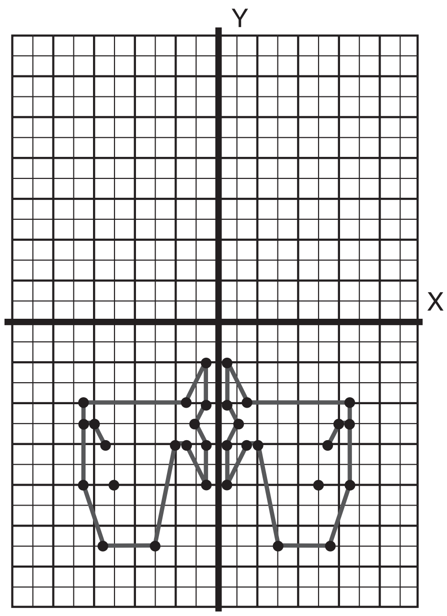


5. The coordinates of the points in the table below are the new coordinates after Billie was reflected about the *y*-axis from his position in problem 4. The *y*-coordinates stayed the same, but the new *x*-coordinates are the opposite of the corresponding *x*-coordinates in the previous table.

Rule in words: A reflection of a point with coordinates (*x*, *y*) about the *y*-axis results in a point with coordinates (opposite of the *x*-coordinate, same *y* coordinate),

Rule in symbols: $(x, y) \rightarrow (-x, y)$

		Billie
Point	A	(-.5, -2)
	B	(-.5, -4)
	C	(-1, -5)
	D	(-.5, -6)
	E	(-.5, -8)
	F	(-1.5, -6)
	G	(-2, -6)
	H	(-3, -11)
	I	(-5.5, -11)
	J	(-6.5, -8)
	K	(-6.5, -4)
	L	(-1.5, -4)
		(connect *L* to *A*)
		(start over)
	M	(-5.5, -6)
	N	(-6, -5)
	O	(-6.5, -5)
		(start over)
	P	(-5, -8)



Extension Questions

- A few years have passed and Billie has grown. A dilation is a transformation that moves a figure and changes its size to create a similar figure. Dilate Billie in the coordinate plane so that his new dimensions are $4\frac{1}{2}$ times the original dimensions. List the coordinates for each vertex on Billie's figure. Explain why the enlarged figure is a dilation.

To dilate Billie to his new size, multiply each number in the ordered pair (x, y) by $4\frac{1}{2}$. The results are listed in a table below.

		Billie
Point	A	(2.25, 9)
	B	(2.25, 18)
	C	(4.5, 22.5)
	D	(2.25, 27)
	E	(2.25, 36)
	F	(6.75, 27)
	G	(9, 27)
	H	(13.5, 49.5)
	I	(24.75, 49.5)
	J	(29.25, 36)
	K	(29.25, 18)
	L	(6.75, 18)
		(connect *L* to *A*)
		(start over)
	M	(24.75, 27)
	N	(27, 22.5)
	O	(29.25, 22.5)
		(start over)
	P	(22.5, 36)

The enlarged figure is a dilation because it meets the following criteria:

- *The transformation has preserved similarity. This can be verified by comparing the lengths of corresponding sides and corresponding angles. Corresponding sides (enlarged to original) have a ratio of 4.5 : 1 and corresponding angles are congruent.*
- *Corresponding sides are parallel.*

- *Since this transformation is not a translation, there is a center of dilation. In this case, the center of dilation is at (0, 0). This can be determined by drawing lines through corresponding vertices. These lines all intersect at the center of dilation.*
- *Each side has been enlarged by a scale factor of* $4\frac{1}{2}$ *.*
- *Orientation of the figures is the same.*

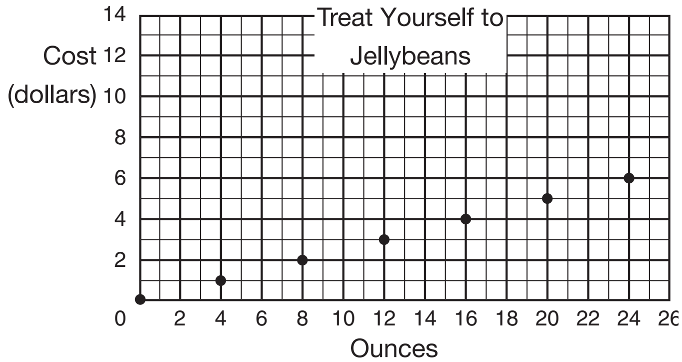
- Describe how this transformation has affected the perimeter and area of the original figure.

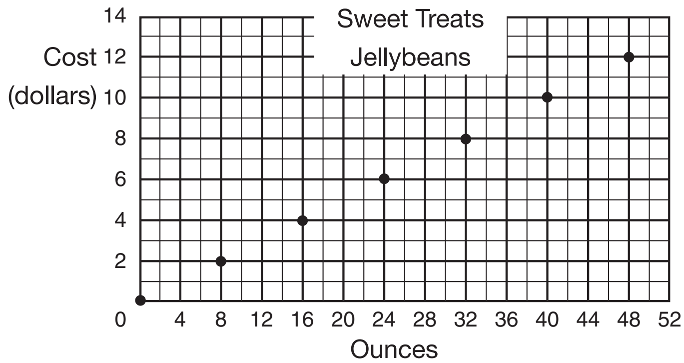
  Perimeter: Since Billie's figure has been scaled up by a factor of 4.5, the enlarged figure is similar to the original figure, and the ratio of corresponding sides (enlarged to original) is 4.5 : 1. This ratio is the same as the ratio of corresponding perimeters. Therefore, the ratio of corresponding perimeters (enlarged figure to original figure) is 4.5 : 1.

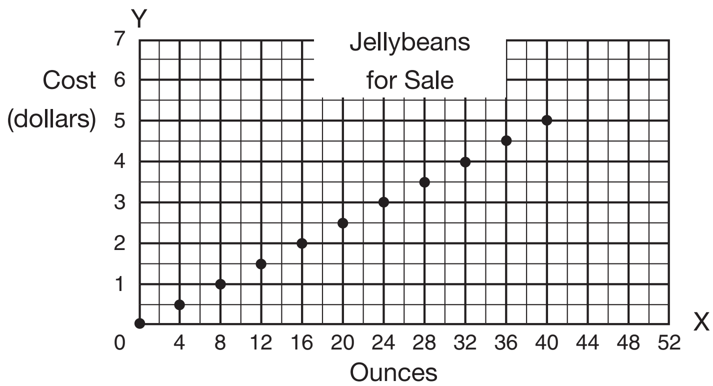
  Area: Since the enlarged figure formed by the dilation is similar to the original figure, the ratio of the areas (enlarged to original) is the square of the ratio of corresponding sides. The ratio of the area of the enlarged figure to the area of the original figure is $(4.5)^2 : 1^2$ *or 20.5 : 1.*

Sweet Trip to the Candy Shop
grade 6

Jellybeans at Deet's Sweet Treats candy shop are priced at $2 for 8 ounces and are sold in 4-ounce and 8-ounce packages. The manager at the candy shop decides to display a graph showing the cost of jellybeans. He asked three of the employees to each make a graph. Below are the graphs each created.







1. What is being shown to the customer in each graph?
2. The manager wants to display the best graph for his customers. Make recommendations about each graph.
3. To help clerks sell jellybeans, create a price guide for up to 4 pounds and justify your reasoning.
4. Using 4-ounce and 8-ounce packages, how many different ways can you purchase 4 pounds of jellybeans? Explain.

Teacher Notes

Materials

Calculator

Straight edge

Connections to Middle School TEKS

(6.7) Geometry and spatial reasoning. The student uses coordinate geometry to identify location in two dimensions.

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(B) evaluate the effectiveness of different representations to communicate ideas

Scaffolding Questions

- What is the scale of the horizontal axis shown on each graph?
- What will the point (16, 4) mean on the Sweet Treats Jellybeans graph?
- What does each graph show about the cost of jellybeans as the number of ounces increases?
- What is the cost of 16 ounces of jellybeans on each graph? 24 ounces? 48 ounces?
- How many ounces of jellybeans will a customer be able to purchase with $2? $4? $5?
- What is the relationship between the number of ounces of jellybeans and the cost in dollars?
- If a customer purchases 24 ounces of jellybeans, how can you find the number of 4-ounce packages he or she can buy? How many 8-ounce packages? How many combinations of 4-ounce and 8-ounce packages?

Sample Solutions

1. Treat Yourself to Jellybeans. This graph shows the following values:

Ounces	**Cost** (dollars)
0	0
4	1
8	2
12	3
16	4
20	5
24	6

The rate is $1 for 4 ounces of jellybeans. The graph shows the cost of 4 ounces to 24 ounces of jellybeans for multiple 4-ounce packages as well as for multiple 8-ounce packages.

Sweet Treats Jellybeans. The information in the graph can be written in table form.

Ounces	**Cost** (dollars)
0	0
8	2
16	4
24	6
32	8
40	10
48	12

This table shows that for every 8 ounces of jellybeans, the cost is $2. The graph shows the cost of 8 ounces to 48 ounces of jellybeans for multiple packages of 8 ounces.

Jellybeans for Sale. The points on the graph result in the following table: This graph shows that for every 4 ounces of jellybeans, the cost is $0.50. The graph shows

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

the cost of 4 ounces to 48 ounces of jellybeans for multiple packages of 4 ounces and 8 ounces.

Ounces	**Cost** (dollars)
0	0
4	.50
8	1
12	1.50
16	20
20	2.50
24	3
28	3.50
32	4
36	4.50

This graph and table show that for every 4 ounces of jellybeans, the cost is $0.50. This is not the correct price given in the problem. The graph shows the cost of 4 ounces to 48 ounces of jellybeans for multiple packages of 4 ounces and 8 ounces.

2. Treat Yourself to Jellybeans. This graph would be a good choice if customers bought small quantities of jellybeans of either 4- or 8-ounce packages.

   Sweet Treats Jellybeans. This graph would be a good choice if most customers bought jellybeans in 8-ounce packages. It does not show that the jellybeans are available in 4-ounce packages. This graph also shows up to 48 ounces of jellybeans.

   Jellybeans for Sale. This graph is not a good choice. It shows 8 ounces of jellybeans for $1. The candy shop sells jellybeans at 8 ounces for $2. This graph shows incorrect information.

3. The following table can be used to create a price guide to help clerks sell up to 4 pounds of jellybeans. The table should show 64 ounces, since

$$16 \text{ oz} = 1 \text{ lb}$$

$$4 \times 16 \text{ oz} = 4 \times 1 \text{ lb}$$

$$64 \text{ oz} = 4 \text{ lb}$$

Jellybean price guide

Ounces	**Cost** (dollars)
4	1
8	2
12	3
16	4
20	5
24	6
28	7
32	8
36	9
40	10
44	11
48	12
52	13
56	14
60	15
64	16

The price guide shows that for every 4-ounce package of jellybeans, the cost is $1. To find the cost of any number of 4-ounce packages, multiply the number of packages by $1. It also shows that for every 8-ounce package of jellybeans the cost is $2. To find the cost of any number of 8-ounce packages, multiply the number of packages by $2.

4. The different options for purchasing 4 pounds or 64 ounces of jellybeans are given in the table below.

Number of 4 oz pkg.	Number of 8 oz pkg.	Process— Total ounces	Total ounces
0	8	0 (4 oz) + 8 (8 oz)	64
2	7	2 (4 oz) + 7 (8 oz)	64
4	6	4 (4 oz) + 6 (8 oz)	64
6	5	6 (4 oz) + 5 (8 oz)	64
8	4	8 (4 oz) + 4 (8 oz)	64
10	3	10 (4 oz) + 3 (8 oz)	64
12	2	12 (4 oz) + 2 (8 oz)	64
14	1	14 (4 oz) + 1 (8 oz)	64
16	0	16 (4 oz) + 0 (8 oz)	64

There are 16 groups of 4 ounces in 64 ounces. Four-ounce packages must be purchased in multiples of 2. It is not possible to purchase an odd number of 4-ounce packages and still make 8-ounce packages.

Extension Questions

- How much is the cost of one ounce of jellybeans at the same rate?

  If Deet's Sweet Treats sells 8 ounces of jellybeans for $2, then the cost of one ounce of jellybeans is $\frac{\$2.00}{8 \text{ ounces}} = \frac{\$0.25}{1 \text{ ounce}}$.

- What would a customer pay for a quarter-pound of jellybeans?

  One pound of jellybeans is the same as 16 ounces of jellybeans, and $\frac{1}{4}$ *of 16 ounces is 4 ounces. Since 16 ounces of jellybeans is $4 and* $\frac{1}{4}$ *of $4 is $1, the cost of one quarter-pound (4 ounces) of jellybeans is $1.*

- Find the cost of 96 ounces of jellybeans.

  The unit rate is $0.25 per ounce.

$$\frac{\$0.25}{1 \text{ ounce}} = \frac{\$0.25 \times 96}{1 \text{ ounce} \times 96} = \frac{\$24}{96 \text{ ounces}}$$

  The cost of 96 ounces is $24.

The Round Table
grade 6

Leo has been asked to design a circular table using $\frac{3}{4}$-inch plywood so that 12 chairs can be placed around the table, with at least 8 inches between chairs. Each chair uses 16 inches of the edge of the table.

1. Will this table fit reasonably in a dining room measuring 12 feet by 14 feet? Explain.

2. Leo is shipping the tabletop. He has added hinges so that the tabletop folds in half. The legs of the table are removable and will be sent separately. Can the tabletop be shipped in a rectangular box measuring 8 feet by 4 feet by 3 inches? Explain your reasoning.

3. Leo decides to make a smaller table. Each chair still uses 16 inches of the edge of the table, and he still wants 8 inches between chairs. He would like the new table to seat 8, that is, $\frac{2}{3}$ the number of people the original table sat. Can Leo take $\frac{2}{3}$ of each measurement of the table to get the size of the new table? Why or why not?

Teacher Notes

Materials

Calculator

Compass

Ruler

String

Connections to Middle School TEKS

(6.6) Geometry and spatial reasoning. The student uses geometric vocabulary to describe angle, polygons, and circles. The student is expected to:

> (C) describes the relationship between radius, diameter, and circumference of a circle

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

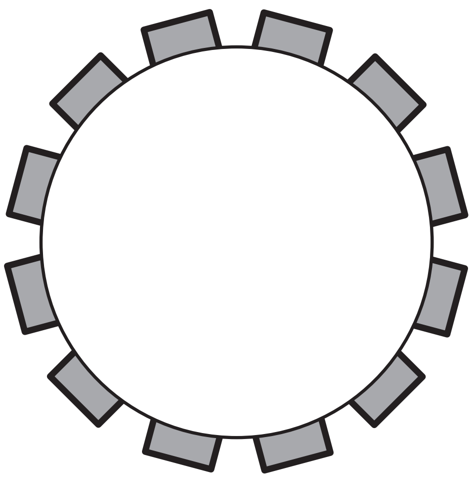
> (A) uses ratios to describe proportional situations
>
> (C) uses ratios to make predictions in proportional situations

Scaffolding Questions

- How would you find the circumference of the table?
- Would it be necessary to find the distance around the entire table?
- What dimensions of the tabletop would he have to consider for the box?
- What are the measures of these dimensions?
- What is the relationship between the circumference and diameter of a circle?
- If you know the circumference, how can you find the diameter?
- What is the relationship between the radius and diameter?
- How can you find the circumference of a circle if you know its diameter?
- How can you find the circumference of a circle if you know its radius?

Sample Solutions

1. Each chair uses 16 inches of space around the table, and each chair will be spaced 8 inches apart.



There will be a total of 12 chairs around the table.

$$12 \times 16 = 192 \text{ inches}$$

There will be a total of 12 spaces between each chair.

$$12 \times 8 = 96 \text{ inches}$$

The circumference of the table is 288 inches. This is the sum of the chairs and the spacing that separates each chair.

$$192 + 96 = 288 \text{ inches}$$

Since C = dπ, then $d = \frac{C}{\pi}$. So, by substitution, $d = \frac{288}{\pi}$ which means $d \approx 91.67$. The diameter of the table Leo will design is approximately 91.67 inches. This means that the distance across the table is approximately 91.67 inches. This distance converted to feet, using 1 foot = 12 inches, is approximately 7.64 feet.

$$91.67 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \approx 7.64 \text{ feet}$$

The table should fit in a dining room measuring 12 feet by 14 feet. The fit will be a little tight.

2. Leo's tabletop has a diameter of 7.64 feet. If the diameter is 7.64 feet, then the radius of the tabletop is half of 7.64 feet. The radius of the tabletop is 3.82 feet. The length of the diameter, 7.64 feet, is the length of the box that Leo will need. The length of the radius, 3.82 feet, is the width of the box. Since the tabletop is made of $\frac{3}{4}$-inch plywood and the table is folded over, the height of the box would need to be at least twice the width of the plywood, or $1\frac{1}{2}$ inches. So yes, Leo could use a rectangular box measuring 8 feet by 4 feet by 3 inches.

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

3. If there is a total of 8 chairs, then Leo will need 8 x 16, or 128 inches of space for the chairs and another 8 x 8, or 64 inches of space between each chair, for a total of 192 inches for the circumference of the table. If Leo finds $\frac{2}{3}$ of the original measure of the circumference, he gets $\frac{2}{3}$ x 288 inches = 192 inches. If the circumference is 192 inches, then the diameter is $\frac{192}{\pi} \approx 61.11\text{ inches}$.

$$61.1\text{ inches} \times \frac{1\text{ foot}}{12\text{ inches}} \approx 5.1\text{ feet}$$

The diameter is approximately 5.1 feet.

If Leo finds $\frac{2}{3}$ of the original measure of the diameter, he gets $\frac{2}{3} \times 7.6\text{ feet} \approx 5.1\text{ feet}$.

If he finds $\frac{2}{3}$ of the original measure of the radius, he gets $\frac{2}{3} \times 3.8\text{ feet} \approx 2.5\text{ feet}$. If the new diameter is 5.1 feet, then the radius is $\frac{1}{2}$ of that, which is approximately 2.5 feet. So Leo could take $\frac{2}{3}$ of the original measurement to find the new dimensions—with the exception of thickness, which remains $\frac{3}{4}$ inch.

Extension Questions

- If the diameter of Leo's table is doubled in length, how is the circumference affected? How does this affect the area of the tabletop surface?

  The diameter of Leo's table is 91.67 inches. If this length is doubled, then it will be 183.34 inches. Circumference is calculated by multiplying the diameter and π. The circumference of the table is 183.34 multiplied by π, which is approximately 576 inches. Doubling the length of the diameter would result in a circumference doubled in size. The area of the original table is approximately 6,600 square inches. The area of the new table is approximately 26,400 square inches. The new area is about 4 times the original area, so the area grows by 4 times.

- If the diameter of Leo's table is halved, how many chairs can be placed around the table?

  Half of the length of the diameter is approximately 45.8 inches. The distance around the table would be calculated by multiplying the diameter and π.

  $$45.8 \times \pi \approx 143.88$$

  The circumference has decreased by half of the original length. The number of chairs placed around the table would also decrease by half. To verify this, 6 x 16 = 96 *and* 8 x 6 = 48*. The distance around the table would be the sum of these lengths,* 96 + 48 = 144 inches*. This distance is half of the original length of 288 inches. Therefore, if the diameter of Leo's table is half the length, then the number of chairs around it would be 6, and each chair would be 8 inches apart.*

Sorting Rectangles
grade 7

The dimensions of seven rectangles are given below.

Rectangle	Width	Length
A	16	28
B	30	36
C	2	3.5
D	8	12
E	2.5	3
F	40	60
G	20	35

1. Sort the rectangles into groups of similar rectangles. Identify which rectangles are in each group.
2. Explain how you decided which rectangles were similar.

Teacher Notes

Materials

Calculator

Construction paper

Grid paper

Ruler

Scissors

Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving proportional relationships such

Scaffolding Questions

- How do you determine whether two rectangles are similar?
- How do corresponding angles of rectangles compare?
- How do corresponding sides of rectangles compare?
- For each rectangle, what is the ratio of width to length?
- How can you use the ratio of width to length to help you sort the rectangles into groups of similar rectangles?

Sample Solutions

1. Group 1: Rectangles A, C, and G

   Group 2: Rectangles B and E

   Group 3: Rectangle D and F

2. For each rectangle, the ratio of width to length can be compared as follows.

Rectangle A: $\frac{16}{28} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{4}{7}$

Rectangle B: $\frac{30}{36} \times \frac{\frac{1}{6}}{\frac{1}{6}} = \frac{5}{6}$

Rectangle C: $\frac{2}{3.5} \times \frac{2}{2} = \frac{4}{7}$

Rectangle D: $\frac{8}{12} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{2}{3}$

Rectangle E: $\frac{2.5}{3} \times \frac{2}{2} = \frac{5}{6}$

Rectangle F: $\frac{40}{60} \times \frac{\frac{1}{20}}{\frac{1}{20}} = \frac{2}{3}$

Rectangle G: $\frac{20}{35} \times \frac{\frac{1}{5}}{\frac{1}{5}} = \frac{4}{7}$

Rectangles A, C, and G have a width-to-length ratio of 4 : 7, rectangles B and E have a width-to-length ratio of 5 : 6, and rectangles D and F have a width-to-length ratio of 2 : 3.

Rectangles A, C, and G are similar because corresponding angles are congruent right angles and they have the same ratio $w : l = 4 : 7$. Rectangles B and E are also similar since corresponding angles are right angles and the ratio $w : l = 5 : 6$. Rectangles D and F can be shown to be similar in the same manner.

In a discussion of similar rectangles, it is important to distinguish between the shape ratio $w : l$ and the size ratio, or scale factor, determined by $l_1 : l_2$ or $w_1 : w_2$. For example, the scale factor from rectangle B to rectangle E is $l_1 : l_2 = 3 : 36$ or $w_1 : w_2 = 2.5 : 30$.

$$l_1 : l_2 = w_1 : w_2 = 1 : 12$$

The scale factor is less than 1 because there is a reduction from rectangle B to rectangle E. The ratio 1 : 12 means that the lengths of the sides of rectangle E are one-twelfth the lengths of the corresponding sides of rectangle B. This size ratio also shows that the lengths of the sides of rectangle B are 12 times the lengths of the corresponding sides of rectangle E.

as similarity, scaling, unit costs, and related measurement units

(7.6) Geometry and spatial reasoning. The student compares and classifies shapes and solids using geometric vocabulary and properties. The student is expected to:

(D) use critical attributes to define similarity

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

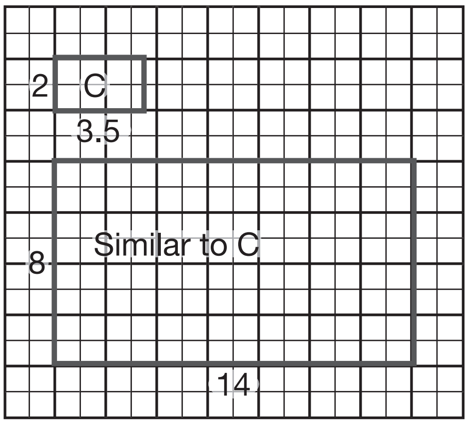
Extension Questions

- Would a rectangle with a width of 18 and a length of 21 belong to any of the groups? Why or why not?

  If the width is compared with the length of this rectangle, the result would be
  $\frac{18}{21} \times \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{6}{7}$*. This shape ratio is not equal to any shape ratio of the groups. Therefore, the given rectangle would not belong to any of the groups.*

- Rectangle C is shown on the grid below. Draw a new rectangle that is similar to rectangle C.

  A possible rectangle can be drawn by using a scale factor of 4. The width will be 4 x 2 units, or 8 units, and the length will be 4 x 3.5 units, or 14 units.



Mighty Mascot!
grade 7

The spirit club for the Mighty Bears projected a picture of the school's mascot on a wall to make a poster for the gym. The original picture was 4 inches wide and 6 inches high.

1. If the wall is 6 feet wide and 8 feet high, what is the largest possible dimension of the projection that will show a complete picture of the mascot? Describe how you solved this problem.
2. What scale factor was used to make the new image? Explain.
3. Would it be possible to use a scale factor of 18? Why or why not?

Teacher Notes

Materials

Calculator

Grid paper

Straight edge

Ruler

Connections to Middle School TEKS

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to applications problems involving proportional relationships such as similarity, scaling, unit costs and related measurement units

(7.6) Geometry and spatial reasoning. The student compares and classifies shapes and solids using geometric vocabulary and properties. The student is expected to:

(D) use critical attributes to define similarity

Scaffolding Questions

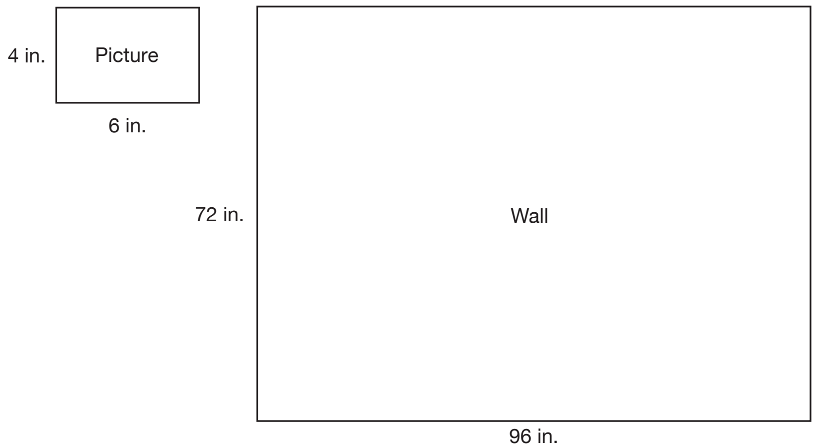
- How do the original picture of the mascot and the projected image compare?
- How can you convert feet to inches?
- What happens to the attributes of a figure when it is changed by a scale factor greater than 1? Less than 1?
- How do the ratios of the corresponding sides compare when a figure is enlarged by a scale factor of 2? 10? 18?

Sample Solutions

1. The dimensions of the wall can be converted to inches. There are 12 inches in 1 foot.

$$6 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 72 \text{ inches}$$

$$8 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 96 \text{ inches}$$



To find the largest dimensions of the picture that can fit into the space on the wall, students can build a table and compare data.

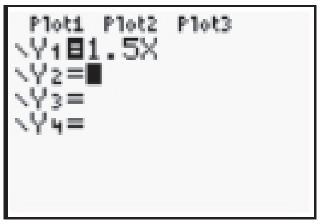
For example:

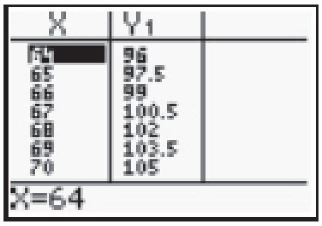
Scale factor	Width in inches	Length in inches
1	4 x 1 = 4	6 x 1 = 6
2	4 x 2 = 8	6 x 2 = 12
3	4 x 3 = 12	6 x 3= 18
10	4 x 10 = 40	6 x 10 = 60
15	4 x 15 = 60	6 x 15 = 90
16	4 x 16 = 64	6 x 16 = 96
17	4 x 17 = 68	6 x 17 = 102
18	4 x 18 = 72	6 x 18 = 108

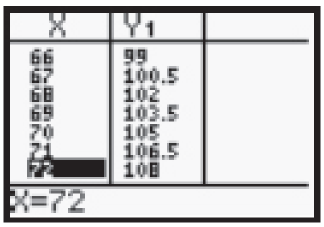
The largest possible dimension of the projection will be 64 inches wide and 96 inches high.

Another solution strategy:

Using the previous table, the ratio *l* : *w* is 6 : 4. Let *y* represent the length in inches and *x*, the width in inches. Write this ratio as *y* : *x* = 3 : 2 or $\frac{y}{x} = \frac{3}{2}$. The equation $y = \frac{3}{2}x$ or *y* = 1.5*x* can be entered into a graphing calculator, and the table feature can be used to find dimensions that will fit on the wall 72 inches wide and 96 inches high.







The maximum length is 96 inches. When *x* = 64, *y* = 96. Therefore, an enlargement 64 inches wide and 96 inches high will fit on the wall.

(7.9) Measurement. The student solves application problems involving estimation and measurement. The student is expected to estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

2. The scale factor used to create the new image is 16. Refer to the table in problem 1.

$$4 \times 16 = 64 \text{ and } 6 \times 16 = 96$$

Since this is an enlargement, the scale factor is greater than 1. Multiplying the original dimensions by 16 gives the dimensions of 64 inches by 96 inches.

Another way to find the scale factor that produced this enlargement is to find the ratio of corresponding widths $w_1 : w_2 = 64 : 4 = 16 : 1$ or the ratio of corresponding lengths $l_1 : l_2 = 96 : 6 = 16 : 1$.

3. It would not be possible to use a scale factor of 18 because multiplying the dimensions of the picture by the scale factor 18 would result in an enlargement that is 72 inches by 108 inches.

$$4 \text{ inches} \times 18 = 72 \text{ inches}$$

$$6 \text{ inches} \times 18 = 108 \text{ inches}$$

The enlargement would be too tall to fit on the wall that is 72 inches wide and 96 inches high.

Extension Questions

- What size surface would be needed to project the picture of the mascot using a scale factor of 20?

  To find the size of the surface, you would multiply the dimensions of the picture by 20.

  $$4 \text{ inches} \times 20 = 80 \text{ inches}$$

  $$6 \text{ inches} \times 20 = 120 \text{ inches}$$

  The new dimensions could be converted to feet.

  $$80 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \approx 6.667 \text{ feet}$$
  $$120 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 10 \text{ feet}$$

  The size of the surface should be about 7 feet by 10 feet, rounding to the nearest foot.

- A projection of a picture on a wall measures 48 inches wide by 72 inches high. The original picture's dimensions have been increased by $33\frac{1}{3}\%$ in this projected image. What were the original dimensions? Explain your reasoning.

If the dimension has been increased by $33\frac{1}{3}\%$*, then the new image is* $100\% + 33\frac{1}{3}\%$ or $133\frac{1}{3}\%$ *of the original image.*

$$133\frac{1}{3}\% = \frac{133\frac{1}{3}}{100} = \frac{133\frac{1}{3}}{100} \times \frac{3}{3} = \frac{4}{3}$$

The dimensions of the projection on the wall are $1\frac{1}{3}$ or $\frac{4}{3}$ *times the original dimensions of the picture. This means that 48 inches is* $\frac{4}{3}$ *times the width (w) of the original picture and 72 inches is* $\frac{4}{3}$ *times the length (l) of the original picture. The following equations can be written and solved to determine the original dimensions.*

$$48 = \frac{4}{3}w \qquad 72 = \frac{4}{3}l$$

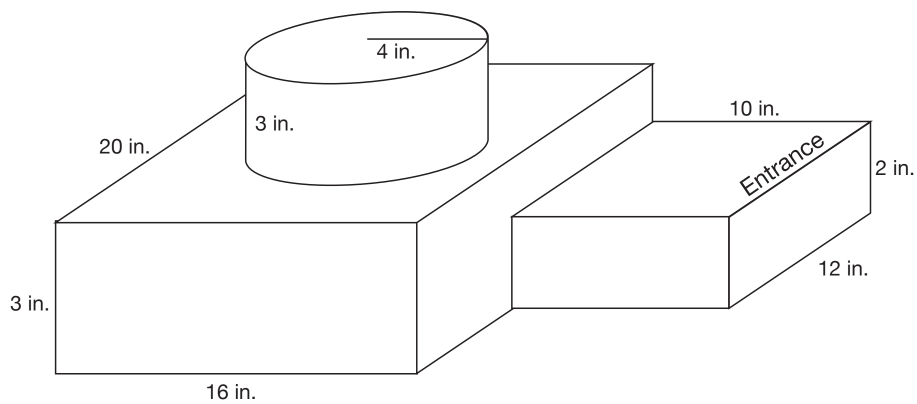
$$\frac{3}{4} \times 48 = \frac{3}{4} \times \frac{4}{3}w \qquad \frac{3}{4} \times 72 = \frac{3}{4} \times \frac{4}{3}l$$

$$36 = w \qquad 54 = l$$

The dimensions of the original picture are width = 36 inches and length = 54 inches.

Javier Builds a Model
grade 8

Javier, an architect, builds a scale model of the new faculty center at the university. A diagram of the scale model is shown below.



1. In the scale model, the entrance is 12 inches wide. If the entrance is actually 60 feet wide, what scale did Javier use to create the model? Explain your answer.
2. Show how to find the actual measurements of the new faculty center at the university. Redraw the model, labeling its actual dimensions.

Teacher Notes

Materials

Calculator

Ruler

Cubes

Connections to Middle School TEKS

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(8.7) Geometry and spatial reasoning. The student

Scaffolding Questions

- What does it mean for two figures to be similar?
- How does the actual measure of the entrance compare with that of the model?
- What does a scale factor tell you about the dimensions of two similar figures?
- How can you find a scale factor from the scale model to the actual building? From the actual building to the scale model?
- How can you find the dimensions of the actual figure if you know the scale factor used to make the model?
- How does the ratio of the corresponding areas of two similar figures compare to the ratio of their corresponding sides?

Sample Solutions

1. To calculate the scale used, begin with the relationship between the number of inches in the scale drawing and the number of feet in the actual building.

$$\frac{\text{60 feet in the actual building}}{\text{12 inches in the scale model}} = \frac{\text{5 feet in the actual building}}{\text{1 inch in the scale model}}$$

Therefore, Javier used a scale of 1 inch = 5 feet.

2. To convert from the dimensions in the scale model to the dimensions of the actual building, the rate $\frac{\text{5 feet in the actual building}}{\text{1 inch in the scale model}}$ may be used. For example, convert 2 inches to the actual size.

$$\frac{\text{5 feet in the actual building}}{\text{1 inch in the scale model}} \times \text{2 inches in the scale model} = \text{10 feet in the actual building}$$

If the rate is $\frac{\text{5 feet in the actual building}}{\text{1 inch in the scale model}}$, the rule that defines the relationship is $y = 5x$, where y represents the number of feet in the actual building and x, the number of inches in the scale model. The equation $y = 5x$ states that the number of feet in the actual building is 5 times the number of inches in the scale drawing.

This equation, $y = 5x$, can be used to find the actual dimensions of the new faculty center.

Large rectangular prism: Length: $y = (5)(20) = 100$ ft

Width: $y = (5)(16) = 80$ ft

Height: $y = (5)(3) = 15$ ft

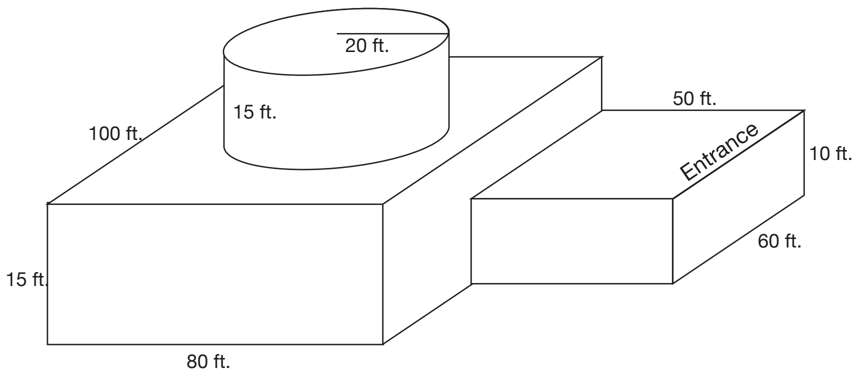
Cylinder: Radius: $y = (5)(4) = 20$ ft

Height: $y = (5)(3) = 15$ ft

Small rectangular prism: Length: $y = (5)(10) = 50$ ft

Width: $y = (5)(12) = 60$ ft

Height: $y = (5)(2) = 10$ ft



uses geometry to model and describe the physical world. The student is expected to:

(B) use geometric concepts and properties to solve problems in fields such as art and architecture

(8.8) Measurement. The student uses procedures to determine measures of solids. The student is expected to:

(C) estimate answers and use formulas to solve application problems involving surface area and volume

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

Extension Questions

- The university is purchasing material for a roof of the building. How much area will be covered?

  The area of the roof of the building is the area of the top of the large rectangular prism and the top of the small rectangular prism. The roof of the large rectangular prism consists of a part of the rectangle that is around the circular portion of the cylinder and a circular portion (base of cylinder). Since the bases of a cylinder are congruent circles, the top of the cylinder has the same area as the bottom. Therefore, the area of the roof of the large rectangular prism is the area of a rectangle with dimensions 100 feet by 80 feet. The area of the smaller section is 50 feet by 60 feet. The total roof area is the sum of these two areas.

  $$\text{Area} = (100)(80) + (50)(60)$$

  $$\text{Area} = 8{,}000 + 3{,}000 = 11{,}000 \text{ square feet}$$

- How much material did Javier use to make the roof of the model?

  The area of the roof of the model is the area of the top of the large rectangular prism and the top of the small rectangular prism.

  $$\text{Area} = (20)(16) + (10)(12)$$

  $$\text{Area} = 320 + 120 = 440 \text{ square inches.}$$

- How does the area of the roof of the model compare with the area of the roof of the actual building?

  A comparison of the areas of the two roofs requires a conversion of feet to inches or inches to feet. Unit conversion can be used to find the number of square feet in 440 square inches.

  $$440 \text{ in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} \approx 3.055 \text{ ft}^2$$

  The ratio of the number of square feet in the roof of the model to the number of square feet in the roof of the actual building is 3.055 : 11,000 or 1 : 3,600. The *ratio of corresponding dimensions (scale model : actual building) is 12 inches : 60 feet or 1 ft : 60 ft. The ratio of the number of square feet in the roof of the model to the number of square feet in the roof of the actual building is* 1 : 3,600 = $1^2 : 60^2$.

The ratio of the number of feet in corresponding dimensions (scale model : actual building) = 1 : 60.

It follows that the ratio of the areas of two similar figures is the square of the ratio of their corresponding dimensions.

- If Javier used a scale of 1 inch = 10 feet, how would the dimensions of the model change?

The scale has been changed from $\frac{\text{5 feet in the actual building}}{\text{1 inch in the scale model}}$ *to* $\frac{\text{10 feet in the actual building}}{\text{1 inch in the scale model}}$*. The rule that models the relationship between the number of feet in the actual building, y, and the number of inches in the model, x, is y = 10x.*

The values of the actual building, y are known. The dimensions of the new model may be found by solving the rule for y.

Large rectangular prism:	*Length:*	$100 = 10x$
		$x = 10$ *in*
	Width:	$80 = 10x$
		$x = 8$ *in*
	Height:	$15 = 10x$
		$x = 1.5$ *in*
Cylinder:	*Radius:*	$20 = 10x$
		$x = 2$ *in*
	Height:	$15 = 10x$
		$x = 1.5$ *in*

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Small rectangular prism:	*Length:*	*50 = 10*x
		x = 5 in
	Width:	*60 = 10*x
		x = 6 in
	Height:	*10 = 10*x
		x = 1 in

The dimensions of the model would be half as large as the dimensions of the original scale model

Student Council President
grade 8

Aricela is running for Student Council president at her school. She used her computer to make an $8\frac{1}{2}" \times 11"$ flyer with her picture and campaign slogan.

1. Aricela would like to enlarge the flyer to maximize size. She wants to create posters that are similar to the original flyer. She would like to display these new posters on doors that measure 3 feet by 6 feet. What size should the posters be? Explain your thinking.

2. Aricela wants to make additional posters using a copy machine. The largest paper she can use is 11 inches by 17 inches. What is the scale factor she must use? Explain your thinking.

3. Can Aricela make a poster with dimensions 13 inches by 15 inches similar to her original flyer of 8.5 by 11 inches? Why or why not?

4. Aricela would like to reduce the flyer to make campaign badges to distribute to students. If the height of each badge is 2 inches, what will the width be in order to make a similar image? Describe how you solved this problem.

5. What scale factor was used for the reduction? Explain your answer.

Material

Graphing calculator

Connections to Middle School TEKS:

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(A) select and use appropriate operations to solve problems and justify the selections

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic

Teacher Notes

Scaffolding Questions

- What makes two figures similar?
- How does an enlargement or reduction affect the shape of a figure? How does it affect the size?
- What is a scale factor?
- How do you find the scale factor of an enlargement? A reduction?
- How does the new figure compare with the original figure in an enlargement? In a reduction?

Sample Solutions

1. Aricela thinks big posters will get more attention; therefore, she wants to enlarge her flyer as much as possible. She will make the largest possible poster by finding a scale factor that will enlarge the poster so that it fits on the door that is 3 feet by 6 feet. Convert the dimensions to inches.

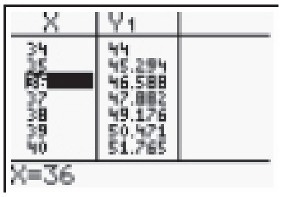
$$3 \text{ feet x } \frac{12 \text{ inches}}{1 \text{ foot}} = 36 \text{ inches}$$

$$6 \text{ feet x } \frac{12 \text{ inches}}{1 \text{ foot}} = 72 \text{ inches}$$

The dimensions in inches are 36 inches by 72 inches. By building a table, possible dimensions can be explored and generalizations can be made.

Scale factor	Width in inches	Length in inches
1	$8\frac{1}{2}$	11
2	17	22
3	$25\frac{1}{2}$	33
4	34	44
5	$42\frac{1}{2}$	55

The ratio of length to width for each row in the table is equivalent to $\frac{11}{8.5}$ and can be expressed as $\frac{y}{x} = \frac{11}{8.5}$. This is a constant ratio or constant of proportionality. The equation $y = \frac{11}{8.5}x$ expresses a proportional relationship between the length and width of each rectangle. From the table, a scale factor between 4 and 5 will enlarge the poster to a width between 34 inches and $42\frac{1}{2}$ inches. Enter $y = \frac{11}{8.5}x$ into a graphing calculator and use the table feature to find $x = 36$. When $x = 36$, $y = 46.6$.



A poster that is 36 inches wide and 46.6 inches high will be similar to the original and will maximize the surface of the door that is 36 inches wide and 72 inches high. To find the scale factor (4.24) that produced this enlargement, find the ratio of corresponding widths or the ratio of corresponding lengths.

$$\frac{36}{8.5} \approx 4.24$$

$$\frac{46.6}{11} \approx 4.24$$

The size of the poster will be 36 inches by 46.6 inches.

2. With a scale factor of 1.29, she would get the largest width possible but would have to trim the extra height. Since Aricela is making an enlargement of her flyer, she will need a scale factor greater than 1. Enter $y = \frac{11}{8.5}x$ into a graphing calculator and scroll to find $x = 11$ and $y = 14.235$.

representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(8.6) Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. The student is expected to:

(A) generate similar shapes using dilations including enlargements and reductions

(8.9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:

(B) use proportional relationships in similar shapes to find missing measurements

Texas Assessment of Knowledge and Skills

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

X	Y_1
7	9.0588
8	10.353
9	11.647
10	12.941
11	14.235
12	15.529
13	16.824

X=11

To find the scale factor used for this enlargement, use the ratio of corresponding widths or the ratio of corresponding heights.

$$\frac{11}{8.5} \approx 1.29$$

$$\frac{14.235}{11} \approx 1.29$$

The following computations can be used to check the value of the scale factor.

$$8\frac{1}{2} \times 1.29 \approx 11$$
$$11 \times 1.29 \approx 14.2$$

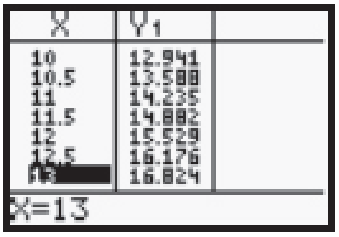
Using a scale factor of 1.29, or 129%, on a copy machine will give a poster that is 11 inches by 14.2 inches.

3. Aricela will not be able to enlarge her flyer to a poster that is 13 inches by 15 inches. Find a scale factor that will scale up the width to 13 inches by finding a ratio of the width of the enlargement to the width of the original flyer. Then multiply this scale factor by the original height to find the new height.

$$13 \div 8\frac{1}{2} \approx 1.5$$
$$11 \times 1.5 = 16.5$$

Using a scale factor of 1.5 will increase the width to 13 inches, but the height would be 16.5 inches, which is too high.

Another strategy would be to enter the equation $y = \frac{11}{8.5}x$ into a graphing calculator and use the table feature to find x = 13. The corresponding value of y is 16.8, which will not meet the dimensions of the 13- by 15-inch poster.



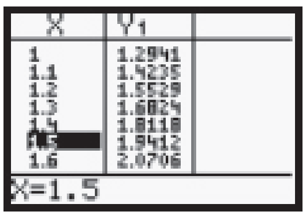
She could make a poster that was 11.5 inches by 14.88 inches that would fit on the 13-inch by 15-inch paper.

4. To find the width of the badge, a proportion of width to height can be written and solved.

$$\frac{8.5}{11} = \frac{w}{2}$$
$$\frac{8.5 \times 2}{11} = w$$
$$\frac{17}{11} = w$$
$$1.5 \approx w$$

The width of the badge is about 1.5 inches.

Another strategy involves the use of the graphing calculator. Enter the equation $y = \frac{11}{8.5}x$ and use the table feature. Scroll down the *y*-column to 1.9412 and read the corresponding *x*-value of 1.5.



5. To find the scale factor used for the reduction, the new dimensions are divided by the corresponding original dimensions. Since this is a reduction, the scale factor is less than 1.

$$1.5 \div 8.5 \approx 0.18$$
$$2 \div 11 \approx 0.18$$

The scale factor used for the reduction is 0.18.

Extension Questions

- On most copy machines, users are allowed to reduce or enlarge documents by specifying a percentage between 50% and 200%. How would you use the copy machine to reduce Aricela's flyer to 25% of the original document?

  A user would need to make a 50% reduction and then reduce the half-size flyer by another 50%. Let x be a dimension of the original document.

  Reduction by 50% means the new image is 0.50x.

  Reducing by 50% again means the next image is 50% of 0.50x or 0.50(0.50x) = 0.25x or 25% of x.

Student Work Sample

This student's work shows the use of scale factor.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Describes mathematical relationships
- Recognizes and applies proportional relationships
- Develops and carries out a plan for solving a problem that includes understand the problem, select a strategy, solve the problem, and check
- Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations
- Evaluates the reasonableness or significance of the solution in the context of the problem
- Demonstrates an understanding of mathematical concepts, processes, and skills
- Communicates clear, detailed, and organized solution strategy

1.) In order to enlarge the poster I have to increase the size by multiplying, I'm going to start with 2

8.5 x 2 = 17
11 x 2 = 22

8.5 x 3 = 25.5
11 x 3 = 33

8.5 x 4 = 34
11 x 4 = 44

8.5 x 5 = 42.5 — too big
11 x 5 = 55

The scale factor has to be between 4 and 5.

So I began multiplying the dimension by 4.1:

8.5 x 4.1 = 34.85
11 x 4.1 = 45.1

8.5 x 4.2 = 35.7
11 x 4.2 = 46.2

8.5 x 4.3 = 36.55
11 x 4.3 = 47.3

The poster should be 35.7" by 46.2".

2.) The largest size paper used for a copier is 11" by 17" since multiplying 2 by the dimension is too big the scale factor has to be between 1 and 2.

I will start will 1.1:

8.5 x 1.1 = 9.35
11 x 1.1 = 12.10

8.5 x 1.2 = 10.2
11 x 1.2 = 13.20

8.5 x 1.3 = 11.05
11 x 1.3 = 14.30

The scale factor has to between 1.2 and 1.3

So I begin by 1.25:

8.5 x 1.25 = 10.625
11 x 1.25 = 13.75

8.5 x 1.26 = 10.71
11 x 1.26 = 13.86

8.5 x 1.27 = 10.795
11 x 1.27 = 13.97

8.5 x 1.28 = 10.88
11 x 1.28 = 14.08

8.5 x 1.29 = 10.965
11 x 1.29 = 14.19

The scale factor is 1.29

To find the percent I have to multiply the scale factor by hundred.

1.29 x 100 = 129

To enlarge the flyer Aricela should select the 129% on the copier.

3) Since 2 is too big the scale factor has be between 1 and 2. I'm going to start by 1.5

$8.5 \times 1.5 = 12.75$
$11 \times 1.5 = 16.5$
$8.5 \times 1.6 = 13.6$
$11 \times 1.6 = 17.6$
> both too big

And if I go lower to 1.4....
$8.5 \times 1.4 = 11.9$
$11 \times 1.4 = 15.4$
it's too short and long

Aricela can not make a poster similar to her original flyer that has dimensions 13" by 15", because the outcome will not be exactly 13" by 15".

4.) Since the length is 2", I'm going to figure out what number times 11 comes out to 2".

$$\frac{11 \cdot n}{11} = \frac{2''}{11}$$

$n = .18$

So the scale factor is .18

I'm going to take .18 and multiply it to 8.5

$8.5 \times .18 = 1.53$

So the width will be 1.53" in order to make a similar image.

5. The scale factor that was used for the reduction was .18.

Since
$8.5 \times .18 = 1.53$
$11 \times .18 = 1.98$

The reduced badge will be 1.5" in width and 2" in length.