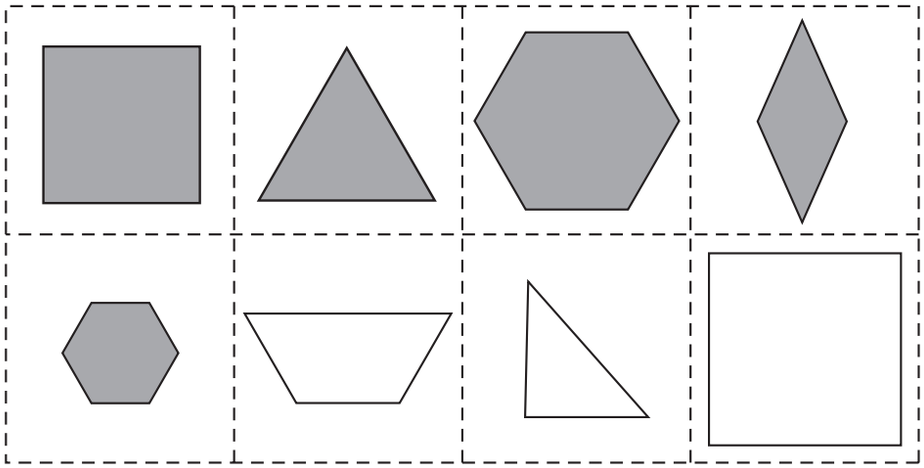
Chapter 5:
Probability and Statistics

Perplexing Polygons
grade 6



Cut along dotted lines

In Mrs. Mac's class each student is given a set of polygon cards to cut out. The polygons are either shaded or not shaded, as shown above. Juan and Monica cut out each of their cards and put them in their own brown paper bag. Answer the following questions.

1. Determine the following probabilities. Justify your answers.

   a. Juan draws a triangle from his bag.

   b. Juan draws a polygon that is not a triangle from his bag.

2. If Juan draws from the bag 60 times and replaces the polygon each time before drawing again, how many times would you expect the polygon to be shaded?

3. Juan and Monica both put their polygons in one bag. If Juan draws from the bag 60 times and replaces the polygon each time before drawing again, how many times would you expect the polygon to be shaded?

4. Juan puts his shaded polygons in one bag and his unshaded polygons in another bag. If Juan draws one polygon from each bag, what is the sample space? What is the probability of drawing a shaded hexagon and a nonshaded square?

Teacher Notes

Materials

Calculator

Connections to Middle School TEKS

(6.9) Probability and statistics. The student uses experimental and theoretical probability to make predictions. The student is expected to:

(A) construct sample spaces using lists, tree diagrams, and combinations

(B) find the probabilities of a simple event and its complement and describe the relationship between the two

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(C) use ratios to make predictions in proportional situations

Scaffolding Questions

- How can you determine the probability of an event?
- Which of the polygons are quadrilaterals? How do you know?
- Explain the relationship between the questions in 1a and 1b.
- How do you find the probability of drawing a shaded polygon from the bag?
- Describe the effect of pulling a polygon from the bag 60 times.
- What is the effect of putting two sets of polygons together in a bag?
- What does *sample space* mean?

Sample Solutions

1. a. There are two triangles in the set of eight polygons.

$$P(\text{drawing a triangle}) = \frac{2 \text{ triangles}}{8 \text{ polygons}} = \frac{1}{4}$$

b. The probability of not drawing a triangle is the complement of drawing a triangle.

$$P(\text{not drawing a triangle}) = 1 - \frac{1}{4} = \frac{3}{4} \quad \text{or} \quad \frac{6 \text{ nontriangles}}{8 \text{ polygons}} = \frac{3}{4}$$

2. There are 5 shaded polygons in the bag and 8 total polygons in the bag.

$$P(\text{drawing a shaded polygon}) = \frac{5 \text{ shaded polygons}}{8 \text{ polygons}} = \frac{5}{8}$$

The experimental probability would be about $\frac{5}{8}$. If students know that a shaded polygon would be drawn 5 out of 8 attempts, then they can find an equivalent ratio for 60 attempts.

$$\frac{5}{8} = \frac{?}{60}$$

The number you multiply 8 by to get 60 is 7.5.

$$\frac{5}{8} \times \frac{7.5}{7.5} = \frac{37.5}{60}$$

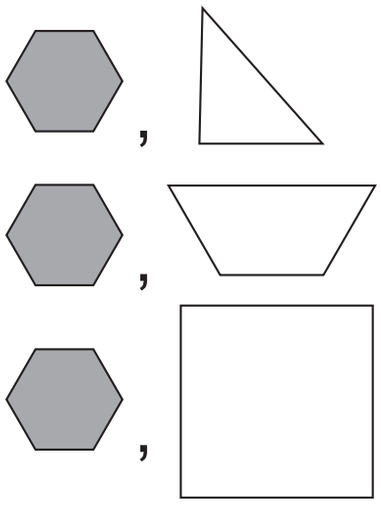
A shaded polygon would be drawn between 37 and 38 times.

3. The probability is not affected by putting two sets of polygons in the bag, because the probability of drawing a shaded polygon is still $\frac{5}{8}$. There are now 10 shaded polygons out of 16 polygons.

$$\frac{10}{16} = \frac{5}{8}$$

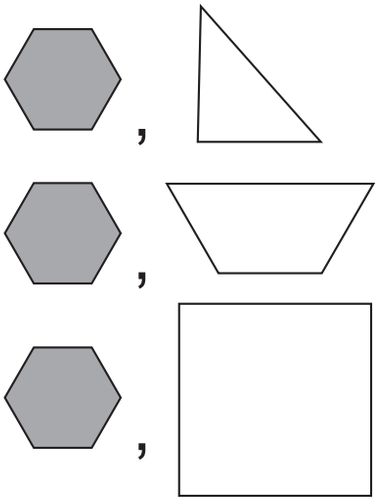
As in problem 2, a shaded polygon would be drawn between 37 and 38 times.

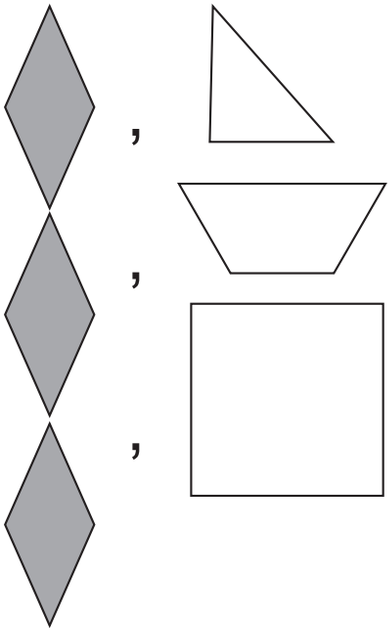
4. The *sample space* is the set of all possible combinations of figures. Because there are 5 shaded polygons and three nonshaded polygons, there are 5 times 3, or 15 possible combinations.

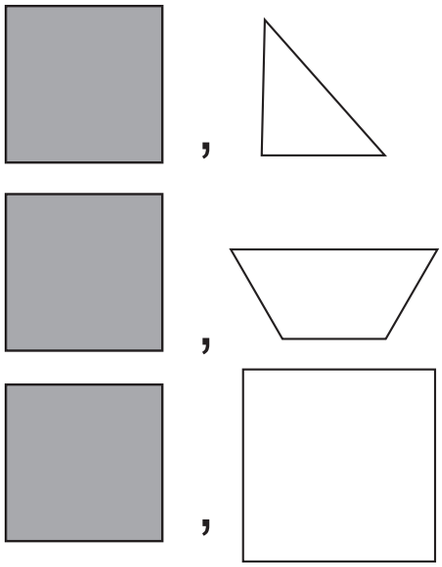


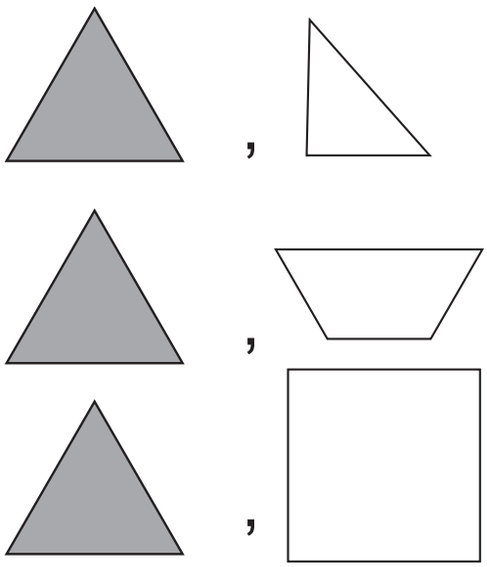
Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.









Two out of the 15 possibilities have a shaded hexagon and a nonshaded square. The probability of drawing a shaded hexagon and a nonshaded square is $\frac{2}{15}$.

Extension Questions

- Describe what could be done to Juan's set of polygons so that the probability of drawing a triangle is $\frac{1}{3}$.

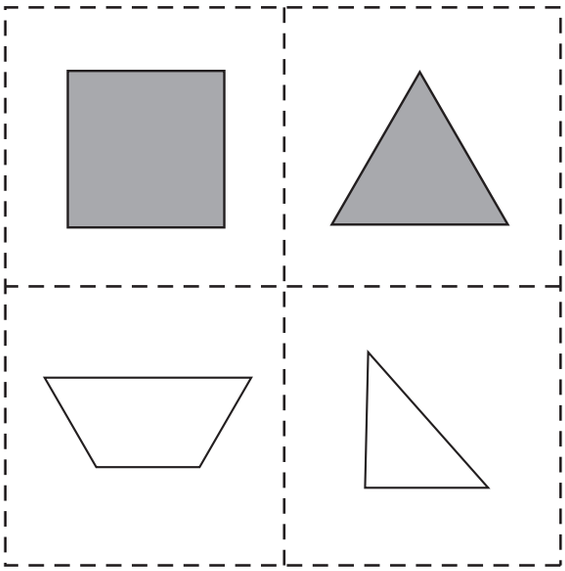
  One possibility is to leave the two triangles in the set of polygons and remove two other polygons from the set. The total number of polygons would then be 6.

$$\frac{2 \text{ triangles}}{6 \text{ polygons}} = \frac{1}{3}$$

- Consider the situation described in problem 4. Describe how removing the triangles from both bags would change your answer.

  The number of shaded polygons would be 4, and the number of nonshaded polygons would be 2. The total number of combinations would be 4 times 2, or 8. There would still be two desired outcomes. The probability would be $\frac{2}{8} = \frac{1}{4}$.

Perplexing Polygons
grade 7



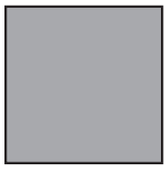
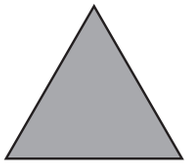
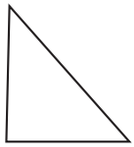
Cut along dotted lines

In Mrs. Mac's class each student is given a set of polygon cards to cut out. The polygons are either shaded or not shaded, as shown above. Juan and Monica cut out each of their cards and put them in their own brown paper bag. Answer the following questions.

1. What is the probability that Juan will draw a shaded polygon from his bag and then Monica will draw a triangle from her bag? Show a sample space to support your answer. If they each draw from the bag 40 times and replace the polygon each time before drawing again, how many times would Juan be expected to draw a shaded polygon and Monica to draw a triangle?

2. Juan draws twice from his bag of polygons and after the first draw he does not put the polygon back into the bag. What is the probability that he will first draw a square and then a triangle? Show a sample space to support your answer.

3. Four of the students each put three of their polygons into the same bag. They do not tell which of the polygons they placed into the bag. A student draws from the bag. The results are shown below.

Type				
Number of times drawn	11	18	10	21

Based on these results, what is the experimental probability of drawing each of the polygons from the bag? What polygon will most likely be drawn next?

Teacher Notes

Materials

Calculator

Connections to Middle School TEKS

(7.10) Probability and statistics. The student recognizes that a physical or mathematical model can be used to describe the probability of real-life events. The student is expected to:

(A) construct sample spaces for compound events (dependent and independent)

(B) find the approximate probability of a compound event through experimentation

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

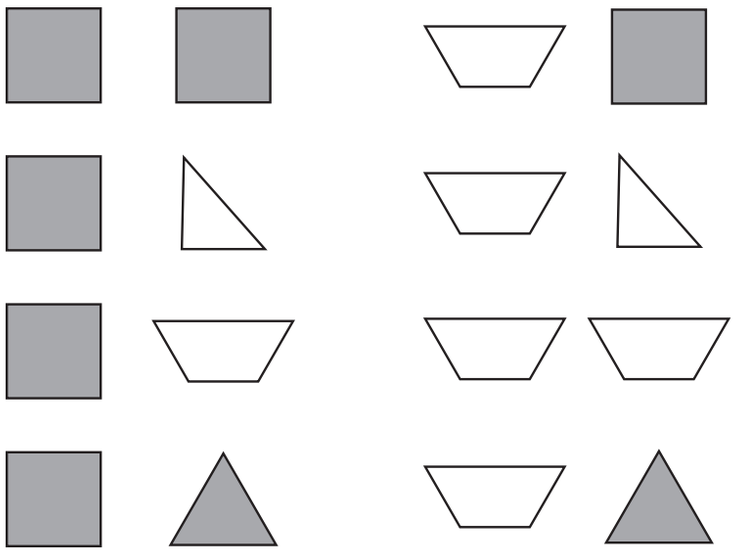
(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

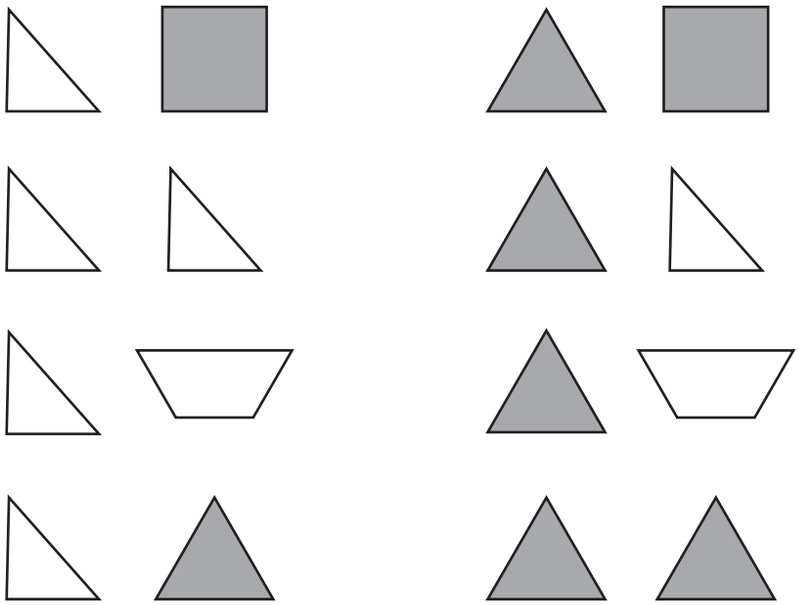
Scaffolding Questions

- How can you determine the probability of an event?
- How do you find the probability of drawing a shaded polygon from the bag?
- How do you find the probability of drawing a triangle from the bag?
- Is the event described in problem 1 simple or compound?
- Is the event dependent or independent?
- Describe the effect of pulling a polygon from the bag 40 times.
- What is the effect of putting four sets of polygons together in a bag?
- Explain how to find out how many polygons are in a bag.
- Based on the table of results in problem 3, how can you find out many times the experiment was conducted?

Sample Solutions

1.





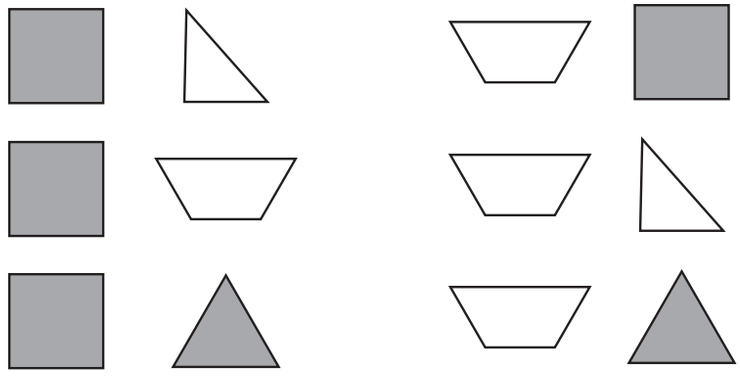
The sample space above shows that the probability of Juan drawing a shaded polygon from his bag and then Monica drawing a triangle from her bag is $\frac{4}{16} = \frac{1}{4}$.

If the experiment is conducted 40 times, the number of times the desired situation might occur is predicted by multiplying the probability by 40.

$$40 \times \frac{1}{4} = \frac{40}{4} = 10$$

It would be expected to happen 10 times.

2.



Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.

In the sample space above, the probability that Juan will first draw a square and then a triangle is 2 out of 12, or $\frac{2}{12}$.

The probability of drawing a square is $\frac{1}{4}$. If the polygon is not replaced there are now 3 polygons in the bag. The probability of drawing a triangle is now 2 out of 3, because there could be 2 triangles left in the bag.

The probability of drawing a square and then a triangle is

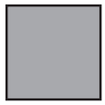
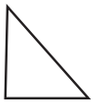
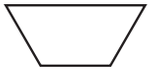
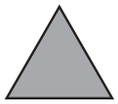
$$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$$

3. The total number of times the experiment was conducted is the sum of the numbers in the table.

$$11 + 21 + 18 + 10 = 60$$

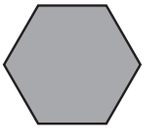
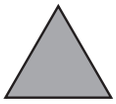
The experiment was conducted 60 times, so to find the probability of drawing each shape, find the number of favorable outcomes and divide by the total number of trials.

$$\frac{\text{number of favorable outcomes}}{\text{total number of trials}}$$

Type				
Probability	$\frac{11}{60}$	$\frac{18}{60} = \frac{3}{10}$	$\frac{10}{60} = \frac{1}{6}$	$\frac{21}{60}$

Based on the experimental probabilities found for drawing each shape, the equilateral triangle has the best probability of being selected.

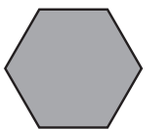
Extension Questions

Type					
Number of times drawn	10	6	5	18	21

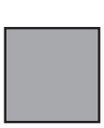
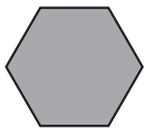
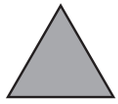
- Four students each put 3 polygons in a bag and conduct an experiment drawing shapes from the bag. The results are recorded above. Predict the number of each type of polygon in the bag.

  If 4 students each put 3 polygons in the bag, there are 12 polygons in the bag.

  Multiply each probability by 12.

Type					
Probability	$\frac{10}{60} = \frac{1}{6}$	$\frac{6}{60} = \frac{1}{10}$	$\frac{5}{60} = \frac{1}{12}$	$\frac{18}{60} = \frac{3}{10}$	$\frac{21}{60}$
Fraction times 12	$\frac{1}{6}(12) = 2$	$\frac{1}{10}(12) = 1.2$	$\frac{1}{12}(12) = 1$	$\frac{3}{10}(12) = 3.6$	$\frac{21}{60}(12) = 4.2$

There cannot be a fractional number of polygons in a bag, so the numbers must be rounded to whole numbers. One possible way to round is as follows:

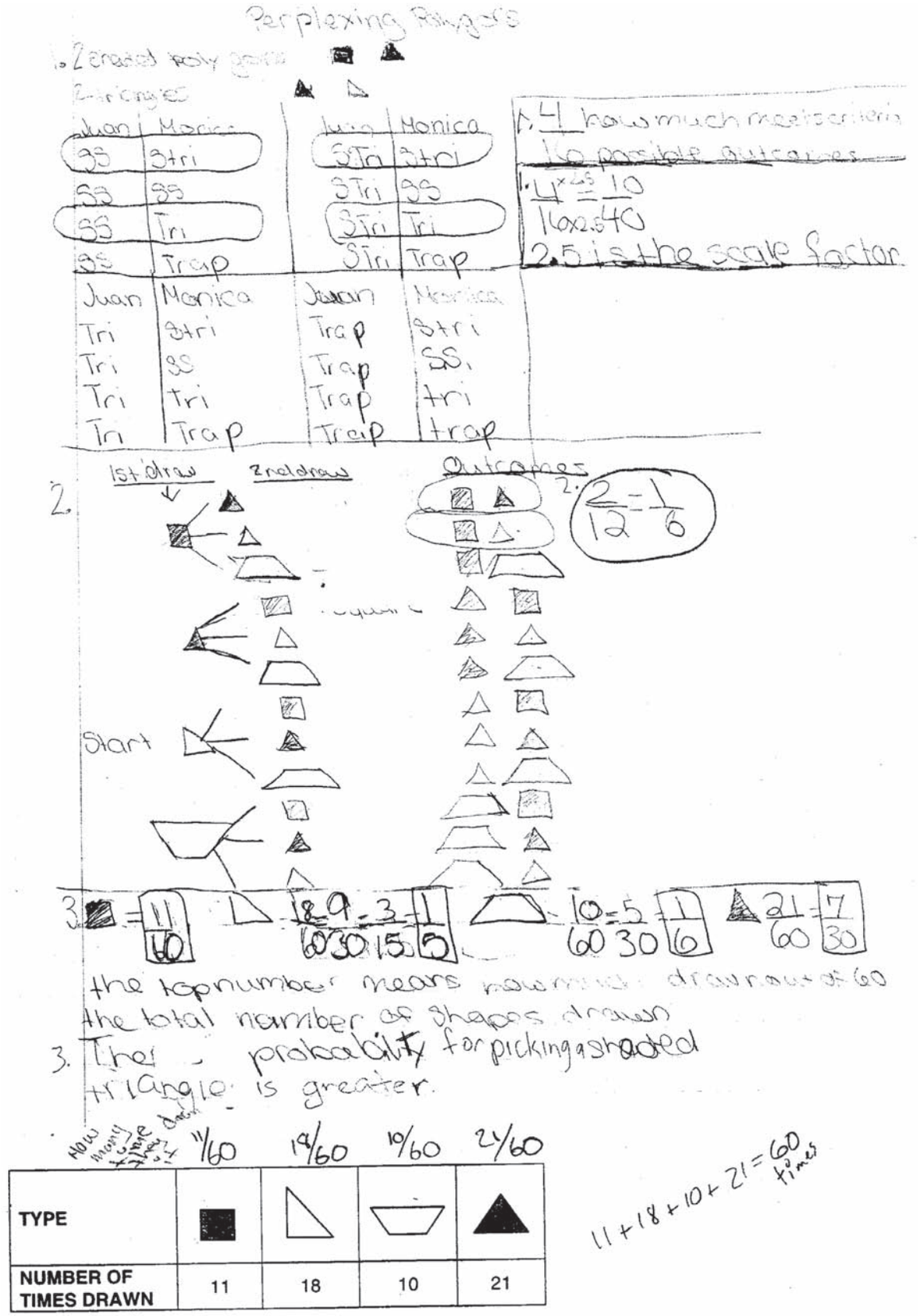
Type					
Possible number	2	1	1	4	4

Student Work Sample

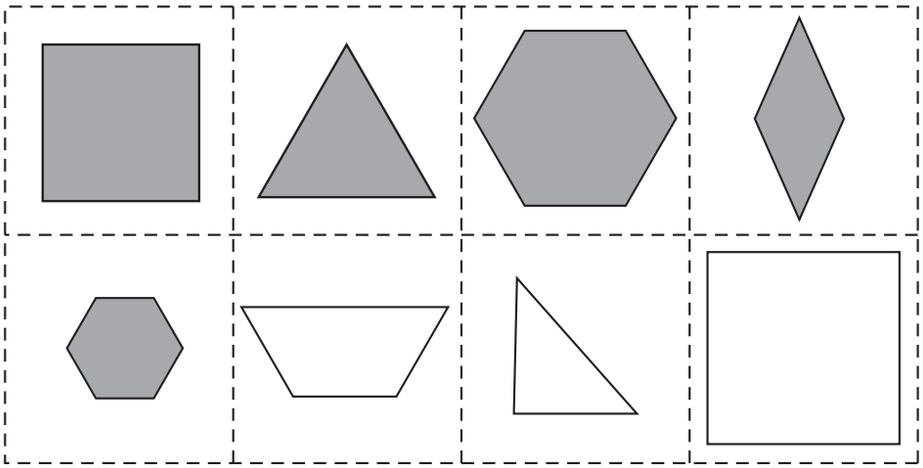
This student's work shows the use of different graphic organizers to find solutions.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Recognizes and applies proportional relationships
- Develops and carries out a plan for solving a problem that includes understand the problem, select a strategy, solve the problem, and check
- Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations
- Demonstrates an understanding of mathematical concepts, processes, and skills
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them
- Communicates clear, detailed, and organized solution strategy


Perplexing Problems
1. 2 created poly gons
2 triangles
Juan | Monica
3S | 3tri
SS | 3S
SS | Tri
3S | Trap
Juan | Monica
Tri | 3tri
Tri | 3S
Tri | Tri
Tri | Trap
Juan | Monica
3Tri | 3tri
3Tri | 3S
3Tri | Tri
3Tri | Trap
Juan | Monica
Trap | 3tri
Trap | SS
Trap | tri
Trap | trap
1. 4 how much meet criteria
16 possible outcomes
1. 4 x 2.5 = 10
16 x 2.5 40
2.5 is the scale factor
2.
1st draw
2nd draw
outcomes
2. 2/12 = 1/6
Start
3. 11/60
18/60 = 9/30 = 3/15 = 1/5
10/60 = 5/30 = 1/6
21/60 = 7/30
the top number means how many drawn out of 60
the total number of shapes drawn
3. The probability for picking a shaded triangle is greater.
How many times drawn?
11/60 18/60 10/60 21/60
11+18+10+21=60 times
TYPE
NUMBER OF TIMES DRAWN
11
18
10
21

Perplexing Polygons
grade 8



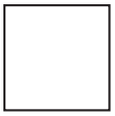
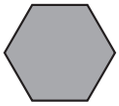
Cut along dotted lines

In Mrs. Mac's class each student is given a set of polygon cards to cut out. The polygons are either shaded or not shaded, as shown above. Juan and Monica cut out each of their cards and put them their own brown paper bag. Answer the following questions.

1. Juan draws a polygon from his bag and then Monica draws a polygon from her bag. If they each draw from the bag 50 times and replace the polygon each time before drawing again, how many times would Juan be expected to draw a shaded polygon and Monica to draw a triangle?

2. Juan draws twice from his bag of polygons and after the first draw he does not put the polygon back into the bag. Predict the number of times that he might draw a square and then a triangle if he conducts the experiment 24 times.

3. Five of the students each put four of their polygons into the same bag. They do not tell which of the polygons they placed into the bag. They conduct an experiment to see if they can tell which of the polygons were placed in the bag. The results are shown below. Predict the contents of the bag. Justify your reasoning.

Type								
Number of times drawn	10	14	11	23	6	9	0	27

4. Eight of the students in the class place all of their polygons into a bag. What is the probability of drawing a hexagon? They are asked to remove 16 polygons from the bag. How can they do this so that the probability of drawing a hexagon remains the same?

5. Eight of the students in the class place all of their polygons into a bag. Describe how to remove polygons from this bag so that the probability of drawing a hexagon is $\frac{1}{5}$.

Teacher Notes

Materials

Calculator

Connections to Middle School TEKS

(8.11) Probability and statistics. The student applies concepts of theoretical and experimental probability to make predictions.The student is expected to:

(A) find the probabilities of compound events (dependent and independent)

(B) use theoretical probabilities and experimental results to make predictions and decisions

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

Scaffolding Questions

- How do you determine the probability of an event?
- How do you find the probability of drawing a triangle from the bag?
- Is the event described in problem 1 simple or compound? Explain how you know.
- Is the event dependent or independent? Describe how you know.
- Describe the effect of pulling a polygon from the bag 50 times.
- What is the effect of putting four sets of polygons together in a bag?
- Explain how to find out how many polygons are in the bag.
- Based on the results in the table in problem 3, how can you find out many times the experiment was conducted?
- Based on the results in the table in problem 3, how many times was a shaded square drawn from the bag? What does that mean about the possible number of shaded squares in the bag?

Sample Solutions

1. This is a problem about two independent events. The probabilities should be multiplied.

   P(drawing a shaded polygon from Juan's bag) •
   P(drawing a triangle from Monica's bag) =

$$\frac{5}{8} \bullet \frac{2}{8} = \frac{10}{64} = \frac{5}{32}$$

If the experiment is conducted 50 times, the number of times the desired situation might occur is predicted by multiplying the probability by 50.

$$50 \bullet \frac{5}{32} = \frac{250}{32} = 7.8125$$

It would be expected to happen between 7 and 8 times.

2. The probability of drawing a square the first time is 2 out of 8 or $\frac{1}{4}$.

   If the polygon is not replaced, there are now 7 polygons in the bag. The probability of drawing a triangle is now 2 out of 7, because there could be 2 triangles left in the bag.

   The probability of drawing a square and then a triangle is

$$\frac{1}{4} \bullet \frac{2}{7} = \frac{2}{28} = \frac{1}{14}$$

   If Juan conducts the experiment 24 times, he will probably repeat the desired outcome between 1 and 2 times because $24 \bullet \frac{1}{14} \approx 1.7$.

3. There are 5 groups of 4, or 20 polygons in the bag. The experiment was conducted 100 times, because the sum of the numbers in the table is 100.

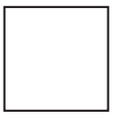
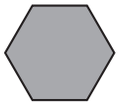
   The probabilities may be computed.

Type								
Number of times drawn	10	14	11	23	6	9	0	27
Probability	$\frac{10}{100} = \frac{1}{10}$	$\frac{14}{100} = \frac{7}{50}$	$\frac{11}{100}$	$\frac{23}{100}$	$\frac{6}{100} = \frac{3}{50}$	$\frac{9}{100}$	0	$\frac{27}{100}$

   To determine the possible number of each type, multiply the probability by the number of polygons, 20.

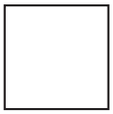
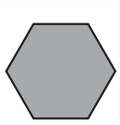
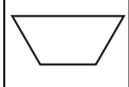
Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.

Type				
Number of times drawn	10	14	11	23
Probability	$\frac{10}{100} = \frac{1}{10}$	$\frac{14}{100} = \frac{7}{50}$	$\frac{11}{100}$	$\frac{23}{100}$
Possible number	$\frac{1}{10}(20) = 2$	$\frac{7}{50}(20) = 2.8$	$\frac{11}{100}(20) = 2.2$	$\frac{23}{100}(20) = 4.6$

Type				
Number of times drawn	6	9	0	27
Probability	$\frac{6}{100} = \frac{3}{50}$	$\frac{9}{100}$	0	$\frac{27}{100}$
Possible number	$\frac{6}{100}(20) = 1.2$	$\frac{9}{100}(20) = 1.8$	0	$\frac{27}{100}(20) = 5.4$

The number of polygons in the bag cannot be a fractional amount. The numbers should be rounded so that the total number of polygons is 20. One possible answer is as follows:

Type								
Possible number of polygons	2	3	2	5	1	2	0	5

4. There are 8 groups of 8, or 64 polygons in the bag. In each group there are 2 hexagons, one large and one small. There are 16 hexagons in the bag. So the probability of drawing a hexagon is $\frac{16}{64} = \frac{1}{4}$. To remove 16 polygons and keep the probability of drawing a hexagon the same, one-fourth of the polygons removed must be hexagons. So 4 hexagons must be removed.

$$16 \times \frac{1}{4} = 4$$

The probability of selecting a hexagon becomes $\frac{12}{48} = \frac{1}{4}$.

5. There are 8 groups of 8, or 64 polygons in the bag. In each group there are 2 hexagons, one large and one small. There are 16 hexagons in the bag. For the probability to be $\frac{1}{5}$, 4 of the hexagons could be removed from the bag. The probability would then be $\frac{12}{60} = \frac{1}{5}$.

Extension Questions

- Construct a bag where the probability of drawing a shaded square is $\frac{1}{4}$ and the probability of drawing a right triangle is $\frac{1}{10}$. You must have at least 1 of each of the 8 shapes in your bag.

  There are many possible solutions. One solution is to have a bag that consists of 10 shaded squares, 4 right triangles, 5 trapezoids, 8 large squares, 2 large hexagons, 4 small hexagons, 5 equilateral triangles, and 2 rhombi. There would be a total of 40 polygons, which would make the probability of drawing a shaded square $\frac{10}{40}$ *, or* $\frac{1}{4}$*, and the probability of drawing a right triangle* $\frac{4}{40}$*, or* $\frac{1}{10}$*.*

Science Quiz
grade 6

Ms. Ross's sixth-grade class at Longhorn Middle School took a science quiz and did not do very well. Below are the grades 18 students received.

10	30	50
60	70	60
70	40	30
40	50	10
40	60	20
40	30	30

Ms. Ross decided to find the median, mode, and range of the grades. She did not like what she discovered, so she decided she would adjust the grades.

1. What were the median, mode, and range that Ms. Ross found? Explain how she found them.

2. Mr. Gray suggested that Ms. Ross just add 30 points to everyone's grade. If she does this, what will the new grades be? Will this change the median, mode, and range of the grades? What are the new median, mode, and range?

3. Ms. Brown suggested that Ms. Ross increase everyone's grade by 20% instead. If Ms. Ross decides to do this, what will the new grades be? Will this change the median, mode, and range of the grades? What are the new median, mode, and range?

4. How are the median, mode, and range affected by the change Mr. Gray suggested? How are they affected by Ms. Brown's suggestion? How are these changes alike and how are they different?

Teacher Notes

Connections to Middle School TEKS

(6.10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to:

(B) use median, mode, and range to describe data

(D) solve problems by collecting, organizing, displaying, and interpreting data

Scaffolding Questions

- How do you find the median of the data?
- How do you find the mode of the data?
- How do you find the range of the data?
- If you made a 70, how would Mr. Gray's suggestion affect your grade?
- If you made a 70, how would Ms. Brown's suggestion affect your grade?
- What are the grades if 30 points are added to each?
- What are the grades if 20% of the grade is added to each?
- Do you get more points when 30 points are added or when 20% of the grade is added?

Sample Solutions

1. Ms. Ross needs to order the grades from least to greatest and determine the grade for which half the grades received are below that grade and half the grades are above that grade.

   Here are the grades in order from least to greatest:

   10,10,20,30,30,30,30,40,40,40,40,50,50,60,60,60,70,70

   Since there are 18 grades, two of these grades, 40 and 40, fit the requirement. Average those two grades and the median grade will be 40:

   $$40 + 40 = 80$$

   $$80 \div 2 = 40$$

   The mode of the grades is the grade that most students received. In this data there are 2 modes, since the grades, 30 and 40 appear 4 times each on the list.

The range of the data can be described as the interval from the least grade to the greatest grade or the difference between the greatest grade received and the least grade received. The range of the grades will be 10 to 70, or 60.

Median: 40

Mode: 30 and 40

Range: 10 to 70, or 60

2. If Ms. Ross takes Mr. Gray's suggestion, the grades will be as follows:

40	60	80
90	100	90
100	70	60
70	80	40
70	90	50
70	60	60

Here are the new grades in order from least to greatest:

40,40,50,60,60,60,60,70,70,70,70,80,80,90,90,90,100,100

The median and mode will each increase by 30 points, but the range will not increase.

Median: 70

Mode: 60 and 70

Range: 40 to 100, or 60

Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.

3. If Ms. Ross takes Ms. Brown's suggestion, 20% of the grade will be added to the grade, or each grade will be multiplied by 120% or 1.2. The grades will be as follows:

12	36	60
72	84	72
84	48	36
48	60	12
48	72	24
48	36	36

Here are the new grades in order from least to greatest:

12,12,24,36,36,36,36,48,48,48,48,60,60,72,72,72,84,84

The median, mode, and range will all increase from the original grades.

Median: 48

Mode: 36 and 48

Range: 12 to 84, or 72

4. With Mr. Gray's suggestion, all grades are increased by 30 points; therefore, the median and the mode also increased by 30. The range was not affected because everyone's grade shifted when 30 points was added to each grade.

With Ms. Brown's suggestion, each grade increases depending on the original grade. Grades increased 2 points for every 10 points of the original grade. The new median, mode, and range were increased by 20%.

When 30 points are added to each grade, all the grades increase by the same amount; therefore, the median and mode also increase by the same amount. The range is not affected because the spread of the data remains the same.

However, if each grade is increased by a rate of 20%, not only are the median and mode increased by 20%, but the spread of the data is also increased by 20% because individual grades increased not by the same amount but by a rate of 20%.

Extension Questions

- Ms. Ross likes Ms. Brown's suggestion, but she wants at least $\frac{1}{2}$ of her students to pass. Help Ms. Ross achieve her goal.

  If Ms. Ross wants to use a suggestion like Ms. Brown's, the grades must all increase by the same percentage. The grades will have to increase so that $\frac{1}{2}$ of the students pass. If you look at the median grade of 40, that grade will have to increase by 30 points to a 70. Fifty percent of 40 is 20, which is not enough points. Ms. Ross would need another 10 points to get the original grade to 70. If 50% adds 20 points to the grade, another 25% would add 10 points; therefore, Ms. Ross needs to increase the percentage to 75%.

  75% of 40 is 30 and 30 + 40 = 70

Big Money Prizes
grade 7

A local radio station is planning a contest. Each winner will select a money envelope. The radio station is planning on having 150 winners and giving away $6,000. Below is the plan for filling the envelopes.

Number of envelopes	Amount of money
1	$5,000
2	250
4	50
12	10
6	5
25	2
100	1

1. What is the typical amount of money a winner will receive? What are the mean, median, mode, and range of the amount won?

2. The sponsors decide to double the amount of contest money they give away. The station manager wants to double the amount of money in each of the envelopes. How would this affect the typical amount of money a winner would receive? How would this affect the mean, median, mode, and range of the amount won?

3. The DJs think it would be better to double the number of winners rather than doubling the amount of money in each envelope. They want to double the numbers of envelopes containing each money amount. How would this affect the typical amount of money a winner would receive? How would this affect the mean, median, mode, and range of the amount won?

Teacher Notes

Materials

Calculator (optional)

Connections to Middle School TEKS

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

(7.12) Probability and statistics. The student uses measures of central tendency and range to describe a set of data. The student is expected to:

(A) describe a set of data using mean, median, mode, and range

(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation

Scaffolding Questions

- What does the mean of the data tell you? What do the median, mode, and range tell you?
- How many envelopes contain $50?
- How much money will be given away in $50 envelopes?
- How many envelopes contain $2?
- How much money will be given away in $2 envelopes?
- How can you find the mean of the data?
- How can you find the median of the data?
- How can you find the mode of the data?
- How can you find the range of the data?

Sample Solutions

1. The typical amount of money can be described by using the mean, median, or mode. The mean is the total amount of money given away divided by the number of winners.

   The number of winners is the sum of the number of envelopes, or 150 winners.

   To verify that $6,000 is given away, consider the amount times the number of people who received that amount.

   $$1(\$5{,}000) + 2(\$250) + 4(\$50) + 12(\$10) + 6(\$5) + 25(\$2) + 100(\$1) = \$6{,}000$$

   Since $6,000 will be the amount given to 150 winners, the mean amount of money will be $6,000 ÷ 150 winners = $40 per winner. The median amount of money will be the amount for which half the winners receive at least that amount and half the winners receive at most that amount. One-half of 150 is 75. Since 100 people receive

the least amount of money, $1, the median amount will be $1. The mode amount of money will be the amount of money that most winners will receive. There are 100 people receiving $1. The mode amount of the money will be $1. The range of the data can be described as the interval from the least amount won to the greatest amount or the difference between the greatest amount won and the least amount won. The range of the winnings will be $1 to $5,000, or $4,999.

2. If the amount of money in each of the envelopes is doubled, the typical amount of money will double. The mean is $12,000 divided by 150, or $80. The mean and the median would be the minimum amount, or $2. The mean, median, and mode will double. The mean will be $80, the median will be $2, and the mode will be $2. The range of the new amounts will be $2 to $10,000, or $9,998.

3. If the number of winners is doubled by doubling the number of envelopes containing each amount of money, the typical amount of money a winner will receive stays the same as in the original plan. Consider the amount times the number of people who received that amount.

$$2(\$5{,}000) + 4(\$250) + 8(\$50) + 24(\$10) + 12(\$5) + 50(\$2) + 200(\$1) = \$12{,}000$$

The mean will be $12,000 ÷ 300 winners or $40 per winner. The median amount will be $1. The mode amount of the money will be $1. The range of the winnings will be $1 to $5,000, or $4,999.

Extension Questions

- Is there another way of giving away the extra $6,000 so that the mean doubles but the mode and median stay the same?

  A possible scenario would be to give the top winner $11,000 while the rest of the winnings remain the same.

- If the radio station decides to triple the amount in each of the envelopes, how would this affect the

Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.

typical amount of money a winner will receive? What if they quadruple the amount? Increase the amount by 5 times? Describe how scaling the amount in each envelope affects the typical amount of money a winner will receive.

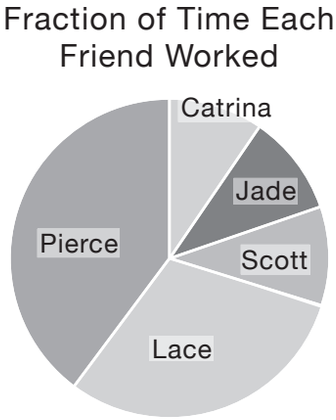
To examine the effects, the student can create a table as follows:

Scaling of the amount in envelope	Mean	Median	Mode
1	40	1	1
2	80	2	2
3	120	3	3
4	160	4	4
5	200	5	5

The mean, median, and mode are increased by the same factor as the amounts in the envelopes.

Five Friends
grade 8

Five friends got an after-school job cleaning out Mr. Hill's garage. The circle graph below shows the amount of the time each friend worked.



Catrina, Jade, and Scott all worked the same length of time. Pierce and Catrina together worked $\frac{1}{2}$ of the total amount of the time the group worked. Lace worked a total of 3 hours; the amount of time she worked was equal to the amount of time that Catrina, Jade, and Scott worked all together.

1. Find the mode, median, mean, and range for the number of hours each worked.
2. If the group got paid $40, how much should each get paid? What will the mode, median, mean, and range for the amount each earned be if the friends split the $40 proportional to the amount of time they worked?
3. Mr. Hill decided to give the friends a bonus of $10 because they did such a good job. If they split the bonus evenly, how would this affect the amount each friend earned? How would this affect the mode, the median, the mean, and the range of the amount that each earned?
4. Pierce said, "It's not fair to split the bonus evenly. I think we should split it proportional to the amount of time each of us worked." If the friends follow Pierce's plan, how much of the bonus will each earn? If the bonus is divided this way, how do the mode, median, mean, and range of the total amount that each earned change due to the bonus? How does this compare with the change from the even split?

Teacher Notes

Materials

Scissors

Connections to Middle School TEKS

(8.12) Probability and statistics. The student uses statistical procedures to describe data. The student is expected to:

(A) select the appropriate measure of central tendency to describe a set of data for a particular purpose

(B) draw conclusions and make predictions by analyzing trends in scatterplots

Scaffolding Questions

- How can you find the mode of a set of data?
- How can you find the median of a set of data?
- How can you find the mean of a set of data?
- How can you find the range of a set of data?
- What does the circle graph represent?
- What do you get when you add up the different fractional parts of the circle graph?
- How do you find what each part of the circle graph represents?
- Who has worked the greatest amount of time?
- Who has worked the least amount of time?
- Do you see any relationships among the amount of time each friend worked?
- How do you divide the money evenly?
- How do you divide the money based on the amount of hours worked by each friend?

Sample Solutions

1. Using the circle graph and the information given, students can establish the amount of time each person worked. First, Catrina, Jade, and Scott all worked an equal amount of time. To verify this using the circle graph, students can cut out the portion of time Catrina worked and overlay this portion of the circle graph on Jade's and Scott's portion of the graph. The three friends had the same portion of the circle graph. If Lace worked 3 hours and this was equivalent to the amount of time Catrina, Jade and Scott worked, then Catrina, Jade, and Scott must have each worked one hour.

By looking at the circle graph, students can figure out that $\frac{1}{2}$ of the time worked by the group equals 5 hours because Jade's 1 hour + Scott's 1 hour + Lace's 3 hours = 5 hours. If $\frac{1}{2}$ the time was equal to 5 hours, then Pierce must have worked 4 hours, because Catrina's 1 hour + Pierce's time must equal 5 hours, $\frac{1}{2}$ the time. Therefore, Catrina worked 1 hour, Jade worked 1 hour, Scott worked 1 hour, Lace worked 3 hours, and Pierce worked 4 hours.

Based on the amount of time found above, the mode, median, mean, and range could now be found. Since 3 of the 5 friends worked 1 hour each, this amount of time was most often worked; therefore, the mode is 1 hour.

The median of the data may be found by ordering the data from the least amount of time worked to the greatest amount of time worked then finding the data point for which half of the values of the data fall below it and half the values of the data are above it. When this is done, the median is found.

1, 1, 1, 3, 4

The median of the data is 1 hour.

The mean of the data can be found by adding the amounts of time each worked and dividing the sum by 5, because there were 5 amounts.

$1 + 1 + 1 + 3 + 4 = 10$

$10 \div 5 = 2$

The mean of the data is 2 hours.

Finding the least amount of time worked and the greatest amount of time worked will give the spread of the data or the range. The range can also be the difference of these amounts. The least amount of time worked was 1 hour and the greatest amount of time worked was 4 hours, so the range of the data is 1 hour to 4 hours, or 3 hours.

2. If the friends decided to divide the money evenly, then each would get \$40 ÷ 5, or \$8. If this were done the

Texas Assessment of Knowledge and Skills

Objective 5: The student will demonstrate an understanding of probability and statistics.

mode would be $8. The median would be $8, the mean would be $8, and there would be no difference in the range of amounts of money each received, so the range would be from $8 to $8, or 0. However, if the friends decided to divide the money based on the amount of time each worked, this would change the mode, median, mean, and range. This may be done by finding an amount of money each should get per hour worked, $40 ÷ 10 hours = $4 per hour. Since Catrina, Jade, and Scott worked 1 hour each would get $4. Lace worked 3 hours so she would get $12, and Pierce, who worked 4 hours, would get $16.

The amounts of money received are $4, $4, $4, $12, and $16.

With this data the mode would be $4, the median would be $4, the mean would be $8, and the range would be from $4 to $16, or $12. The mean would stay the same because even though each friend earned a different amount of money, they still received a total of $40 for the five to split up.

3. If they split the bonus evenly, each friend would get $2 more. If they divided the original amount by 5, then each person would receive $8 + $2, or $10. If they were paid proportional to the amount of time they worked, each person would get $2 more, which means Catrina, Jade, and Scott would get $6, Lace would get $14, and Pierce would get $18.

   The amounts of money received are $6, $6, $6, $14, and $18.

   In this case, the mode would be $6, the median would be $6, the mean would be $10, and the range would be from $6 to $18, or $12. If each friend got $2 more, the mode, median, and mean would each increase by $2; however, the range would stay the same since the least amount of money earned would be $6 and the greatest amount would be $18. The range of the data shifted, but the spread or difference of the range remained $12.

4. If the friends decided to divide the bonus based on the amount of time each worked, they could find the hourly rate for the bonus and give each friend that amount based on the amount of time worked. The hourly bonus rate would be $10 ÷ 10 hours worked equals $1 per hour; therefore, Catrina, Jade, and Scott would each get $1 added to their $4 earned, for a total of $5 each. Lace would get $3 added because she worked 3 hours originally, which would make her total earnings $15, and Pierce would get $4 of the bonus since he originally worked 4 hours, which would make his total earnings $20.

   The amounts of money received are $5, $5, $5, $15, and $20.

   The mode would now be $5, the median would be $5, the mean would be the same as when the bonus is divided evenly, or $10, and the range would be from $5 to $20, or $15.

   Notice that when the bonus was distributed proportionally, each friend's earnings were

increased by a scale factor of $\frac{5}{4}$ because the amount of money with the bonus is \$50, or $\frac{5}{4}(\$40)$.

The mode, median, mean, and range were all increased by that same scale factor.

Mean or Median	Mean	Range
$\frac{5}{4}(\$4) = \$5$	$\frac{5}{4}(\$8) = \$10$	$\frac{5}{4}(\$12) = \$15$

This would not be the case if the bonus were split evenly. The mean was the same for each scenario because the total amount earned for the job stayed at \$50 split among 5 friends.

The range changed because now each of the amounts the friends earned grew proportionally. The friends who worked the least amount of time received the least amount of bonus, and the friends who worked more received a larger amount of the bonus, so the range of the data became greater.

Extension Questions

- If the total number of hours increased to 30 hours and the friends still worked proportionally the same amount of time, how many hours did each friend work?

  If each friend still works proportionally the same amount of time, then the amount of time they work for 30 hours will increase proportionally. If the total hours worked increased from 10 hours to 30 hours, the time was increased by 3 times the amount. Therefore Catrina, Jade, and Scott each worked 3 times the original amount of time, 3 x 1 hour = 3 hours. Lace worked 3 x 3 hours = 9 hours, and Pierce worked 3 x 4 hours = 12 hours.

  3 hours + 3 hours + 3 hours + 9 hours + 12 hours = 30 hours

- If Jade worked a total of 10 hours and everyone else still worked proportionally the same amount of time, how many total hours did the group work?

  If Jade worked 10 hours, this means that Catrina and Scott also worked 10 hours each. Lace worked the same number of hours as did Catrina, Jade, and Scott all together, so Lace worked a total of 30 hours. Jade, Scott, and Lace worked $\frac{1}{2}$ the time, which means they worked 10 hours + 10 hours + 30 hours = 50 hours. Since Catrina's hours plus Pierce's hours also equals $\frac{1}{2}$ the amount of the total time the group worked, then Pierce had to have worked 40 hours. Another way to find the amount of time each worked is to find the scale at which Jade increased the number of hours worked. Jade went from 1 hour to 10 hours, a scale factor of 10. Since everyone worked proportionally, this means everyone worked 10 times the amount

of time they had originally worked. Catrina, Jade, and Scott worked 1 hour x 10 = 10 hours. Lace worked 3 hours x 10 = 30 hours, and Pierce worked 4 hours x 10 = 40 hours.

- If each data point in a data set is doubled, will the mode, median, mean, and range of the resulting data set all be two times the values for the original data set? How would doubling each point affect the shape of a line plot of the data?

  Yes, scaling each data point results in all of the measures of center and range scaled by the same factor. The line plot will have a similar shape but will be dilated (rescaled).