

Chapter 4:
Measurement

Lights at the Marleys' and Farleys' grade 6

OVERARCHING
THE MARLEYS AND
FARLEYS

The Marley family and the Farley family are neighbors who live across the street from each other on Boxgarden Light Lane. They have been good friends for many years. Frequently, their projects for home improvement turn out to involve the same part of their homes.

For Halloween, both families like to hang lights on their windows. Last year both families bought orange lights. The Marley family bought a string of orange lights 20 feet long. The Farley family bought a string of orange lights 30 feet long.

This year, each family decided to decorate one additional window at each of their homes. Fortunately, additional orange lights were available in just the sizes they needed by the same company. The Marleys increased their orange light supply to 30 feet and the Farleys increased their orange light supply to 40 feet.

1. Which family increased their orange light supply more? Explain your thinking.
2. What was the percentage increase by length of lights for each of the families? Explain your thinking.
3. The neighbors in the other 12 homes on the Marley side of the street decide to decorate their homes with orange lights exactly like the Marleys. The neighbors in the other 10 homes on the Farley side of the street decide to decorate their homes with orange lights exactly like the Farleys. Is 280 yards of orange lights enough to light Boxgarden Light Lane this year? How do you know?

Teacher Notes

Materials

Calculator

Strips of paper or adding machine tape

Scissors

Glue stick or tape

Connections to Middle School TEKS

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(B) use addition and subtraction to solve problems involving fractions and decimals

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) represent ratios and percents with concrete models, fractions, and decimals

(6.4) Patterns, relationships, and algebraic thinking. The student uses letters as

Scaffolding Questions

- By how many feet of lights did the Marley family increase their orange light supply?
- By how many feet of lights did the Farley family increase their orange light supply?
- What are some different ways to make numerical comparisons between the Marley orange light supply last year and this year? Between the Farley orange light supply last year and this year?
- How are feet and yards related?

Sample Solutions

1. If we consider the number of feet in the increase, neither family increased their light supply more than the other. Both families increased their orange light supply by 10 feet. This absolute change can be determined by subtraction.

$$\begin{aligned}30 \text{ ft} - 20 \text{ ft} &= 10 \text{ ft} \\40 \text{ ft} - 30 \text{ ft} &= 10 \text{ ft}\end{aligned}$$

2. The Farleys increased their orange light supply from 20 feet to 30 feet, or 50% of their original 20 feet. This relative change can be demonstrated using strips of paper as follows:

Use a strip of paper or adding machine tape to represent the original 20 feet. Fold the paper strip in half. Each half now represents 10 feet, or 50% of the original length.

10 feet 50% of the original 20 feet	10 feet 50% of the original 20 feet
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Cut another strip of paper the length of one of the halves and tape it to the end of the original strip of paper. This new strip of paper represents 20 feet plus 10 feet, or 100% plus 50%. This model shows that the Marleys

increased their original orange light supply by 10 feet, or 50%.

10 feet 50% of the original 20 feet	10 feet 50% of the original 20 feet	additional 10 feet equal to 50% of the original 20 feet
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Another strategy would be to write a ratio of the number of feet in the increase to the original number of feet and express it as an equivalent ratio, 50 : 100.

$$10 \text{ ft} : 20 \text{ ft} = 50 \text{ ft} : 100 \text{ ft}$$

$$\frac{50}{100} = 50\%$$

The Marleys increased their orange light supply from 30 feet to 40 feet, or $33\frac{1}{3}\%$ of their original 30 feet. This relative change can be determined by the ratio of the number of feet in the increase to the original number of feet and a percentage.

$$\frac{10}{30} = 33\frac{1}{3}\%$$

Strips of paper can be used to model this situation. Let one strip of paper represent the Marleys' original 30 feet of orange light supply. Fold the paper strip into thirds and label each strip with feet and percentage as shown below.

10 feet $33\frac{1}{3}\%$ of the original 30 feet	10 feet $33\frac{1}{3}\%$ of the original 30 feet	10 feet $33\frac{1}{3}\%$ of the original 30 feet
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Since the Marleys have increased their orange light supply by 10 feet, cut another strip of paper the length of one of the folded sections and tape it onto the end of the original strip of paper to show 40 feet of lights.

10 feet $33\frac{1}{3}\%$ of the original 30 feet	10 feet $33\frac{1}{3}\%$ of the original 30 feet	10 feet $33\frac{1}{3}\%$ of the original 30 feet	additional 10 feet $33\frac{1}{3}\%$ of the original 30 feet
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variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, perimeter, area, etc.

(6.8) Measurement.

The student solves application problems involving estimation and measurement of length, area, time, temperature, capacity, weight, and angles. The student is expected to:

(A) estimate measurements and evaluate reasonableness of results

(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, capacity, and weight

(D) convert measures within the same measurement system (customary and metric) based on relationships between units

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

The model now shows 40 feet of orange lights and an increase of $33\frac{1}{3}\%$ over the original light supply.

Since 50% is greater than $33\frac{1}{3}\%$, the Farleys increased their orange light supply by a greater percentage of their original amount.

3. Yes, 280 yards of lights is enough to light Boxgarden Light Lane this year.

For the Marley side of the street, there are 12 homes plus the Marley home, each to be lit with 30 feet of lights for a total of 390 feet.

$$13 \times 30 \text{ ft} = 390 \text{ ft}$$

For the Farley side of the street, there are 10 homes plus the Farley home, each to be lit with 40 feet of lights for a total of 440 feet.

$$11 \times 40 \text{ ft} = 440 \text{ ft}$$

Together, 830 feet of lights are needed to light Boxgarden Light Lane.

$$390 \text{ ft} + 440 \text{ ft} = 830 \text{ ft}$$

Since there are 3 feet in one yard, there are 840 feet in 280 yards.

$$280 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 840 \text{ feet}$$

Since 830 feet is less than 840 feet, 280 yards of lights will be enough to light Boxgarden Light Lane.

Extension Questions

- If the number of houses varied, what rule could be used to show the relationship between the total number of feet of lights needed and the number of homes displaying lights in the same way as the Marleys' home?

$$f = 30h$$

The variable f represents the number of feet of lights needed, and h represents the number of homes that will display lights.

- If the number of houses varied, what rule could be used to show the relationship between the total number of feet of lights needed and the number of homes displaying lights in the same way as the Farleys' home?

$$f = 40h$$

The variable f represents the number of feet of lights needed, and h represents the number of homes that will display lights.

- For the winter holidays, all 24 homes on Boxgarden Light Lane agree to display 50 feet of white lights. Would 350 yards of white lights be enough to light Boxgarden Light Lane for the winter holidays? How do you know?

No, 350 yards of lights would not be enough. The 24 homes would need 1,200 feet of lights.

$$50 \text{ ft} \times 24 \text{ ft} = 1,200 \text{ ft}$$

Since 1 yard is the same as 3 feet, 350 yards is the same as 1,050 feet.

$$350 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 1,050 \text{ feet}$$

This would be 150 ft less than what is needed.

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

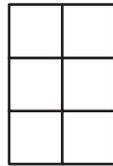
Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

Gardens at the Marleys' and Farleys' grade 7

The Marley family and the Farley family are neighbors who live across the street from each other on Boxgarden Light Lane. They have been good friends for many years. Frequently, their projects for home improvement turn out to involve the same part of their homes. Both the Marleys and the Farleys like fresh vegetables grown in their own rectangular gardens. The following diagram shows the Marleys' garden last summer.



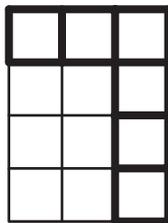
The next diagram shows the Farleys' garden last summer.



For each diagram in this problem, one segment represents one meter.

This summer both families enlarged their gardens.

The Marleys' garden now looks like the following diagram.



The Farleys' garden now looks like the following diagram.



1. What are the dimensions of each garden for last summer and this summer?
2. Which family increased the size of their garden more? Describe how you determined your response.
3. Is your response to problem 2 the only possible answer? Explain.
4. Is the Marleys' new garden mathematically similar to their old garden? Is the Farleys' new garden mathematically similar to their old garden? Explain how you know if they are similar.

Materials

Calculator

Color tiles or grid paper

Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) estimate and find solutions to application problems involving percent

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling,

Teacher Notes

Scaffolding Questions

- What are the dimensions of each garden?
- By how many square meters did the Marleys' garden increase?
- By how many square meters did the Farleys' garden increase?
- What is the percentage increase of the Marleys' garden from last summer to this summer?
- What is the percentage increase of the Farleys' garden from last summer to this summer?
- How can you use a pictorial model to represent the percentage increase of each garden from last summer to this summer?
- What are some different ways to make numerical comparisons of the Marley and Farley gardens?
- What are the critical attributes of similar shape?
- How can you show similarity between the original Marley garden and the enlarged Marley garden?
Between the original Farley garden and the enlarged Farley garden?

Sample Solutions

1. Last summer the Marley garden measured 2 meters by 3 meters. This summer the Marley garden measures 3 meters by 4 meters.

Last summer the Farley garden measured 1 meter by 2 meters. This summer the Farley garden measures 2 meters by 4 meters.

2. If we consider the number of square feet of increase in the garden, neither family increased their garden more than the other from last summer to this summer. Both families increased their garden by 6 square meters. This absolute change is determined by the subtraction of their areas.

Area of the Marley garden last summer

$$3 \text{ m} \times 2 \text{ m} = 6 \text{ m}^2$$

Area of the Marley garden this summer

$$4 \text{ m} \times 3 \text{ m} = 12 \text{ m}^2$$

Difference in areas of the two gardens

$$12 \text{ m}^2 - 6 \text{ m}^2 = 6 \text{ m}^2$$

Area of the Farley garden last summer

$$2 \text{ m} \times 1 \text{ m} = 2 \text{ m}^2$$

Area of the Farley garden this summer

$$4 \text{ m} \times 2 \text{ m} = 8 \text{ m}^2$$

Difference in areas of the two gardens

$$8 \text{ m}^2 - 2 \text{ m}^2 = 6 \text{ m}^2$$

Another way to determine the absolute change in the area of each garden is by counting the additional square meters in this summer's garden compared with last summer's garden for each family.

3. The response to problem 2 is not the only response. Another correct response would be that the Farley garden increased more than the Marley garden.

The Marley garden increased from 6 square meters to 12 square meters. This is a 100% increase in the size of their garden from last summer to this summer. This relative change can be modeled using color tiles as shown.



Each square tile represents 1 square meter. The whole row of 6 square meters represents 100%.

Another row of tiles can be made with 2 different colors.

unit costs, and related measurement units

(7.6) Geometry and spatial reasoning. The student compares and classifies shapes and solids using geometric vocabulary and properties. The student is expected to:

(D) use critical attributes to define similarity

(7.9) Measurement. The student solves application problems involving estimation and measurement. The student is expected to estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences,

to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical,

Six of one color can represent the original area of 6 square meters, and another color can represent the additional square meters. Since 6 tiles represents 100%, twice the number of tiles represents 200%. This model shows that there has been a 100% increase in area.



Another way to determine this relative change is by a ratio and a percentage. The ratio of the amount of increase in the garden to the amount of original area in the garden is $6 \text{ m}^2 : 6 \text{ m}^2$.

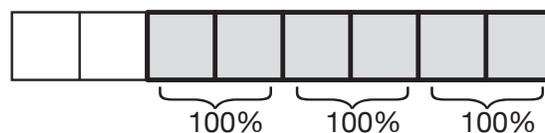
$$\frac{6}{6} = 100\%$$

The Farley garden increased from 2 square meters to 8 square meters. This is a 300% increase in the size of their garden from last summer to this summer. This relative change can also be modeled using color tiles as shown.



Each square tile represents 1 square meter. The row of 2 square meters represents 100%.

Another row of tiles can be made with 2 different colors. Two of one color can represent the original area of 2 square meters, and another color can represent the additional square meters. Since 2 tiles represents 100%, four times the number of tiles represents 400%. This model shows that there has been a 300% increase in area.



This model shows a 300% increase in the size of the Farley garden from last summer to this summer.

This percentage increase can also be determined by a ratio and a percentage. The ratio of the amount of increase in the garden to the amount of original area in the garden is $6 \text{ m}^2 : 2 \text{ m}^2$.

$$\frac{6}{2} = 300\%$$

Since 300% is greater than 100%, the Farley garden increased more from last summer to this summer.

4. The Marley gardens do not represent similar figures. Although corresponding angles are congruent, both corresponding dimensions did not increase by a common scale factor. The width increased from 2 meters to 3 meters by a factor of $1\frac{1}{2}$. The length increased from 3 meters to 4 meters by a factor of $1\frac{1}{3}$.

$$\frac{3 \text{ meters}}{2 \text{ meters}} = \frac{3}{2} = 1\frac{1}{2}$$

$$\frac{4 \text{ meters}}{3 \text{ meters}} = \frac{4}{3} = 1\frac{1}{3}$$

Another justification can be made using the “shape ratio,” comparing length to width for each rectangular garden.

$$\text{Original garden: } \frac{3 \text{ meters}}{2 \text{ meters}} = \frac{3}{2}$$

$$\text{New garden: } \frac{4 \text{ meters}}{3 \text{ meters}} = \frac{4}{3}$$

Since these ratios are not equivalent, these rectangular gardens do not have the same shape and are not similar.

The Farley gardens are similar figures. Corresponding angles are congruent with a measure of 90 degrees. Corresponding dimensions increased by a common scale factor of 2. The width increased from 1 meter to 2 meters. The length increased from 2 meters to 4 meters.

$$\frac{1 \text{ meters}}{2 \text{ meters}} = \frac{2 \times 1 \text{ meters}}{2 \times 2 \text{ meters}} = \frac{2 \text{ meters}}{4 \text{ meters}}$$

physical, or algebraic
mathematical models

Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

By comparing the “shape ratios” for the original and new gardens, it can be shown that the rectangular gardens are similar.

$$\text{Original garden: } l : w = 2 : 1$$

$$\text{New garden: } l : w = 4 : 2$$

$$2 : 1 = 4 : 2$$

Since these ratios are equivalent, the gardens have similar shapes.

Extension Questions

- This summer both families decided to fence in their gardens. Fencing costs \$1.50 per meter. Find the cost of the fence for each garden.

The perimeter of the Marley garden this summer is 14 meters. At \$1.50 per meter, their cost for fencing is \$21.

$$\text{Perimeter: } 2 \times 4 \text{ m} + 2 \times 3 \text{ m} = 14 \text{ m}$$

$$14 \text{ m} \times \$1.50 \text{ per meter} = \$21$$

The perimeter of the Farley garden this summer is 12 meters. At \$1.50 per meter, their cost for fencing is \$18.

$$\text{Perimeter: } 2 \times 2 \text{ m} + 2 \times 4 \text{ m} = 12 \text{ m}$$

$$12 \text{ m} \times \$1.50 \text{ per meter} = \$18$$

- If the Marleys had increased the dimensions of their garden by the same scale factor as the Farleys increased their garden, what would be the dimensions of the Marley garden this summer?

If the dimensions of the Marley garden increased by the same scale factor of 2 as the Farley garden increased, the dimensions of the Marley garden this summer would be 4 meters by 6 meters.

$$2 \text{ m} \times 2 = 4 \text{ m}$$

$$3 \text{ m} \times 2 = 6 \text{ m}$$

- How does the scale factor of 2 for the increase in the dimensions of the Farleys' garden affect the increase in area?

The area of the Farley garden increased from 2 m^2 to 8 m^2 , which is a factor of 4. A factor of 4 is the square of the scale factor 2.

$$\frac{8 \text{ m}^2}{2 \text{ m}^2} = \frac{4}{1}$$

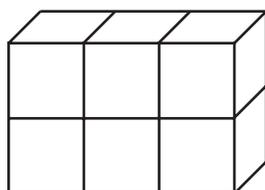
$$2^2 = 4$$

Storage Boxes at the Marleys' and Farleys' grade 8

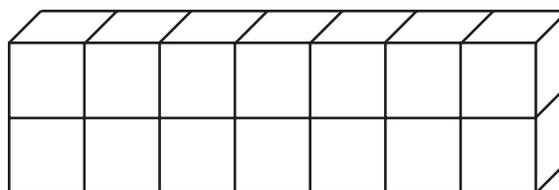
The Marley family and the Farley family are neighbors who live across the street from each other on Boxgarden Light Lane. They have been good friends for many years. Frequently, their projects for home improvement turn out to involve the same part of their homes. Both the Farleys and the Marleys have outdoor garden storage boxes for hoses, tools, etc. Last year, the Farleys' storage box looked like the rectangular prism in the diagram on the left and the Marleys' storage box looked like the rectangular prism in the diagram on the right.

For each storage box in the diagrams, one \square represents one meter.

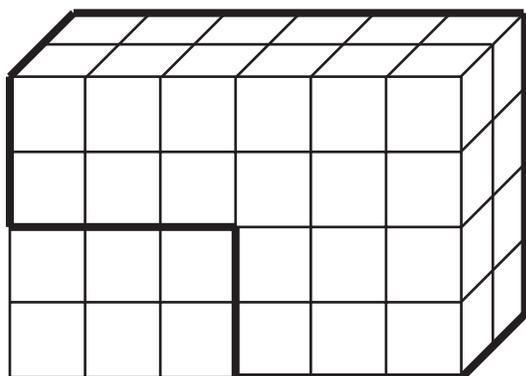
Farleys' storage box last year



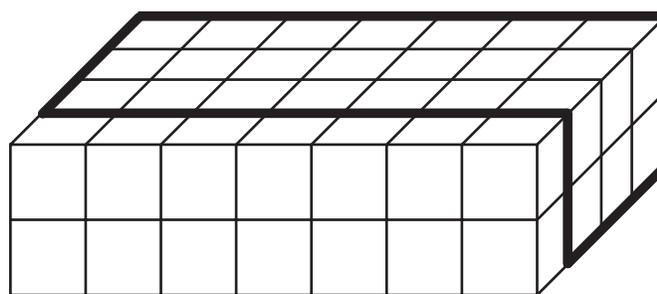
Marleys' storage box last year



This year the Farley family enlarged their storage box to look like the diagram on the left. The Marley family also enlarged their storage box to look like the diagram on the right.



Farleys' storage box this year



Marleys' storage box this year

1. What are the dimensions of each storage box?
2. Which family increased the size of their storage box more? Describe how you determined your response.
3. Is your response to problem 2 the only possible answer? Explain your thinking.
4. Is the Marleys' new storage box mathematically similar to their old storage box? Is the Farleys' new storage box mathematically similar to their old storage box? Explain how you know if they are similar.

Teacher Notes

Materials

Calculator

Cubes or Isometric Dot Paper

Connections to Middle School TEKS

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(A) compare and contrast proportional and non-proportional relationships

Scaffolding Questions

- What are the dimensions of each storage box?
- By how many cubic meters did the Marley storage box increase?
- By how many cubic meters did the Farley storage box increase?
- What are some different ways to make numerical comparisons between the volume of the Marley storage boxes last year and this year? Between the volume of the Farley storage boxes last year and this year?
- What are the critical attributes of similar rectangular prisms?

Sample Solutions

1. The dimensions of the Farleys' storage box last year were 3 meters by 1 meter by 2 meters. The dimensions of the Farleys' storage box this year are 6 meters by 2 meters by 4 meters.

The dimensions of the Marleys' storage box last year were 7 meters by 1 meter by 2 meters. The dimensions of the Marleys' storage box this year are 7 meters by 4 meters by 2 meters.

2. Neither family increased the volume of their storage box more than the other this year. Both families increased the volume of their storage boxes by 42 cubic meters. This absolute change is determined by the difference of their volumes.

$$\begin{aligned}3 \text{ m} \times 1 \text{ m} \times 2 \text{ m} &= 6 \text{ m}^3 \\6 \text{ m} \times 2 \text{ m} \times 4 \text{ m} &= 48 \text{ m}^3 \\48 \text{ m}^3 - 6 \text{ m}^3 &= 42 \text{ m}^3\end{aligned}$$

$$\begin{aligned}7 \text{ m} \times 1 \text{ m} \times 2 \text{ m} &= 14 \text{ m}^3 \\7 \text{ m} \times 2 \text{ m} \times 4 \text{ m} &= 56 \text{ m}^3 \\56 \text{ m}^3 - 14 \text{ m}^3 &= 42 \text{ m}^3\end{aligned}$$

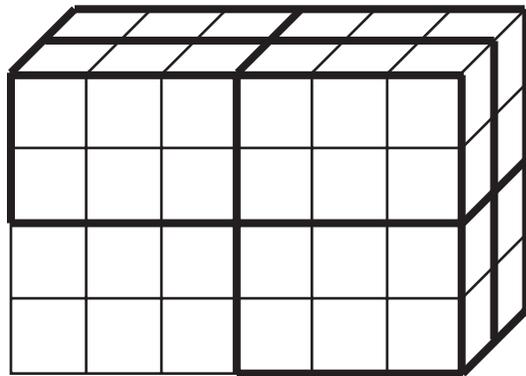
Cubes could also be used to model each box (Farleys' box last year and Farleys' box this year). A comparison of the number of cubes needed to build each Farley box would show a difference of 42 cubes. A comparison of the number of cubes needed to build each Marley box would show a difference of 42 cubes. Therefore, neither family increased the volume of their storage box more than the other family.

3. The response to problem 2 is not the only response. Another correct response would be that the volume of the Farley storage box increased more than the volume of the Marley storage box from last year to this year.

The Farley storage box increased from 6 cubic meters to 48 cubic meters. This is a 700% increase in the size of their storage box from last year to this year. This relative change can be determined using a visual model.

Use six cubes of one color to build the Farley storage box last year. Since the storage box increased by 42 cubic meters, use 42 cubes of another color to build as many storage boxes as possible that are the same as the one from last year.

Because 8 times 6 is 48, there should be 8 sets of 6 cubes altogether. The first set of 6 cubes represents the original volume of 6 cubic meters and 100%. There will be 7 more sets of 6 cubes each representing 100%, for a total of 700%. This set of 42 cubes of a different color represents a 700% increase in the volume of this year's box compared with the volume of last year's box.



(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

(B) use geometric concepts and properties to solve problems in fields such as art and architecture

(8.8) Measurement. The student uses procedures to determine measures of solids. The student is expected to:

(B) connect models to formulas for volume of prisms, cylinders, pyramids, and cones

(C) estimate answers and use formulas to solve application problems involving surface area and volume

(8.9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:

(B) use proportional relationships in similar shapes to find missing measurements

(8.10) Measurement. The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates

Another way to show this relative change is by using a ratio expressed as a fraction and its equivalent percentage. The ratio is the ratio of the number of cubic meters of increase in volume to the original volume.

$$\frac{42 \text{ m}^3}{6 \text{ m}^3} = \frac{7}{1} = \frac{700}{100} = 700\%$$

The volume of the Marley storage box increased from 14 cubic meters to 56 cubic meters. This is a 300% increase in the size of their storage box from last year to this year. This relative change can be modeled with cubes as described in the Farley problem. Use 14 cubes of one color to build the Marley storage box for last year. Then use 42 cubes of a different color to build 3 more boxes.

Because 4 times 14 is 56, there should be 4 sets of 14 cubes altogether. The first set of 14 cubes represents the original volume of 14 cubic meters and 100%. There will be 3 more sets of 14 cubes, each representing 100%, for a total of 300%. This set of 42 cubes of a different color represents a 300% increase in the volume of this year's box compared to the volume of last year's box.

This can also be demonstrated using the ratio of the number of cubic meters of increase in volume to the original volume.

$$\frac{42 \text{ m}^3}{14 \text{ m}^3} = \frac{3}{1} = \frac{300}{100} = 300\%$$

4. The Marley storage boxes do not represent similar solids. Although corresponding angles are congruent, all three corresponding dimensions do not increase by a scale factor. The length and height did not change; however, the width changed from 1 meter to 4 meters, a factor of 4. If the solids were similar the dimensions would have all changed by the same scale factor.

The Farley storage boxes do represent similar solids. Corresponding angles are 90 degrees because the storage boxes are rectangular prisms. Corresponding dimensions increased by a scale factor of 2. The

length increased from 3 meters to 6 meters. The width increased from 1 meter to 2 meters, and the height increased from 2 meters to 4 meters. The ratios of the lengths of corresponding sides are equal.

$$\frac{\text{this year's box length}}{\text{last year's box length}} = \frac{6}{3} = \frac{2}{1}$$

$$\frac{\text{this year's box depth}}{\text{last year's box depth}} = \frac{2}{1}$$

$$\frac{\text{this year's box height}}{\text{last year's box height}} = \frac{4}{2} = \frac{2}{1}$$

Extension Questions

- If the Marleys had increased their storage box by the same scale factor as the Farleys increased their storage box, what would be the dimensions of the Marleys' storage box this year?

If the Marleys had increased the dimensions of their storage box by the same scale factor of 2 as the Farleys' storage box, the dimensions of the Marleys' storage box this year would be 14 meters by 2 meters by 4 meters.

$$7 \text{ m} \times 2 = 14 \text{ m}$$

$$1 \text{ m} \times 2 = 2 \text{ m}$$

$$2 \text{ m} \times 2 = 4 \text{ m}$$

- How does a change in dimensions by a scale factor of 2 affect the surface area of the Farleys' storage box? The volume of the Farleys' storage box?

We can express this relationship by comparing the scale factor for the changes in dimensions and the scale factor for surface areas of the two boxes. The corresponding dimensions of the Farleys' boxes from last year to this year changed by a scale factor of 2. The corresponding surface areas from last year to this year changed by a scale factor of 4. Therefore, a scale factor change of 2 in corresponding dimensions of the two similar boxes results in a scale factor change of 2 squared in corresponding surface areas of the boxes.

understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

Surface area of the Farleys' storage box last year

$$2(3 \text{ m} \times 2 \text{ m}) + 2(3 \text{ m} \times 1 \text{ m}) + 2(2 \text{ m} \times 1 \text{ m}) = \\ 2(6 \text{ m}^2 + 3 \text{ m}^2 + 2 \text{ m}^2) = 22 \text{ m}^2$$

Surface area of the Farleys' storage box this year

$$2(6 \text{ m} \times 4 \text{ m}) + 2(6 \text{ m} \times 2 \text{ m}) + 2(4 \text{ m} \times 2 \text{ m}) = \\ 2(24 \text{ m}^2 + 12 \text{ m}^2 + 8 \text{ m}^2) = 88 \text{ m}^2$$

Ratio of surface areas

$$\frac{88 \text{ m}^2}{22 \text{ m}^2} = \frac{4}{1}$$

The ratio of the surface areas is 4 : 1 or 2² : 1.

- How does the scale factor of 2 for the increase in the dimensions of the Farleys' storage box affect the increase in the volume of the storage box?

The volume of the Farleys' storage box increased by a factor of 8. This is the cube of 2, the scale factor for the increase in corresponding dimensions.

Volume of the Farleys' storage box last year

$$3 \text{ m} \times 1 \text{ m} \times 2 \text{ m} = 6 \text{ m}^3$$

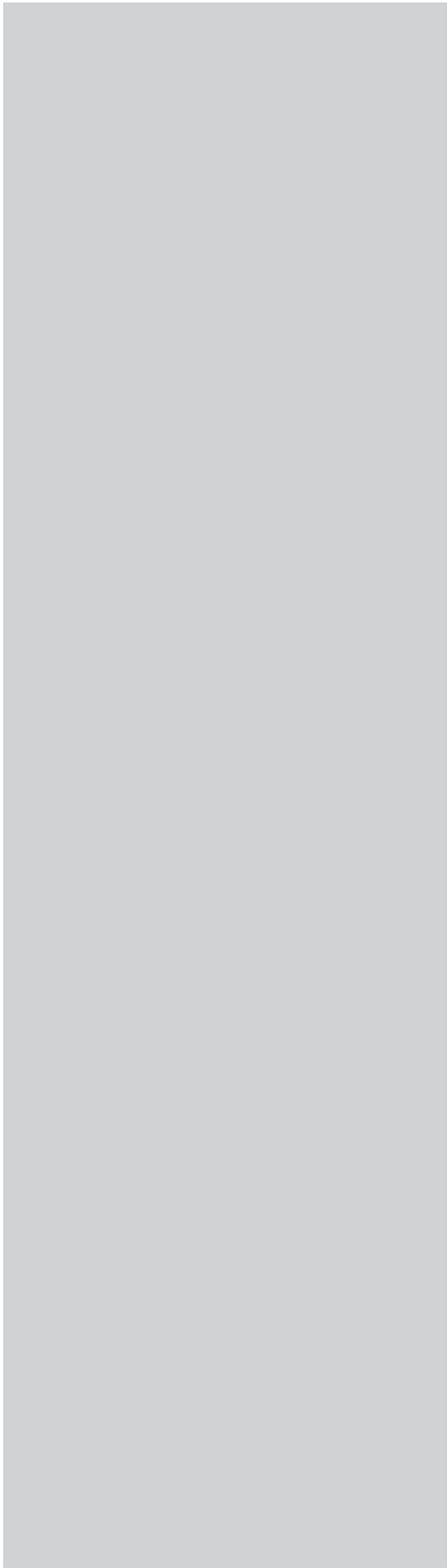
Volume of the Farley's storage box this year

$$6 \text{ m} \times 2 \text{ m} \times 4 \text{ m} = 48 \text{ m}^3$$

Ratio of the volumes

$$\frac{48 \text{ m}^3}{6 \text{ m}^3} = \frac{8}{1}$$

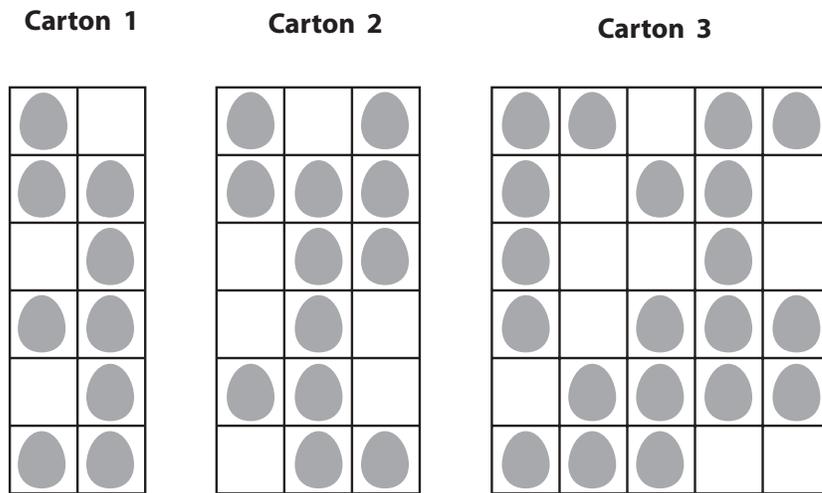
The ratio of the surface areas is 8 : 1 or 2³ : 1.



Extravaganza grade 6

Festive cascarones (hollowed out eggs filled with confetti) are believed to date back to the Renaissance days. Today, children bump the cascarones on the heads of others and make a wish. If the eggshell breaks and showers the recipient's head with confetti, it is said the wish will come true.

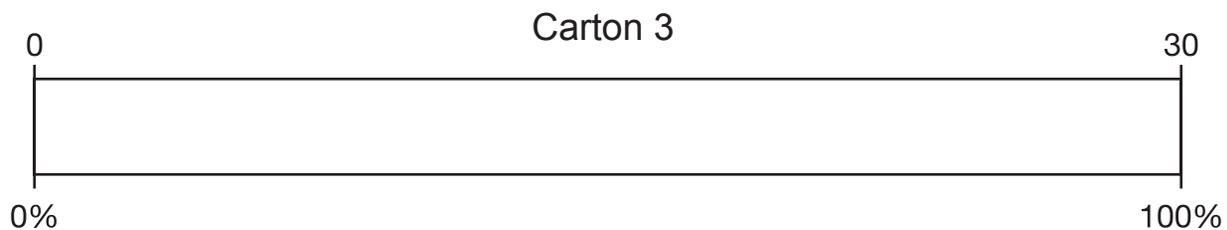
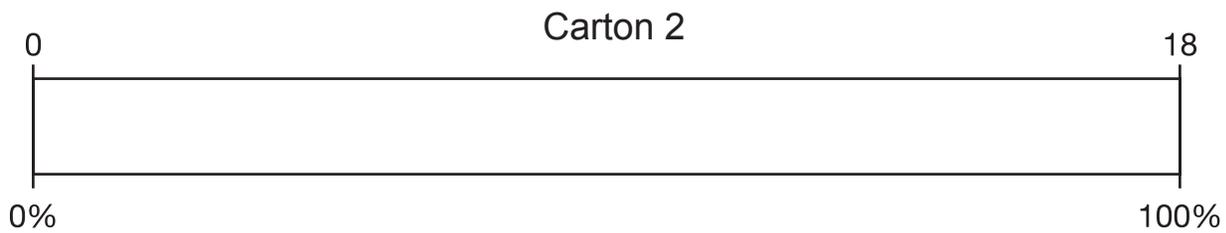
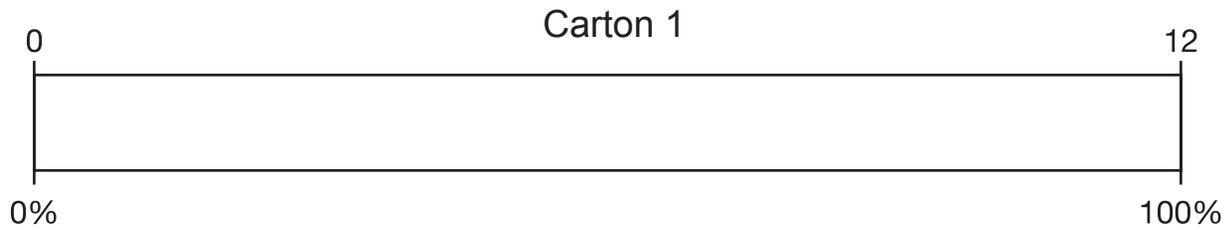
Juanita and her friends are creating cascarones for a party. They are filling each egg with 625 milligrams of confetti. The friends are using recycled egg cartons of different sizes to hold their cascarones. Carton 1 can hold one dozen cascarones. Carton 2 can hold one-and-a-half dozen cascarones. Carton 3 can hold two-and-a-half dozen cascarones. The diagrams below show how much of the cartons have been filled at this time.



1. Complete the following information about each carton.

Carton number	Carton 1	Carton 2	Carton 3
Number of spaces with an egg			
Number of spaces without an egg			
Number of spaces in carton			
Fraction of spaces with an egg			
Fraction of spaces without an egg			

2. Shade each bar to show the amount of cascarones currently in each carton.



3. Which carton has more eggs? Explain your answer using the information from the diagrams and problems 1 and 2.
4. What is the total number of grams of confetti needed if all spaces in all three cartons are filled?
5. Suppose that 6 more cartons, labeled 4 through 9, were also available. Complete the table so that the number of spaces with an egg and the number of spaces without an egg in cartons 4 through 9 are proportional to the number of spaces with an egg and the number of spaces without an egg in carton 2.

Carton number	2	4	5	6	7	8	9
Number of spaces with an egg	12				30		2
Number of spaces without an egg	6		30			14	
Total number of spaces in carton	18	6		24			

Teacher Notes

Materials

Calculator

Connections to Middle School TEKS

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations

(B) represent ratios and percents with concrete models, fractions, and decimals

(C) use ratios to make predictions in proportional situations

(6.4) Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

A) Use tables and symbols to represent and describe proportional and

Scaffolding Questions

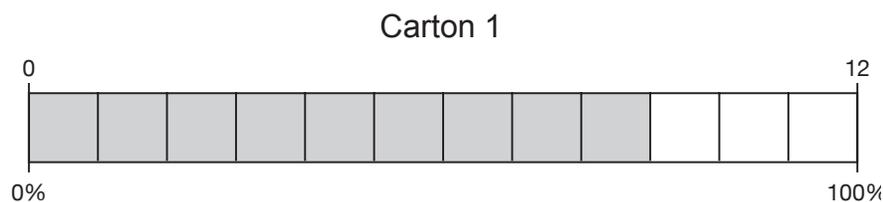
- How many individual eggs are in a dozen eggs?
- How many individual eggs are in $1\frac{1}{2}$ dozen eggs?
- How many individual eggs are in $2\frac{1}{2}$ dozen eggs?
- Where would the benchmark fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ be located on the fraction bar?
- What percentages are equivalent to the benchmarks $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$?
- How can a comparison be made among the three egg cartons using fractions and percentages?
- How many milligrams are equivalent to one gram?

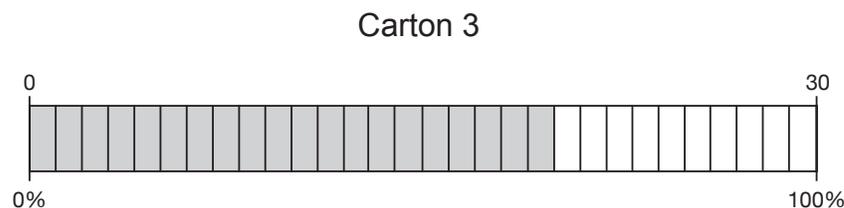
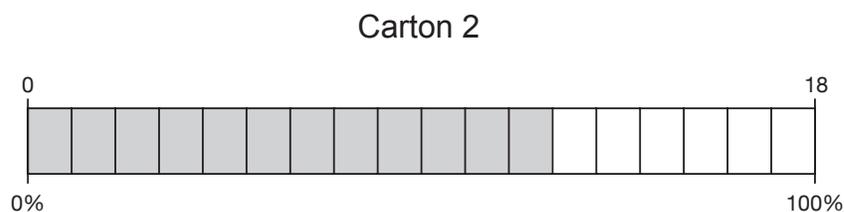
Sample Solutions

1.

Carton number	Carton 1	Carton 2	Carton 3
Number of spaces with an egg	9	12	20
Number of spaces without an egg	3	6	10
Number of spaces in carton	12	18	30
Fraction of spaces with an egg	$\frac{9}{12}$	$\frac{12}{18}$	$\frac{20}{30}$
Fraction of spaces without an egg	$\frac{3}{12}$	$\frac{6}{18}$	$\frac{10}{30}$

2. The bars are marked with the number of spaces available in each carton. The number of spaces shaded is the number of eggs in the carton.





3. One correct response is that carton 3 has more eggs because it has 20 eggs, while cartons 2 and 1 have 12 eggs and 9 eggs respectively, and $20 > 12 > 9$. Another correct response is that carton 1 has more eggs proportionally. Carton 1 is $\frac{3}{4}$ or 75% filled, while both cartons 2 and 3 are $\frac{2}{3}$, or $66\frac{2}{3}\%$ filled.

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{2}{3} = \frac{8}{12}$$

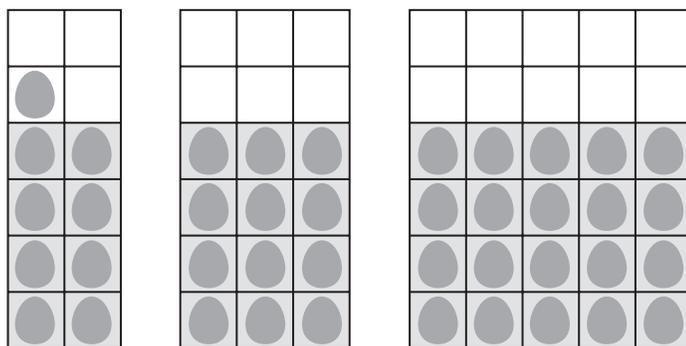
$$\frac{9}{12} > \frac{8}{12}$$

$$\frac{3}{4} > \frac{2}{3}$$

$$75\% > 66\frac{2}{3}\%$$

The proportional comparison can also be shown visually by altering the cascarones' positions in the egg cartons.

The shaded parts represent $\frac{2}{3}$ of each carton.



other relationships involving conversions, sequences, perimeter, area, etc.

(6.8) Measurement. The student solves application problems involving estimation and measurement of length, area, time, temperature, capacity, weight, and angles. The student is expected to:

(D) convert measures within the same measurement system (customary and metric) based on relationships between units

Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

4. There are $12 + 18 + 30$, or 60 spaces available. Since each cascarone gets filled with 625 mg of confetti, the following proportions can be used to find that 37.5 grams of confetti are needed if all spaces in all three cartons are filled.

$$\frac{1 \text{ egg}}{625 \text{ mg of confetti}} = \frac{60 \text{ eggs}}{x \text{ mg of confetti}}$$

$$\frac{1 \text{ egg}}{625 \text{ mg of confetti}} \times \frac{60}{60} = \frac{60 \text{ eggs}}{37,500 \text{ mg of confetti}}$$

$$\frac{1,000 \text{ mg}}{1 \text{ gram}} = \frac{37,500 \text{ mg}}{y \text{ grams}}$$

$$\frac{1,000 \text{ mg}}{1 \text{ gram}} \times \frac{37.5}{37.5} = \frac{37,500 \text{ mg}}{37.5 \text{ grams}}$$

5. If there is a proportional relationship between the number of eggs in carton 2 and the number in cartons 4 through 9, there is a scale factor that can be determined for each set according to what information is given in the table.

Carton 4:

$$\frac{\text{number of spaces in carton 4}}{\text{number of spaces in carton 2}} = \frac{6}{18} = \frac{1}{3}$$

The scale factor is $\frac{1}{3}$.

Multiply the amounts in carton 2 by $\frac{1}{3}$ to determine the unknown amounts in carton 4.

$$12 \times \frac{1}{3} = 4 \qquad 6 \times \frac{1}{3} = 2$$

Carton 5:

$$\frac{\text{number of spaces without an egg in carton 5}}{\text{number of spaces without an egg in carton 2}} = \frac{30}{6} = \frac{5}{1}$$

The scale factor is 5.

Multiply the amounts in carton 2 by 5 to determine the unknown amounts in carton 4.

$$12 \times 5 = 60 \qquad 18 \times 5 = 90$$

Carton 6:

$$\frac{\text{number of spaces in carton 6}}{\text{number of spaces in carton 2}} = \frac{24}{18} = \frac{4}{3}$$

The scale factor is $\frac{4}{3}$.

Multiply the amounts in carton 2 by $\frac{4}{3}$ to determine the unknown amounts in carton 6.

$$12 \times \frac{4}{3} = 16 \qquad 6 \times \frac{4}{3} = 8$$

Carton 7:

$$\frac{\text{number of spaces with an egg in carton 7}}{\text{number of spaces with an egg in carton 2}} = \frac{30}{12} = \frac{5}{2}$$

The scale factor is $\frac{5}{2}$.

Multiply the amounts in carton 2 by $\frac{5}{2}$ to determine the unknown amounts in carton 7.

$$6 \times \frac{5}{2} = 15 \qquad 18 \times \frac{5}{2} = 45$$

Carton 8:

$$\frac{\text{number of spaces without an egg in carton 8}}{\text{number of spaces without an egg in carton 2}} = \frac{14}{6} = \frac{7}{3}$$

The scale factor is $\frac{7}{3}$.

Multiply the amounts in carton 2 by $\frac{7}{3}$ to determine the unknown amounts in carton 8.

$$12 \times \frac{7}{3} = 28 \qquad 18 \times \frac{7}{3} = 42$$

Carton 9:

$$\frac{\text{number of spaces with an egg in carton 9}}{\text{number of spaces with an egg in carton 2}} = \frac{2}{12} = \frac{1}{6}$$

The scale factor is $\frac{1}{6}$.

Multiply the amounts in carton 2 by $\frac{1}{6}$ to determine the unknown amounts in carton 9.

$$6 \times \frac{1}{6} = 1 \qquad 18 \times \frac{1}{6} = 3$$

The table has been completed using this information.

Carton number	2	4	5	6	7	8	9
Number of spaces with an egg	12	4	60	16	30	28	2
Number of spaces without an egg	6	2	30	8	15	14	1
Total number of spaces in carton	18	6	90	24	45	42	3

Extension Questions

- If there were two cartons filled exactly like carton 1, two cartons filled exactly like carton 2, and two cartons filled exactly like carton 3, which cartons would have more cascarones?

A chart describing the new situation is shown below.

Carton number	Carton 1	Carton 2	Carton 3
Number of spaces with an egg	18	24	40
Number of spaces without an egg	6	12	20
Number of spaces in carton	24	36	60
Fraction of spaces with an egg	$\frac{18}{24} = \frac{9}{12}$	$\frac{24}{36} = \frac{12}{18}$	$\frac{40}{60} = \frac{20}{30}$
Fraction of spaces without an egg	$\frac{6}{24} = \frac{3}{12}$	$\frac{12}{36} = \frac{6}{18}$	$\frac{20}{60} = \frac{10}{30}$

Using absolute thinking, it can be found that the two number 3 cartons have more cascarones, since $40 > 24 > 18$.

Using relative thinking, it can be found that the fractions for spaces with eggs are equivalent to the fractions when there was only one carton of each size, so the two number 1 cartons have more cascarones proportionally than do the two number 2 cartons or the two number 3 cartons.

$$\frac{18}{24} = \frac{9}{12} = \frac{3}{4} = 75\%$$

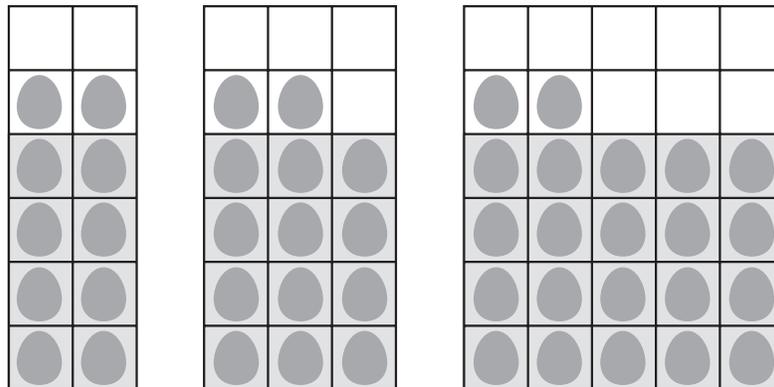
$$\frac{24}{36} = \frac{12}{18} = \frac{2}{3} = 66\frac{2}{3}\%$$

$$\frac{40}{60} = \frac{20}{30} = \frac{2}{3} = 66\frac{2}{3}\%$$

$$75\% > 66\frac{2}{3}\%$$

- If one more cascarone was added to carton 1 and two more cascarones were added to cartons 2 and 3, which carton would have more cascarones proportionally?

The new cartons could look like these diagrams.



The following table shows the statistics for the new diagrams.

Carton number	Carton 1	Carton 2	Carton 3
Number of spaces with an egg	10	14	22
Number of spaces without an egg	2	4	8
Number of spaces in carton	12	18	30
Fraction of spaces with an egg	$\frac{10}{12}$	$\frac{14}{18}$	$\frac{22}{30}$
Fraction of spaces without an egg	$\frac{2}{12}$	$\frac{4}{18}$	$\frac{8}{30}$

Comparing the fraction of spaces with an egg, carton 1 has more eggs proportionally.

$$\text{Carton \#1: } \frac{10}{12} = \frac{5}{6} = \frac{150}{180}$$

$$\text{Carton \#2: } \frac{14}{18} = \frac{7}{9} = \frac{140}{180}$$

$$\text{Carton \#3: } \frac{22}{30} = \frac{11}{15} = \frac{132}{180}$$

$$\frac{150}{180} > \frac{140}{180} > \frac{132}{180}$$

$$\frac{10}{12} > \frac{14}{18} > \frac{22}{30}$$

Matchmaker grade 6

Follow the pattern to complete each table.

Cut out each verbal description, table, graph, and equation. For each measurement relationship, match the verbal description, the table, the graph, and the rule that correspond to each other. Glue the corresponding representations on another sheet of paper.

State at least three reasons why the relationships are proportional. Use examples from a variety of representations.

The number of grams is equal to 1000 times the number of kilograms.

The number of centimeters is equal to 100 times the number of meters.

The number of fluid ounces is equal to 8 times the number of cups.

The number of inches is equal to 36 times the number of yards.

x	y
1	8
2	16
3	
4	
5	
6	48

$$y = 36x$$

x	y
1	1000
2	2000
3	
4	
5	5000
6	

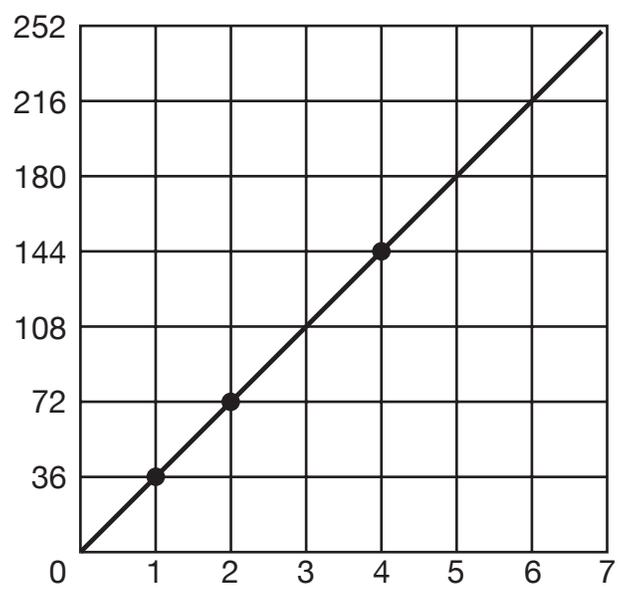
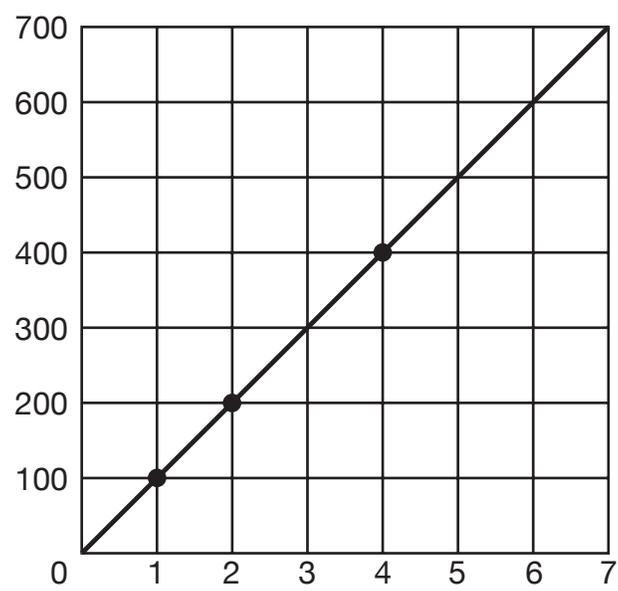
$$y = 100x$$

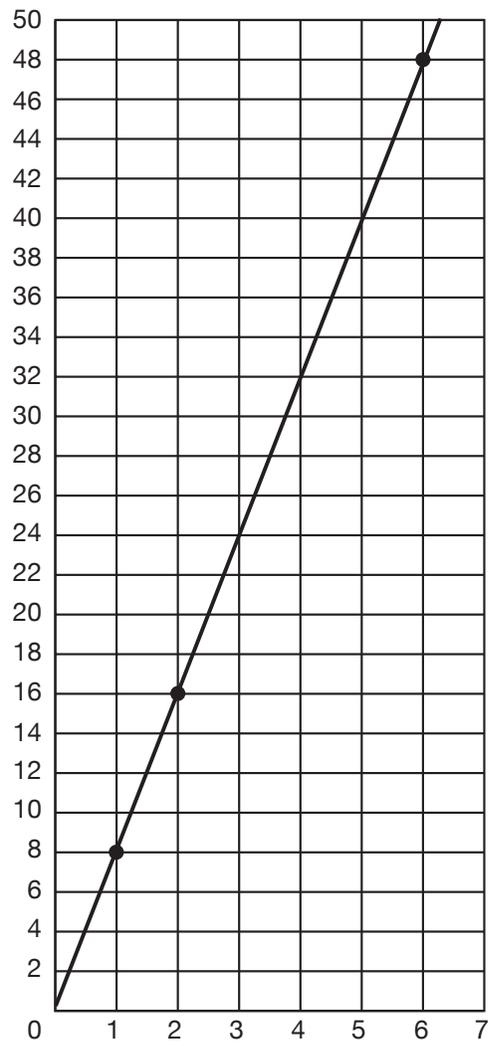
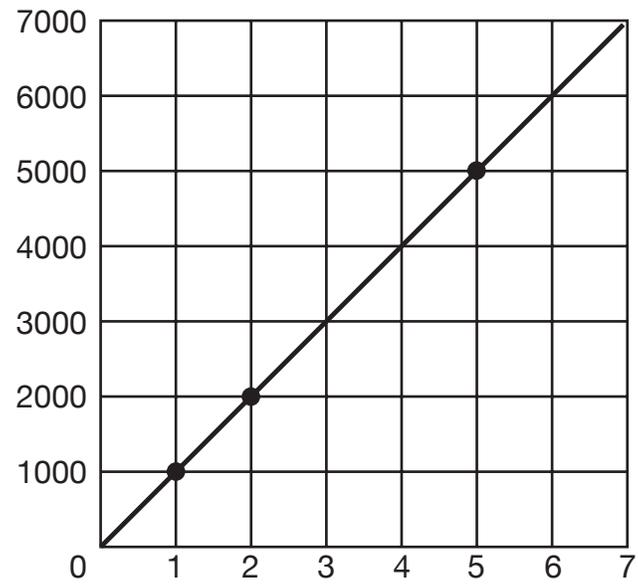
x	y
1	100
2	200
3	
4	400
5	
6	

$$y = 8x$$

x	y
1	36
2	72
3	
4	144
5	
6	

$$y = 1000x$$





Materials

Calculator

Connections to Middle School TEKS

(6.8) Measurement. The student solves application problems involving estimation and measurement of length, area, time, temperature, capacity, weight, and angles.

(D) convert measures within the same measurement system (customary and metric) based on relationships between units

(6.10) Probability and statistics. The student uses statistical representations to analyze data.

(D) solve problems by collecting, organizing, displaying, and interpreting data

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models.

(A) communicate mathematical ideas using language, efficient

Teacher Notes

Scaffolding Questions

- In the table, what is the ratio $y : x$ for each ordered pair (x, y) ?
- What is the significance of this ratio $y : x$?
- What does the ordered pair $(1, 8)$ mean in words?
- How do you graph the ordered pair $(2, 16)$?
- What is the relationship between the number of kilograms and the number of grams?
- How many centimeters are there in 1 meter? 2 meters? 3 meters?
- How many fluid ounces are there in 1 cup? 2 cups? 3 cups?
- How many inches are there in 1 yard?
- How can you find the number of inches in 2 yards? 3 yards?
- How could you write an equation that expresses the relationship between the number of inches and the number of yards? The number of fluid ounces and the number of cups?
- How can you identify a proportional relationship from a table? Graph? Equation?

Sample Solutions

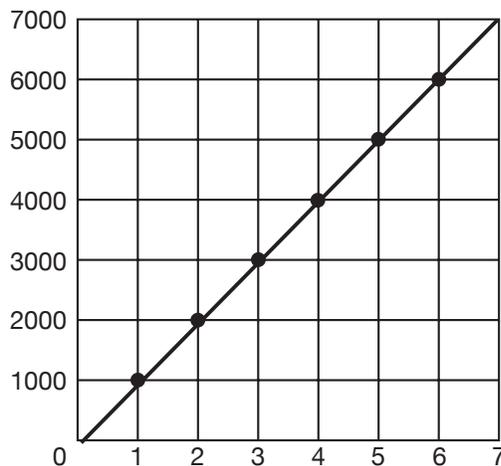
The first match with the graph, verbal description, equation, and table is given below. Each representation shows the relationship between the number of grams and the number of kilograms. The verbal description states that the number of grams is 1,000 times the number of kilograms. The formula $y = 1,000x$ follows from this statement, where y represents the number of grams and x , the number of kilograms. The rate is 1,000 kilograms per gram. The formula or rule can also be derived from the table using a process column to show $1,000(1)$, $1,000(2)$, $1,000(3)$. . . $1,000(x)$.

Each ordered pair of the graph shows this multiplicative relationship.

x	y
1	1,000
2	2,000
3	3,000
4	4,000
5	5,000
6	6,000

The number of grams is equal to 1,000 times the number of kilograms.

$$y = 1,000x$$



The second match shows the connections among the different representations for the relationship between the number of fluid ounces and the number of cups. In the table, the ratio of $y : x$ is $8 : 1$ and can be expressed as $\frac{y}{x} = \frac{8}{1}$. Since $\frac{y}{x} = \frac{8}{1}$, the equation $y = 8x$ can be written, where y represents the number of fluid ounces and x , the number of cups. The equation $y = 8x$ states that the number of fluid ounces is 8 times the number of cups. The rate is 8 ounces per cup. Each ordered pair on the graph lies on the line $y = 8x$. Any point in the first quadrant on this line would represent this relationship. For example, the point $(1.5, 12)$ means that 1.5 cups contains 12 ounces.

tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(6.13) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions.

(B) validate his/her conclusions using mathematical properties and relationships

Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

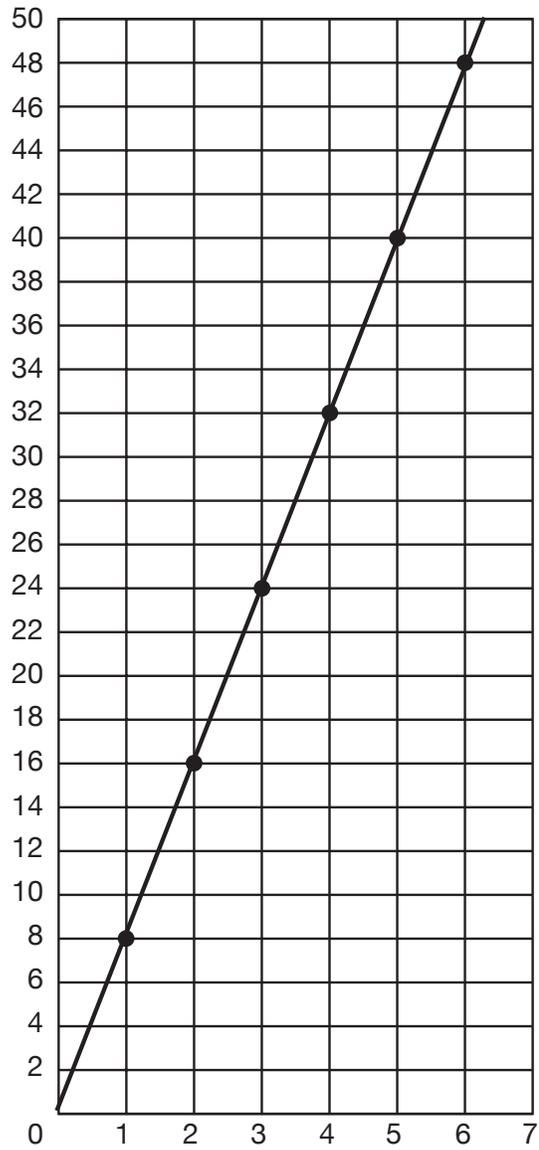
Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

Objective 5: The student will demonstrate an understanding of probability and statistics.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

The number of fluid ounces is equal to 8 times the number of cups.

$$y = 8x$$

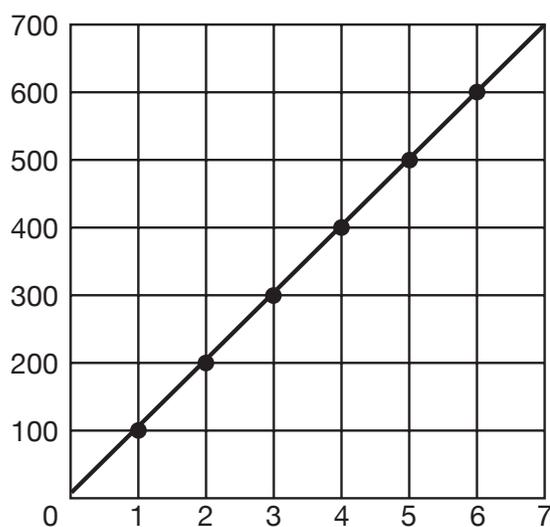


The third match makes connections among the different representations, all of which show the relationship between the number of centimeters and the number of meters. Each ordered pair on the graph represents an ordered pair of measurements that have a multiplicative relationship. The y -value is 100 times the x -value on the graph because the number of centimeters y is 100 times the number of meters x . The rate is 100 centimeters per meter. This relationship can be observed in the verbal statement, table, graph, and rule.

x	y
1	100
2	200
3	300
4	400
5	500
6	600

The number of centimeters is equal to 100 times the number of meters.

$$y = 100x$$

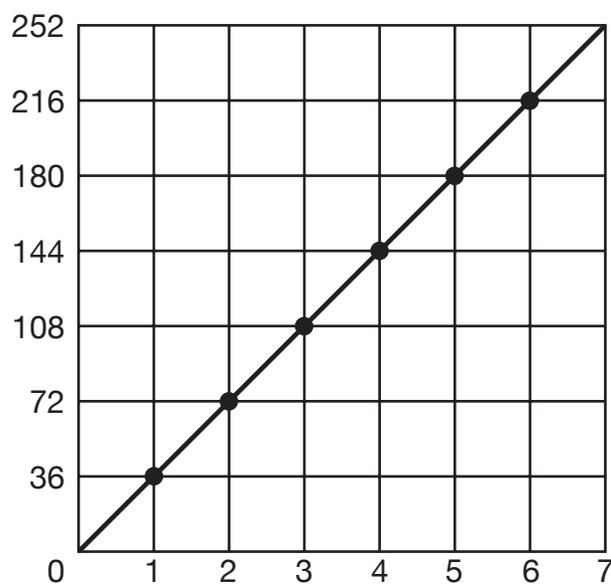


The fourth match shows the relationship between the number of inches and the number of yards. In each representation, the number of inches is 36 times the number of yards. The rate is 36 inches per yard. The graph shows that for each increase of 1 yard there is an increase of 36 inches. This can also be observed in the table by making vertical comparisons of consecutive values for x and y .

x	y
1	36
2	72
3	108
4	144
5	180
6	216

The number of inches is equal to 36 times the number of yards.

$$y = 36x$$



All four of these measurement conversions are proportional relationships because they have the following characteristics of a proportional relationship:

- When the points are connected on each graph with a line, the line contains the point $(0,0)$. In a proportional relationship, the line containing the data points also contains the origin.
- In each of these relationships, the ratio of $y : x$ is a constant k .

number of grams : number of kilograms = 1,000 : 1

number of fluid ounces : number of cups = 8 : 1

number of centimeters : number of meters = 100 : 1

number of inches : number of yards = 36 : 1

This constant k is called the constant of proportionality.

Each of these problems has an equation of the form $y = kx$.

$$y = 1,000x$$

$$y = 100x$$

$$y = 8x$$

$$y = 36x$$

- There is a multiplicative relationship between the numbers in the ordered pairs (x, y) for each of the situations. For example, the ordered pair $(3, 108)$ in the fourth match shows that the number of inches in 3 yards is 3×36 , or 108. Another way to think about this relationship is as follows: The number of yards in 108 inches is $108 \div 36$, or 3.

Extension Questions

- Does the point $(8, 288)$ belong to any of the four relationships? If so, what does it mean?

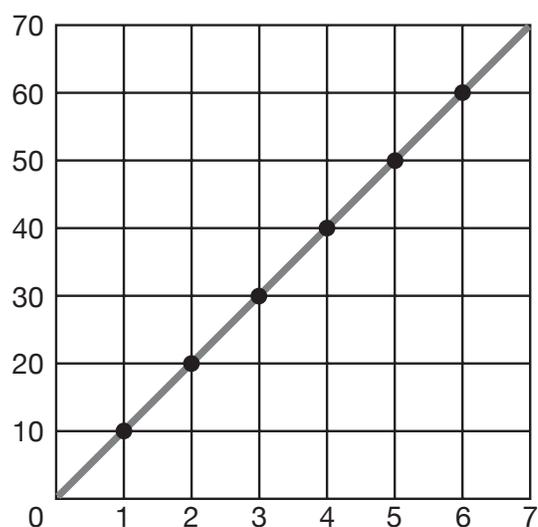
The point $(8, 288)$ belongs to the relationship between yards and inches. Since 1 yard contains 36 inches, 8 times 36 is 288, the number of inches in 8 yards.

- What are verbal, tabular, symbolic, and graphical representations for the relationship between millimeters and centimeters? Explain how each representation models the characteristics of a proportional relationship.

x	y
1	10
2	20
3	30
4	40
5	50
6	60

The number of millimeters is equal to 10 times the number of centimeters.

$$y = 10x$$



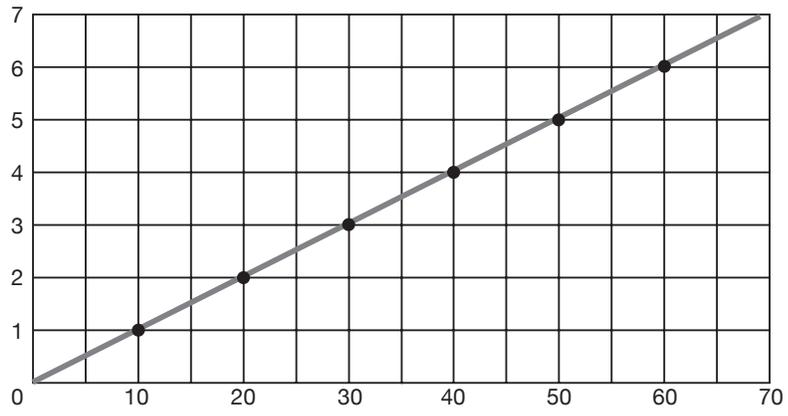
The graph shows a line that passes through the origin. The ratio $\frac{y}{x}$ is a constant 10 for all ordered pairs (x, y) . This ratio $\frac{y}{x} = \frac{10}{1}$ can be expressed as $y = 10x$, which is of the form $y = kx$ for a proportional relationship. The verbal description states a multiplicative relationship between the number of centimeters and the number of millimeters: The number of millimeters is 10 times the number of centimeters.

The following representations show another way of stating the relationship between millimeters and centimeters: The number of centimeters is $\frac{1}{10}$ the number of millimeters where y represents the number of centimeters and x represents the number of millimeters. The ordered pair $(30, 3)$ on the graph shows the multiplicative relationship between the number of millimeters and the number of centimeters: 30 millimeters is 10 times the number of centimeters.

x	y
10	1
20	2
30	3
40	4
50	5
60	6

The number of centimeters is equal to $\frac{1}{10}$ times the number of millimeters.

$$y = \frac{1}{10} x$$

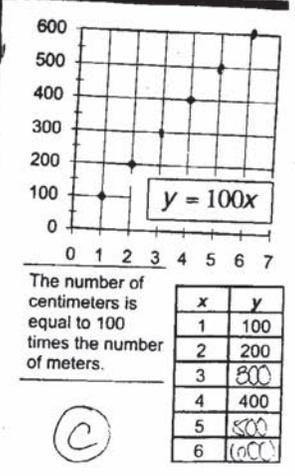
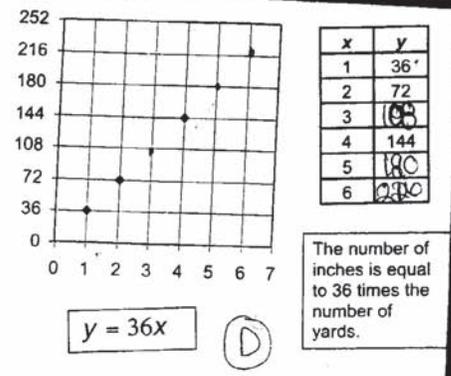
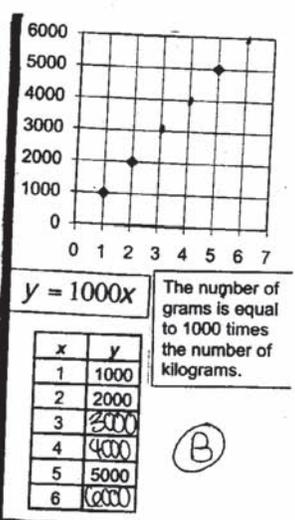
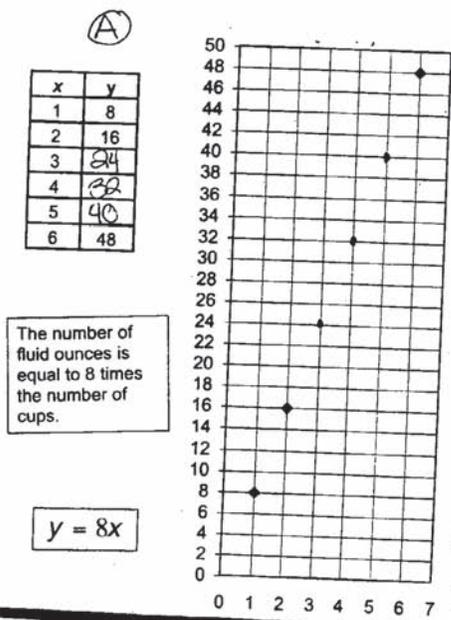
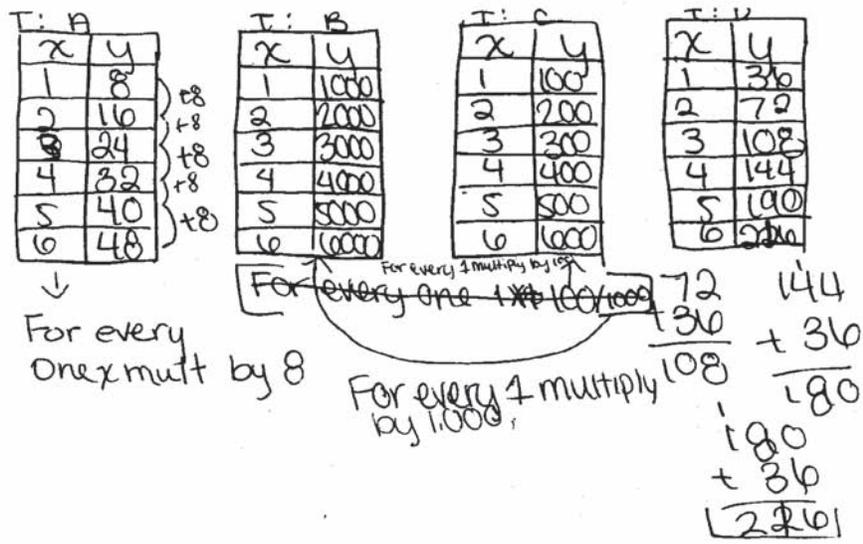


Student Work Sample

This student's work shows that the student recognizes that there is proportional growth in the tables.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Recognizes and applies proportional relationships
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them



There is a number you can multiply x by to get y in every table. ex:

T:A multiply by 8, T:B multiply by multiply 1000, T:C multiply by 100, and T:D multiply by 36. (T:A means table A)

When I connect the points on the graph it forms a line which crosses at (0,0) or the origin.

If you look at the equation the number in front of the x tells you what to multiply by. ex:

$$y = 8x \quad \text{if } x = 9 \quad \text{then } 8 \times 9 = 72$$

Bug Juice grade 7

When campers get thirsty, out comes the well-known camp beverage: Bug Juice!

The Camp Big Sky version of this popular beverage is made with four ounces of Mix A added to tap water to make two quarts of Bug Juice. Four ounces of Mix A costs \$0.78.

The Camp Wild Flowers version of Bug Juice is made from a package of Mix B. It takes 0.14 of an ounce of mix with 4 ounces of sugar and tap water to make two quarts. Mix B costs \$0.20 a package. Sugar costs \$0.36 a pound.

Each camp has 180 campers. During a typical day, each camper drinks two 8-ounce cups of Bug Juice.

1. Each camp has budgeted \$30 for their daily Bug Juice. Is \$30 a day enough? How do you know?
2. Campers begin to complain. They want their Bug Juice “buggier.” How could each camp change their recipe, continue to serve 180 campers two 8-ounce cups of Bug Juice daily, and not spend more than a budget of \$40 a day for Bug Juice? Explain your reasoning.

Teacher Notes

Materials

Calculator

Measurement conversion charts (one per student)

Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions.

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships.

(B) estimate and find solutions to application problems involving proportional

Scaffolding Questions

- How are cups and fluid ounces related?
- How are pounds and solid ounces related?
- How many ounces does each camper drink per day?
- What are some different strategies that you could use to determine how many ounces of Bug Juice 180 campers drink per day?
- What is the cost of 2 quarts of Bug Juice for Camp Big Sky? Camp Wild Flowers?
- How could you use equivalent ratios to find the cost of Bug Juice for 180 campers per day?
- How could you use a proportion to help solve this problem?

Sample Solution

1. The budgeted amount of \$30 is enough for Camp Wild Flowers, but it is not enough for Camp Big Sky.

Multiply both parts of the ratio $\frac{16 \text{ ounces}}{1 \text{ camper}}$ by the scale factor 180 to get an equivalent ratio $\frac{2,880 \text{ ounces}}{180 \text{ campers}}$.

$$\frac{16 \text{ ounces}}{1 \text{ camper}} \times \frac{180}{180} = \frac{2,880 \text{ ounces}}{180 \text{ campers}}$$

The ratio $\frac{2,880 \text{ ounces}}{180 \text{ campers}}$ means 2,880 ounces of Bug Juice are needed for 180 campers who drink 16 ounces of Bug Juice daily.

There are 64 ounces in 1 recipe of Bug Juice (2 quarts).

$$\frac{2 \text{ quarts}}{1 \text{ recipe of Bug Juice}} \times \frac{8 \text{ ounces}}{1 \text{ cup}} \times \frac{2 \text{ cups}}{1 \text{ pint}} \times \frac{2 \text{ pints}}{1 \text{ quart}} = \frac{64 \text{ ounces}}{1 \text{ recipe of Bug Juice}}$$

At Camp Big Sky, 2 quarts, or 64 ounces of Bug Juice costs \$0.78. Knowing that 2,880 ounces are needed for 180 campers, the following proportion can be written to find the cost, c , for 180 campers.

$$\frac{64 \text{ ounces}}{\$0.78} = \frac{2,880 \text{ ounces}}{c}$$

Since $2,880 \div 64 = 45$, the ratio $\frac{64 \text{ ounces}}{\$0.78}$ multiplied by $\frac{45}{45}$ equals $\frac{2,880 \text{ ounces}}{\$35.10}$. The cost for 180 campers at Camp Big Sky to have two 8-ounce cups of Bug Juice using Mix A is \$35.10, which is greater than the \$30 budgeted.

At Camp Wild Flowers, the cost for 2 quarts of their version of Bug Juice is determined by adding the cost of Mix B, \$0.20 for 0.14 of an ounce, to the cost of 4 ounces of sugar. Knowing that sugar costs \$0.36 for 1 pound or 16 ounces, the following proportion can be solved to find the cost of 4 ounces of sugar, s .

$$\frac{\$0.36}{16 \text{ ounces}} = \frac{s}{4 \text{ ounces}}$$

Since $4 \div 16 = \frac{4}{16}$ or $\frac{1}{4}$, the ratio $\frac{\$0.36}{16 \text{ ounces}}$ multiplied by $\frac{1}{4}$ equals $\frac{\$0.09}{4 \text{ ounces}}$.

Thus, 4 ounces of sugar cost \$0.09.

Add the cost of 4 ounces of sugar to the cost of Mix B ($\$0.09 + \$0.20 = \$0.29$) to get the total cost of 2 quarts of Bug Juice.

Knowing that 2,880 ounces are needed for 180 campers, the following proportion can be written to find the cost, c , for 180 campers at Camp Wild Flowers.

$$\frac{64 \text{ ounces}}{\$0.29} = \frac{2,880 \text{ ounces}}{c}$$

relationships such as similarity, scaling, unit costs, and related measurement units

(7.9) Measurement.

The student solves application problems involving estimation and measurement. The student is expected to estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume

Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

Multiplying the ratio $\frac{64 \text{ ounces}}{\$0.29}$ by $\frac{45}{45}$ equals $\frac{2,880 \text{ ounces}}{\$13.05}$. A scale factor of 45 increases from 64 ounces to 2,880 ounces. The cost for 180 campers at Camp Wild Flowers to have two 8-ounce cups of Bug Juice using Mix B is \$13.05, which is less than the budgeted \$30 amount.

2. Answers will vary. Here is one possible solution.

At Camp Big Sky, the Bug Juice can be made slightly “buggier” and still fit the new \$40 budget by adding $\frac{1}{8}$ more of the original Mix A to the tap water. The cost will be $1\frac{1}{8}$ times the original cost of \$0.78, which is \$0.8775, or about \$0.88. The following proportion can be solved to determine the cost, c , for 180 campers at Camp Big Sky to have Bug Juice that is $\frac{1}{8}$ buggier.

$$\frac{64 \text{ ounces}}{\$0.88} = \frac{2,880 \text{ ounces}}{c}$$

Multiplying the ratio $\frac{64 \text{ ounces}}{\$0.88}$ by $\frac{45}{45}$ equals $\frac{2,880 \text{ ounces}}{\$39.60}$. The cost for 180 campers at Camp Big Sky to have Bug Juice that is $\frac{1}{8}$ buggier is \$39.60, which is less than the new budgeted amount of \$40.

At Camp Wild Flowers, Bug Juice can be made considerably buggier than at Camp Big Sky. For example, the amount of Mix B could be tripled from 0.14 of an ounce to 0.42 of an ounce. The cost would change by tripling the cost of 0.14 ounce of Mix B ($3 \times \$0.20 = \0.60) and adding the cost of 4 ounces of sugar (\$0.09). The total cost for the “buggier” juice is $\$0.60 + \$0.09 = \$0.69$.

The following proportion can be solved to find out the cost, c , for 180 campers at Camp Wild Flowers to have Bug Juice that is three times buggier.

$$\frac{64 \text{ ounces}}{\$0.69} = \frac{2,880 \text{ ounces}}{c}$$

Multiplying the ratio $\frac{64 \text{ ounces}}{\$0.69}$ by $\frac{45}{45}$ equals $\frac{2,880 \text{ ounces}}{\$31.05}$.

The cost for 180 campers at Camp Wild Flowers to have Bug Juice that is three times “buggier” is \$31.05, which is less than the new budgeted amount of \$40. However, this may be too “buggy” because no additional sugar is being added in this situation.

Extension Questions

- What is another way for campers at Camp Big Sky and Camp Wild Flowers to have buggier Bug Juice and stay within the \$40 budget?

The number of ounces of water used to make Bug Juice for each camper daily could be reduced to create a “buggier” taste. No additional cost would be incurred for this new version of Bug Juice because no additional mix is added. However, each camper would be served smaller amounts.

- Camp Armadillo makes their version of Bug Juice from Mix C, a liquid concentrate. Twelve ounces of concentrate are added to 36 ounces of water for their Bug Juice recipe. The 12 ounces of liquid concentrate cost \$1.44. Will Camp Armadillo with 180 campers also meet the original \$30 budgeted amount for Bug Juice? How do you know?

Camp Armadillo will not meet the original \$30 budgeted amount for Bug Juice. Actually, Camp Armadillo will nearly triple the budgeted amount.

Knowing that 2,880 ounces are needed for 180 campers, the following proportion can be written to find the cost, c , for 180 campers at Camp Armadillo. The 48 ounces is obtained by adding the 12 ounces of concentrate to three cans or 36 ounces of water for this recipe of Bug Juice.

$$\frac{48 \text{ ounces}}{\$1.44} = \frac{2,880 \text{ ounces}}{c}$$

Since $2,880 \div 48 = 60$, the ratio $\frac{48 \text{ ounces}}{\$1.44}$ can be multiplied by $\frac{60}{60}$ and equals
$$\frac{2,880}{\$86.40} .$$

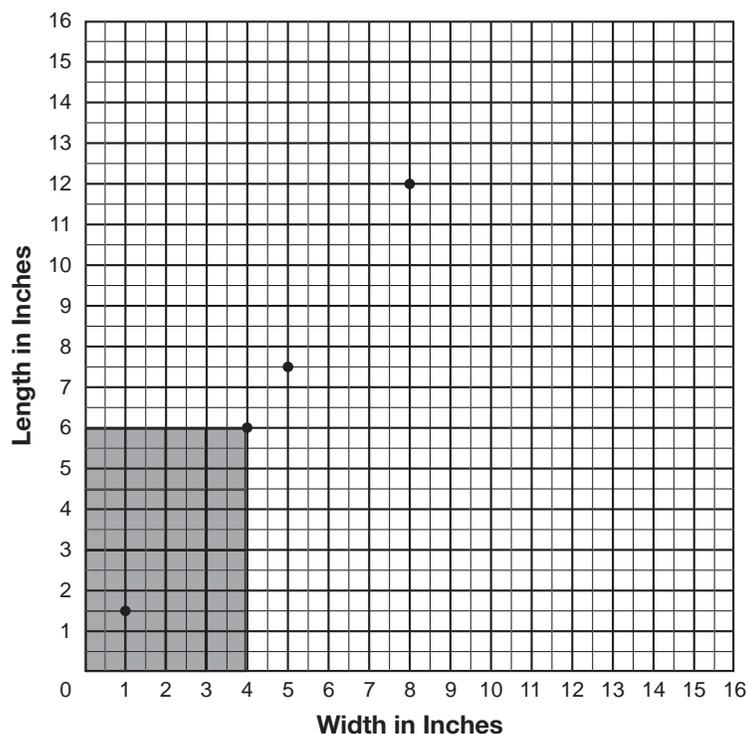
The cost for 180 campers at Camp Armadillo to have two 8-ounce cups of Bug Juice using Mix C is \$86.40, far above the \$30 budgeted amount.

Photographic Memories grade 7

Family photo albums are filled with rectangular pictures of great memories that measure 4 inches in width by 6 inches in length.

At Texan Photo Magic, families bring their film to be processed in a special way. With a push of a button, a photo technician can view the picture as it will develop into a 4-inch by 6-inch photo, or he or she can choose to make enlargements or reductions that are mathematically similar. Amazingly, the machinery at Texan Photo Magic also plots a graph of the width and length of the potential similar photos as the technician views them.

The graph below shows the 4-inch by 6-inch photo and three similar photos the technician viewed on the machine.



1. Describe how proportions can be used to show that the three photos are similar to the 4-inch by 6-inch photo.

2. In what other ways can the evidence for similarity be provided?
3. If a similar photo has a length of 15 inches, what will be its width? Explain your reasoning.
4. Will a photo measuring 9 inches by 13 inches follow the same pattern in the graph as the other similar photos? How do you know?
5. How many of the 1-inch by 1.5-inch photos could be printed on the same size paper as the 4-inch by 6-inch photo? How could you use the graph to demonstrate your answer?

Teacher Notes

Materials

Calculator

1-inch graph paper

Markers

Connections to Middle School TEKS

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

(7.4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling

Scaffolding Questions

- What do the ordered pairs on this graph represent? What does the ordered pair (5, 7.5) mean on this graph?
- Do all the points on the graph lie on a line? How do you know?
- What are coordinates of another point that would lie on this line?
- Does this graph represent a proportional relationship? Explain.
- If the length of a photo is 9 inches, how could you use this graph to find its width?
- How can you determine if two rectangles are similar?

Sample Solutions

1. Two rectangles are similar if the corresponding sides are proportional and corresponding angles are congruent. Since the angles of any rectangle are right angles, the corresponding angles are congruent. Equivalent ratios can be used to show corresponding sides proportional by demonstrating that both dimensions of a new photo can be multiplied by the same scale factor to produce the corresponding dimensions in the original photo. The points on the graph that represent width in inches and length in inches are (1, 1.5), (4, 6), (5, 7.5), and (8, 12). The following shows the scale factor that is used to demonstrate equivalent ratios:

$$\begin{aligned}\frac{1.5 \text{ inches}}{1 \text{ inch}} \times \frac{4}{4} &= \frac{6 \text{ inches}}{4 \text{ inches}} \\ \frac{7.5 \text{ inches}}{5 \text{ inches}} \times \frac{0.8}{0.8} &= \frac{6 \text{ inches}}{4 \text{ inches}} \\ \frac{12 \text{ inches}}{8 \text{ inches}} \times \frac{0.5}{0.5} &= \frac{6 \text{ inches}}{4 \text{ inches}}\end{aligned}$$

Corresponding sides can also be shown to be proportional by using the ratio $l : w$. The ratio $l : w$ is $6 : 4$ for the original photo and is equivalent to corresponding ratios for the other photos represented on the graph: $6 : 4 = 12 : 8 = 7.5 : 5 = 1.5 : 1$. Since all the photos represented on the graph have the same ratio $l : w$, they have the same shape. Scale factors affect the size of the rectangular photos and can be used to “size up” or “size down” the photos.

- The line that contains the points on the graph will also contain the origin. This is one of the characteristics of a proportional relationship. Therefore, the rectangles with dimensions represented by the ordered pairs (w, l) will have corresponding sides in proportion. Since corresponding angles are also congruent (all 4 right angles of a rectangle are congruent), the graph in the first quadrant represents the set of all similar rectangles having $l : w = 1.5 : 1$. For every inch of width on the graph, the length is 1.5 inches. All photos similar to the original 4-inch by 6-inch photo must lie on this straight line (quadrant 1 only) that passes through the origin.
- The width of a similar photo with a length of 15 inches will be 10 inches. A scale factor is determined by $15 \div 6 = 2\frac{1}{2}$ or 2.5. This scale factor of 2.5 will scale up the 6-inch length to 15 inches. Since the photos will be similar, the 4-inch width must also be scaled up by this same factor as shown below. The ratio $10 : 15$ is equivalent to the ratio $4 : 6$, the ratio of the number of inches in the width to the number of inches in the length.

$$\frac{6 \text{ inches}}{4 \text{ inches}} \times \frac{2.5}{2.5} = \frac{15 \text{ inches}}{10 \text{ inches}}$$

- A photo measuring 9 inches by 13 inches will not follow the same pattern on the graph as the other similar photos. In the set of similar photos on the graph, the ratios of $l : w$ are equivalent to $6 : 4$ or $3 : 2$. The ratio of $13 : 9$ is not equivalent to $6 : 4$ or $3 : 2$. Therefore, the photo indicated by the point $(9, 13)$ is not on the straight line containing the given points of the similar photos.

Another strategy involves the use of a graphing calculator. Since the ratio of length to width in the set of

(7.6) Geometry and spatial reasoning. The student compares and classifies shapes and solids using geometric vocabulary and properties. The student is expected to:

(D) use critical attributes to define similarity

(7.7) Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane. The student is expected to:

(A) locate and name points on a coordinate plane using ordered pairs of integers

(7.8) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

(C) use geometric concepts and properties to solve problems in fields such as art and architecture

(7.9) Measurement. The student solves application problems involving estimation and measurement. The student is expected to

estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

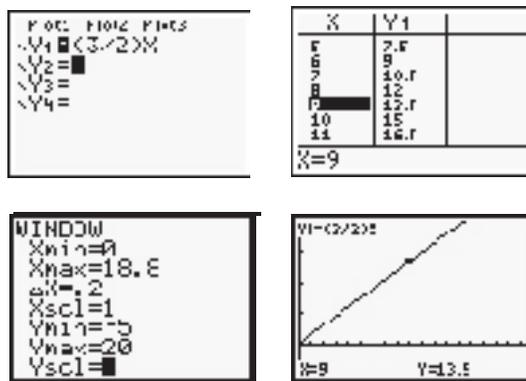
(7.11) Probability and statistics. The student understands that the way a set of data is displayed influences its interpretation. The student is expected to:

- (B) make inferences and convincing arguments based on an analysis of given or collected data

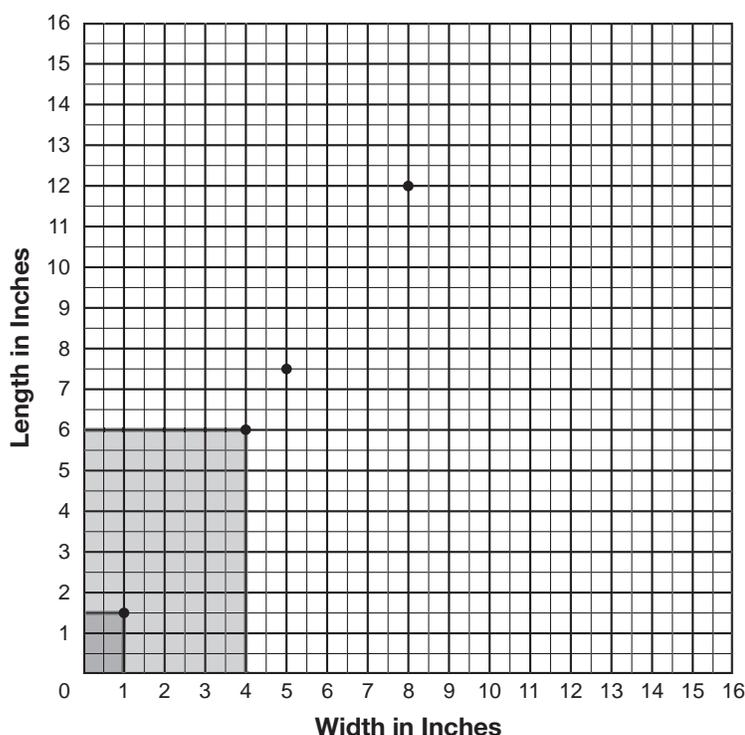
(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics
- (C) select or develop an appropriate

similar rectangles is $3 : 2$, $\frac{y}{x} = \frac{3}{2}$ can be written, where y represents the length and x , the width. Another way to write this equation is $y = \frac{3}{2}x$. Enter the equation $y = \frac{3}{2}x$ in a graphing calculator and use the table feature to find $x = 9$ and read the corresponding y -value of 13.5. The trace feature of the graphing calculator may also be used to find the point on the graph for which $x = 9$. A rectangle with a width of 9 inches must have a length of 13.5 to be similar to the other rectangles on the graph. The photo measuring 9 inches by 13 inches does not fit the pattern.



5. Sixteen photos measuring 1 inch by 1.5 inches could be printed on the same size paper as the 4-inch by 6-inch photo. Graph the points with coordinates $(0,0)$, $(4,0)$, $(4,6)$, and $(0,6)$ to form a rectangle with a width of 4 inches and a length of 6 inches on 1-inch graph paper. Start at the origin and place a mark at 1-inch intervals along the x -axis for a total of 4 inches. Begin at the origin and place a mark at $\frac{1}{2}$ inch intervals along the y -axis for a total of 4 intervals. Draw horizontal and vertical line segments from these marks along the axes to form a set of 16 rectangles with dimensions 1 inch by 1.5 inches. All 16 rectangles completely cover the grid for the 4-inch by 6-inch photo.



Extension Questions

- What are at least 3 other photos similar to the ones already indicated on the graph? How do you know they are similar?

Enter the equation $y = \frac{3}{2}x$ in a graphing calculator and use the table feature to scroll for other ordered pairs (width, length) that are on the graph of similar photos. Examples: (2,3), (7, 10.5), (12,18), (10,15). These new photos with the dimensions given in the ordered pairs above have a length-to-width ratio of 3 : 2.

$$\frac{10.5}{7} = \frac{3}{2} \quad \frac{18}{12} = \frac{3}{2} \quad \frac{15}{10} = \frac{3}{2}$$

This means that the rectangles all have the same ratio.

Another strategy would be to graph the set of rectangles with dimensions given in the ordered pairs (2,3), (7, 10.5), (12,18), and (10,15) on the same grid. Then draw diagonals from the origin for all the

problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

Objective 5: The student will demonstrate an understanding of probability and statistics.

rectangles. The diagonals will all lie on the same line.

Graph the line $y = \frac{3}{2}x$. This line would contain all the ordered pairs whose coordinates represent similar rectangles with the same ratio 3 : 2. The line $y = \frac{3}{2}x$ is the same line as the line containing the diagonals of the given rectangles.

- If you double the dimensions of the 4-inch by 6-inch photo, by what factor does the area change?

When the dimensions of the 4-inch by 6-inch photo are doubled to 8 inches by 12 inches, the area increases from 24 square inches to 96 square inches.

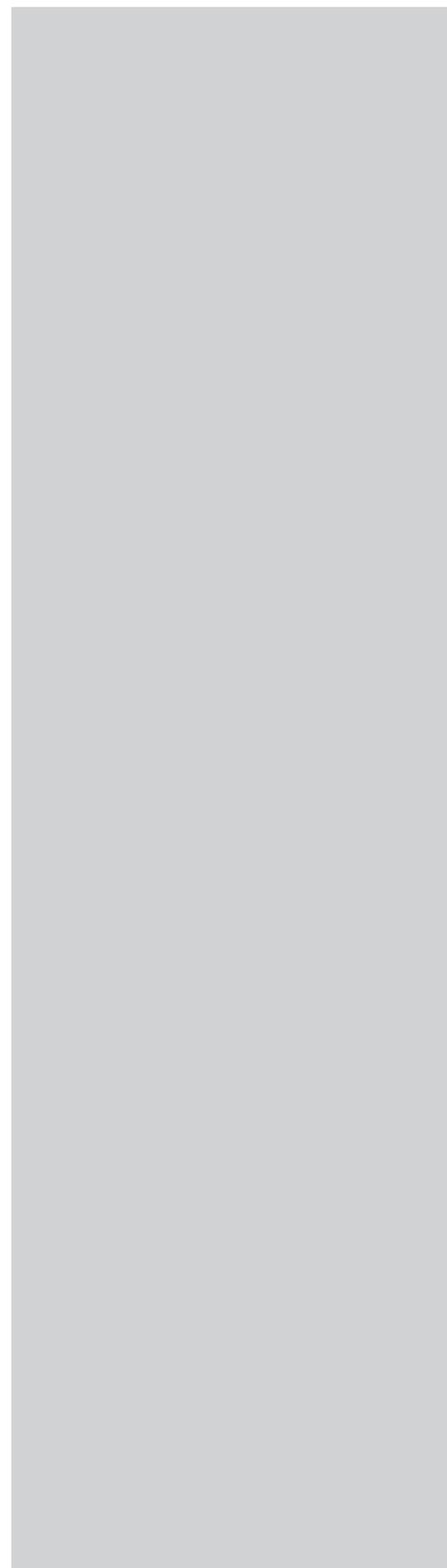
This is an increase by a factor of 4 because $96 \div 24 = 4$. When the dimensions of a figure are changed by a scale factor n , the area is changed by a factor of n^2 , the square of the scale factor.

Student Work Sample

This student's work shows the use of multiple ways of finding percentages, including scaling, division, and knowledge of benchmark fractions and percentages.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Describes mathematical relationships
- Recognizes and applies proportional relationships
- Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations
- Demonstrates an understanding of mathematical concepts, processes, and skills
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them



Photographic Memories

1) $\frac{\text{width } 4}{\text{length } 6} = \frac{8}{12}$ all similar

width $\frac{4 \times 1.25}{6 \times 1.25} = \frac{5}{7.5}$ $\sqrt[3]{\frac{125}{27}} = \frac{5}{3}$
 $\frac{4}{6} = \frac{5}{7.5}$

2) $\frac{8_{ms}}{12_{ms}} = \frac{66\%}{100}$ change to %

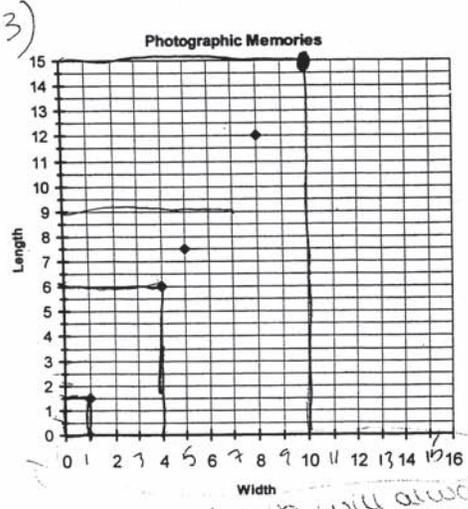
$\frac{5_{ms}}{7.5_{ms}} = \frac{66\%}{100}$

$\frac{1_{ms}}{1.5_{ms}} = \frac{66\%}{100}$

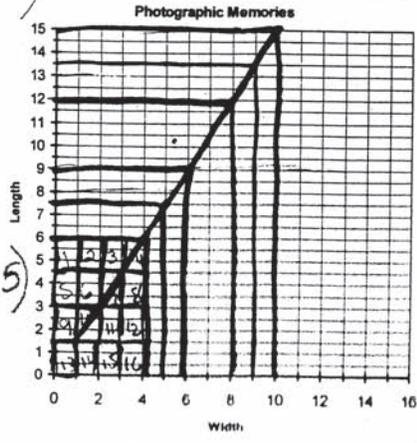
$\frac{4}{6} = \frac{66\%}{100}$ I know $\frac{2}{3} = 66\%$

3) $\frac{4 \times 2.5}{6 \times 2.5} = \frac{10}{15}$ → width see graph 4)

$\frac{9 \times 13.5}{13 \times 10}$ no, because all of the pictures have to be 66% to be similar



Graph means length will always be 1.5 x width.



- extending line
- the photo size.
- 44
- 5) 10 can fit

Photo size	Length	width
4x6	6	4
5x7.5	7.5	5
6x9	9	6
7x10.5	10.5	7
8x12	12	8
9x13.5	13.5	9
10x15	15	10

9x13.5
not
9x13

going up by 1/2 going up by 1

Liberty Enlightening the World!

grade 8

The Statue of Liberty was given to the United States more than 100 years ago by France in recognition of our friendship. Building the Statue of Liberty was a massive task. Lady Liberty holds in her left hand a tablet stating in Roman numerals July 4 (IV), 1776 (MDCCLXXVI). The tablet measures 7.19 meters long, 4.14 meters wide, and 0.61 meter thick.

The distance from the ground to the tip of the torch is 92.99 meters. The distance across one of the statue's eyes is 0.71 meter.

The gift shop sells a miniature Statue of Liberty to take home as a souvenir. The miniature statue is a scaled model of the actual statue.

1. If the distance across Lady Liberty's eye is 0.5 centimeter in the miniature statue, what is the scale factor from the miniature statue to the actual statue? Explain your thinking.
2. What are the approximate dimensions of the tablet in the miniature statue? Show how you determined these.
3. If the Statue of Liberty's tablet is solid, what would be the volume of the tablet? What would be the volume of the tablet in the miniature Statue of Liberty? How do these relate?
4. If you wanted a scaled model of the Statue of Liberty to be as tall as yourself, what information do you need?

Materials

Calculator

Connections to Middle School TEKS

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

Teacher Notes

Scaffolding Questions

- How can you find a scale factor when the actual height and miniature height are known?
- How can you use a scale factor to find the dimensions of the tablet in the miniature statue?
- How do you change meters to centimeters?
- How can you determine the volume of a rectangular prism?
- What is the resulting effect on volume when the dimensions are changed by a scale factor of 2?

Sample Solutions

1. Since the actual distance across Lady Liberty's eye is 0.71 meter, change 0.71 meter to 71 centimeters.

$$0.71 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ meter}} = 71 \text{ cm}$$

To find a scale factor from the miniature dimension of 0.5 cm to the corresponding actual dimension of 71 cm, find the rate.

$$\frac{71 \text{ centimeters on the actual statue}}{0.5 \text{ centimeters on the miniature}} = \frac{142 \text{ centimeters on the actual statue}}{1 \text{ centimeter on the miniature}}$$

Since the miniature is being scaled up, the scale factor will be greater than 1.

2. The actual dimensions of the tablet are 7.19 meters long, 4.14 meters wide, and 0.61 meter thick. The rate is

$$\frac{142 \text{ centimeters on the actual statue}}{1 \text{ centimeter on the miniature}} \quad \text{OR} \quad \frac{1 \text{ centimeter on the miniature statue}}{142 \text{ centimeters on the actual statue}}$$

To convert from the actual dimension to the miniature dimension, multiply by this rate.

$$7.19 \text{ meters} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{1 \text{ centimeter on the miniature statue}}{142 \text{ centimeters on the actual statue}} \approx 5.06 \text{ cm}$$

$$4.14 \text{ meters} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{1 \text{ centimeter on the miniature statue}}{142 \text{ centimeters on the actual statue}} \approx 2.92 \text{ cm}$$

$$0.61 \text{ meters} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{1 \text{ centimeter on the miniature statue}}{142 \text{ centimeters on the actual statue}} \approx 0.43 \text{ cm}$$

3. The volume of the actual tablet is $719 \text{ cm} \times 414 \text{ cm} \times 61 \text{ cm} = 18,157,626 \text{ cm}^3$.

The volume of the tablet in the miniature Statue of Liberty is approximately $5.06 \text{ cm} \times 2.92 \text{ cm} \times 0.43 \text{ cm} = 6.353336 \text{ cm}^3$.

The volume of the actual tablet is approximately 3,000,000 times larger since $18,157,626 \text{ cm}^3 \div 6.353336 \text{ cm}^3 = 2,857,967.216$.

Notice that if you cube the scale factor 142, you find that 142^3 is 2,863,288, or close to 2,857,967.216.

When the dimensions of a figure are changed by a scale factor of c , the volume is changed by c^3 , the cube of the scale factor.

4. To keep a scaled model of the Statue of Liberty in proportion, you need to know your height in order to compute the scale factor for other parts of the statue. Also, a decision would need to be made whether to use the Statue of Liberty data for distance from the heel to the top of the head or to use the height from the base to the torch.

Extension Questions

- What would be the approximate distance from the tip of the torch to the ground for the miniature Statue of Liberty in problem 1?

$$92.99 \text{ meters} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{1 \text{ centimeter on the miniature statue}}{142 \text{ centimeters on the actual statue}} \approx 65 \text{ cm}$$

The distance from the tip of the torch to the ground for the miniature Statue of Liberty is approximately 65 cm, or 0.65 meters.

(8.7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

(B) use geometric concepts and properties to solve problems in fields such as art and architecture

(8.8) Measurement. The student uses procedures to determine measures of solids. The student is expected to:

(C) estimate answers and use formulas to solve application problems involving surface area and volume

(8.9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:

(B) use proportional relationships in similar shapes to find missing measurements

(8.10) Measurement. The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally

Texas Assessment of Knowledge and Skills

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

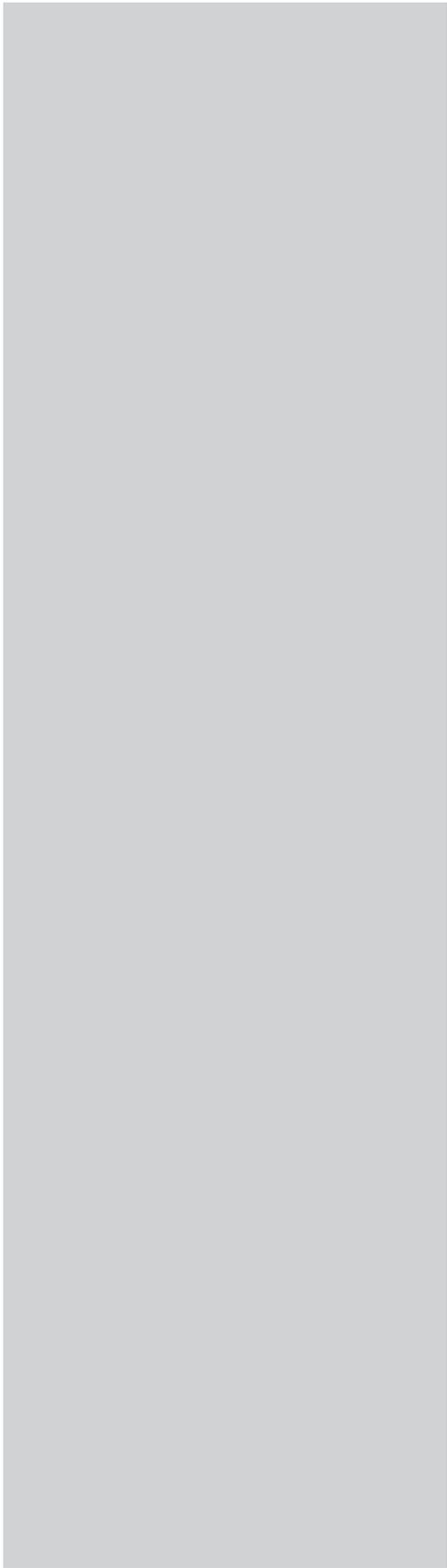
Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

- The distance from the heel to the top of the head of the Statue of Liberty is 33.86 m. What is the scale factor from your “heel to top of the head” height to the corresponding distance of the actual Statue of Liberty?

Answers will vary for this question depending on the height of the person. For example, if a person were 140 cm tall, a scale factor of 24.2 could be determined using division.

$$33.86 \text{ m} = 3,386 \text{ cm}$$

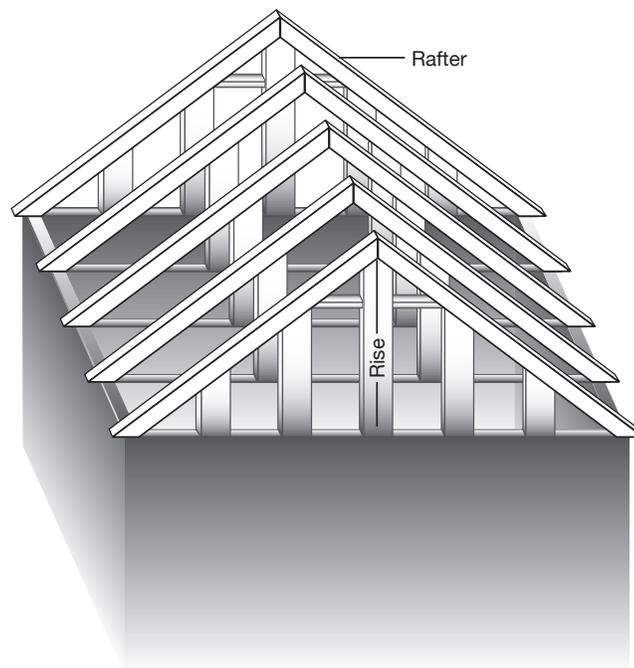
$$3,386 \text{ cm} \div 140 \text{ cm} \approx 24.2$$



In the Rafters grade 8

The Whitman family is building a beautiful home. They want Angel, their loveable dog, to feel very comfortable. For the times Angel must remain outdoors, the Whitmans are also going to build a scaled model of their home, including a matching roof.

The slope of the roof will be determined by the ratio 3 inches of rise (vertical length) for every 4 inches of run (horizontal length).



1. If the run of the roof is 156 inches in the Whitman home, how high will the roof rise?
2. The rafters extend from the top ridge of the roof to the bottom of the lower border that overhangs the walls. There are 5 rafters on the right side of the roof and 5 more on the left. What is the length of each of the rafters at the Whitman home?
3. The scale factor from the run of the roof in Angel's doghouse to the run of the roof of the Whitman home is 13. What will be the lengths of the rise and run of the doghouse?
4. What will be the length of the rafters on Angel's doghouse?

Teacher Notes

Materials

Calculator

Connections to Middle School TEKS

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height,
 $a = 1.4h$

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

Scaffolding Questions

- How could you label the roof of the Whitman home with the measurements that are given?
- How would a triangle similar to one on the roof be labeled to show the relationships between rise and run?
- How do you find the length of the hypotenuse of a right triangle?

Sample Solution

1. The slope of a roof is determined by the ratio of 3 inches of rise for every 4 inches of run. The scale factor is 39 because $156 \text{ in.} \div 4 \text{ in.} = 39$.

$$\frac{3 \text{ inches of rise}}{4 \text{ inches of run}} \times \frac{39}{39} = \frac{117 \text{ inches of rise}}{156 \text{ inches of run}}$$

Therefore, the rise of the Whitman home is 117 feet.

2. A right triangle is formed by the rise, the run, and the connecting rafter, so the Pythagorean Theorem may be used to find the length of one rafter. The legs of the right triangle are the rise and the run. The hypotenuse is the rafter.

$$3^2 + 4^2 = (\text{hypotenuse})^2$$

$$9 + 16 = 25$$

$$25 = 5^2$$

This means that for every 3 inches of rise and every 4 inches of run, the rafter must be 5 inches long.

The proportion $\frac{3 \text{ inches of run}}{5 \text{ inches of rafter}} = \frac{117 \text{ inches of run}}{f}$ can be solved to find f , the number of inches of rafter needed for the Whitman home. Since $117 \text{ in.} \div 3 \text{ in.} = 39$ and $39 \text{ in.} \times 5 \text{ in.} = 195$, 195 inches of rafter are needed for each rafter of the Whitman home.

3. Using the given scale factor of 13, the proportion $\frac{x \text{ inches of rise}}{y \text{ inches of run}} \times \frac{13}{13} = \frac{117 \text{ inches of rise}}{156 \text{ inches of run}}$ can be solved for x and y using division.

$$117 \text{ in.} \div 13 \text{ in.} = 9 \text{ in.} \text{ and } 156 \text{ in.} \div 13 \text{ in.} = 12 \text{ in.}$$

This means that the roof in Angel's home will have a rise of 9 inches and a run of 12 inches.

4. Because $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, each rafter will be 15 inches in length.

Extension Questions

- How many feet of wood are needed for all 10 rafters in Angel's home?

Each rafter is 15 inches and there are 10 rafters, so 150 inches of wood are needed.

$$150 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 12.5 \text{ feet}$$

- As much as possible, rafters are continuous pieces of wood for sturdiness, instead of a lot of pieces of wood patched together. Rafters should be patched at most once per rafter. If rafters are made of boards of wood 10 feet long, how many actual 10-foot boards are needed for all 10 rafters in the Whitman home?

Since one rafter is 195 inches or 16.25 ft. long and $16.25 \div 10 = 1.625$, 1.625 pieces of 10-foot board are needed for one rafter. Since there can only be one patched piece, you will need 20 boards for the rafters.

Each rafter will need one 10-foot board and $\frac{5}{8}$ of a second board. Since there is only $\frac{3}{8}$ of the second board left, this piece will be scrapped.

(8.6) Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. The student is expected to:

(A) generate similar shapes using dilations including enlargements and reductions

(8.7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

(B) use geometric concepts and properties to solve problems in fields such as art and architecture

(8.9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems

(B) use proportional relationships in similar shapes to find missing measurements

Texas Assessment of Knowledge and Skills

Objective 2: The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning.

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement.

