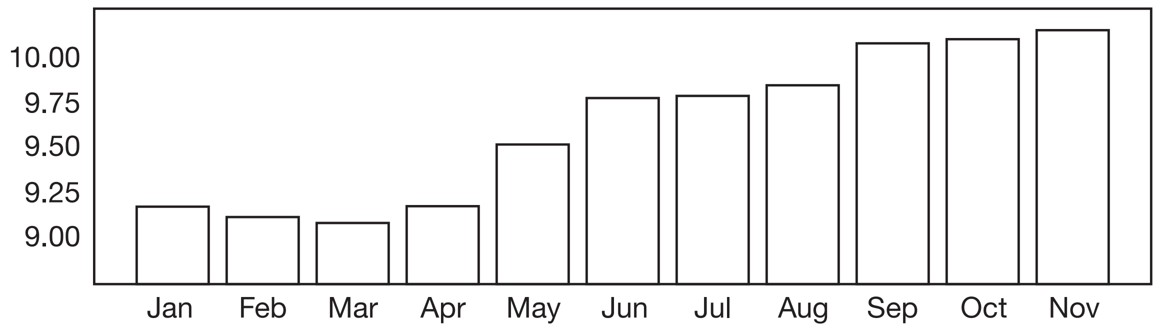
Chapter 6:
*Underlying Processes
and Mathematical Tools*

What's in Your Wallet?
grade 6

Upon returning from a two-week vacation in the year 2002 from Mexico City, the Marcos family visited a bank to exchange their unspent Mexican pesos for U.S. dollars. They had 178 pesos they wished to exchange for U.S. dollars.

Below is the average number of Mexican pesos (MXN) for one U.S. Dollar (1 USD).

Exchange rates for Mexican pesos to 1 U.S. dollar, 2002



January	9.164	MXN
February	9.106	MXN
March	9.073	MXN
April	9.166	MXN
May	9.509	MXN
June	9.767	MXN
July	9.779	MXN
August	9.838	MXN
September	10.071	MXN
October	10.094	MXN
November	10.144	MXN

1. Based on this data, approximately how many USDs should the family expect to receive in exchange for their pesos? Justify your answer.

2. Explain how you could determine which month would have resulted in fewer USDs in exchange for the pesos. Use language and graphical, numerical, physical, and/or algebraic models to communicate your reasoning.

Materials

Calculator

Connections to Middle School TEKS

(6.1) Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms. The student is expected to:

(A) compare and order non-negative rational numbers

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates

(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

Teacher Notes

Scaffolding Questions

- What does the information given in the table mean?
- What is an approximate exchange rate for changing Mexican pesos to USDs?
- Approximately how many Mexican pesos would the family need to exchange in order to receive 1 USD? 2 USDs? 3 USDs?
- How could you organize this information?
- What patterns do you see?
- What is the scale on the vertical axis of the bar graph?

Sample Solutions

1. According to the table, the exchange rate between the U.S. dollar and Mexican peso fluctuated between about 9 and 10 Mexican pesos for every 1 USD. The conversions are organized in the table below.

Lower-end approximate exchange rate:
1 USD ≈ 9 Mexican pesos

USD	1	2	3	4	5	6	7	. . .	10	20	d
Pesos	9	18	27	36	45	54	63	. . .	90	180	$9d$

A pattern exists that for every 1 USD there are about 9 times as many pesos. Therefore, 20 USDs is about 180 pesos (20 x 9 = 180).

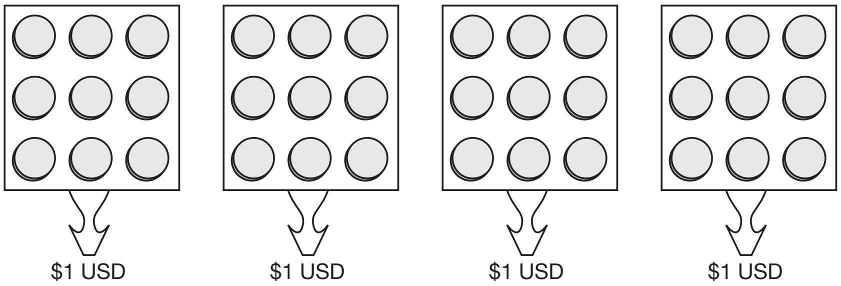
Higher-end approximate exchange rate:
1 USD ≈ 10 Mexican pesos

USD	1	2	3	4	5	6	7	. . .	17	18	19	20	d
Pesos	10	20	30	40	50	60	70	. . .	170	180	190	200	$10d$

Now the pattern states that for every 1 USD there are about 10 times as many pesos. Therefore, 18 USDs is about 180 pesos (18 x 10 = 180).

If the Marcos family exchanged their 178 pesos for USDs, they would receive approximately $18 to $20, depending on the daily exchange rate.

2. According to the bar graph, November had the highest exchange rate with a rate of about 10.14 pesos for 1 USD. This means that it would take more pesos to equate to 1 USD in November than it would in March, when it would take only 9 pesos to equate to 1 USD. Consider the simple example given below, where each circle represents 1 peso.



In March, the 36 pesos above could have been exchanged for about 4 USDs at the rate of 1 USD for every 9 pesos. In November, when the exchange rate was at its highest average, 36 pesos could not be exchanged for 4 USDs. It would take 40 pesos to equate to 4 USDs at the rate of 1 USD for every 10 pesos. Therefore, the higher the exchange rate, the more pesos are needed to make 1 USD, which means you get fewer U.S. dollars for your peso.

Based on the tables given in problem 1, the Marcos family would have received nearly 20 USDs in exchange for their 178 pesos in March. However, in November, the same 178 pesos would have been exchanged for only $18 dollars (approximately).

(A) use ratios to describe proportional situations

(B) represent ratios and percents with concrete models, fractions, and decimals

(C) use ratios to make predictions in proportional situations

(6.10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to:

(B) use median, mode, and range to describe data

(D) solve problems by collecting, organizing, displaying, and interpreting data

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas

Extension Questions

- If the Marcos family exchanged their pesos in the month of May at an exact exchange rate of 9.5 pesos to 1 USD, how could you determine the amount of USDs the family would receive?

Exchange rate: 1 USD = 9.5 Mexican pesos

USD	1	2	3	4	5	6	7	· · ·	10	· · ·	**?**	*d*
Pesos	9.5	19	28.5	38	47.5	57	66.5	· · ·	95	· · ·	**178**	9.5*d*

The pattern shows that to find the number of pesos, you multiply the number of USDs by 9.5.

Use a calculator to see how many groups of 9.5 are in 178 . Divide as shown below:

$$178 \div 9.5 \approx 18.74$$

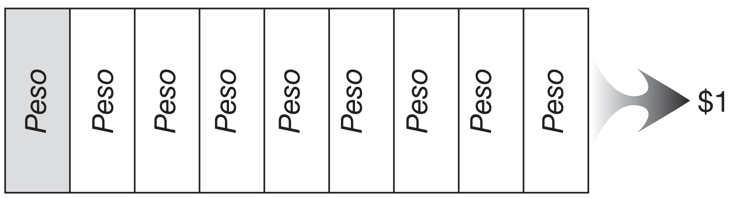
Multiplying the rate $\frac{1\text{ USD}}{9.5\text{ pesos}}$ *by* $\frac{18.74}{18.74}$ *gives an equivalent rate.*

$$\frac{1\text{ USD} \times 18.74}{9.6\text{ pesos} \times 18.74} \approx \frac{18.74\text{ USD}}{178\text{ pesos}}$$

The Marcos family would receive 18.74 USDs in exchange for their 178 Mexican pesos.

- If the average exchange rate for 2002 is approximately 9 pesos for every 1 USD, how many U.S. dollars would 1 peso be worth?

If the exchange rate is 9 pesos for every 1 USD, then 1 peso would be worth $\frac{1}{9}$ *of a dollar. The figure below represents 1 USD. It has been divided into 9 equal parts to represent 9 pesos = 1 USD. Therefore, 1 peso equals* $\frac{1}{9}$ *of the dollar.*



using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(B) evaluate the effectiveness of different representations to communicate ideas

(6.13) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(B) validate his/her conclusions using mathematical properties and relationships

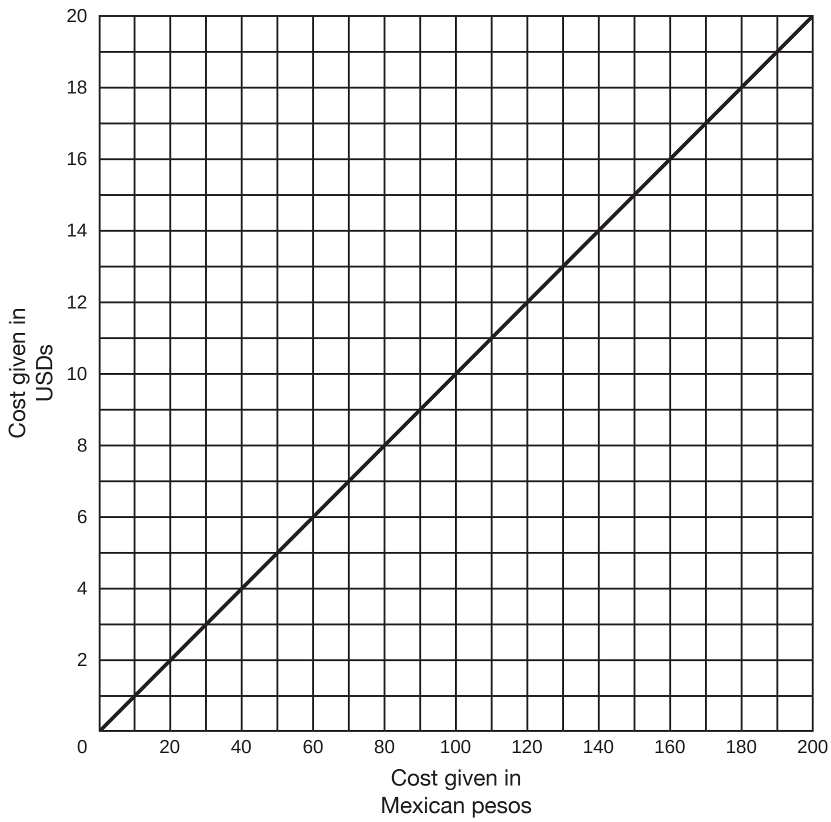
Texas Assessment Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

What's in Your Wallet?
grade 7

During a two-week trip to Mexico, the Marcos family purchased several mementos. John, the youngest son, purchased a T-shirt for 85 pesos. Cindy, the oldest daughter, purchased a leather belt for 150 pesos. Mrs. Marcos brought back a piñata for which she spent 90 pesos.

USD and Mexican peso conversion graph



1. How much did the souvenirs cost in USDs? How did you determine this?

   a. T-Shirt: ________________

   b. Leather belt: ____________

   c. Piñata: _________________

2. What did the exchange rate appear to be at the time of the Marcos family vacation? What is your evidence?

3. To help convert pesos into dollars quickly, the Marcos family wanted to create a table showing the relationship between the USD and the peso. Based on the exchange rate found in question 2 above, find the missing values in the table.

USD and Mexican peso conversion chart

Value in USDs		0.50	1	2		7	
Value in Mexican pesos	1				35		100

4. Describe a rule (in words) that the Marcos family could use to determine the number of USDs they are spending for an item that is priced in pesos.

5. If Mr. Marcos purchased an iron sculpture for 1,250 pesos, would the exchange rate given in USD be found at the point (1,250, 12.50)? Give reasons to validate your answer.

Teacher Notes

Materials

Graphing calculator (optional)

Ruler or straight edge

Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

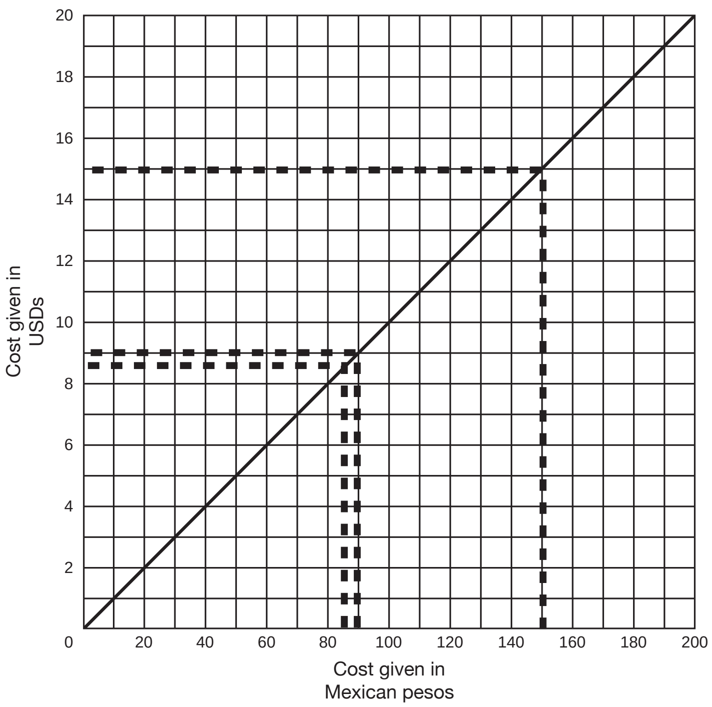
(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

Scaffolding Questions

- How could you use the graph to find the number of pesos that are equal to 1 USD?
- Explain why this rate is important. What is the name of this special rate?
- How could you use this rate to complete the graph?
- Using the graph, how many pesos are equivalent to 2 USDs? 3 USDs? 4 USDs?
- How could you organize this data?
- What patterns do you observe?
- Using the unit rate and graph, how can you validate the cost in USDs of the T-shirt that was purchased for 85 pesos?

Sample Solutions

1. Using the graph, the cost of each item purchased in pesos can be determined in USDs. Find the cost in pesos along the *x*-axis and use a ruler or straight edge while drawing a vertical line segment from this point to the line on the graph.



From this point of intersection, draw a horizontal line segment to the *y*-axis. Read the cost in USDs at this point of intersection on the *y*-axis.

a. T-shirt: $8.50

b. Leather belt: $15.00

c. Piñata: $9.00

2. The exchange rate was approximately 1 USD = 10 pesos. This rate can be determined from the linear pattern of the graph. The graph is a straight line. The relationship between the number of dollars and the number of pesos is proportional. The constant of proportionality is determined by finding a point on the graph (20, 2).

$$\frac{2 \text{ dollars}}{20 \text{ pesos}} = \frac{1 \text{ dollar}}{10 \text{ pesos}} \text{ or 1 dollar for every 10 pesos}$$

(7.4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling

(7.7) Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane. The student is expected to:

(A) locate and name points on a coordinate plane using ordered pairs of integers

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to

everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(7.15) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(B) validate his/her conclusions using mathematical properties and relationships

This rate may also be written

$$\frac{\$1}{10 \text{ pesos}} = \frac{\$1 \div 10}{10 \text{ pesos} \div 10} = \frac{\$0.10}{1 \text{ peso}} \text{ or } \$0.10 \text{ per peso}$$

3. To complete the table, use the conversion rate from problem 2: $0.10 for 1 peso and $1 for 10 pesos. To find other values in the table multiply by the scale factor. The completed table showing the relationship between the USD and the peso is given below.

USD and Mexican peso conversion chart

Value in USDs	0.10	5(0.10)= 0.50	1	2(1) = 2	3.5(1)= 3.5	7(1)= 7	10(1)= 10
Value in Mexican pesos	1	5(1)=5	10	2(10) = 20	3.5(10)= 35	7(10)= 70	10(10)= 100

4. A rate of 1 peso to 0.10 of a USD is determined by the table. This means that for every peso, the value in USDs is $0.10. To find the amount of USDs an item would cost, take the price given in pesos and multiply by $0.10 . Let d be the number of USDs and let p be the number of pesos.

$$d = 0.10p$$

5. No, the point with coordinates (1,250, 12.50) does not represent the correct conversion in USDs for the iron sculpture purchased for 1,250 pesos. Using the graph, if the amount given in USDs is $12.50, its corresponding peso value would be 125 pesos and located at the point with coordinates (125, 12.50). If the sculpture cost 1,250 pesos, the amount of the sculpture given in USDs would be $125 because the value of the peso is $\frac{1}{10}$ the value of the U.S. dollar. Dimensional analysis can be used to determine the amount paid in USDs, as shown below.

$$\frac{1{,}250 \text{ pesos} \times 1 \text{ USD}}{10 \text{ pesos}} = 125 \text{ USD}$$

Extension Questions

- On any given day the exchange rate can fluctuate. Suppose the exchange rate had been 1 USD = 9 pesos. How would this have affected your graph?

  The new ordered pairs (pesos, USDs) would include (9, 1), (85, 9.4), (150, 16.67), and (90, 10). This would have produced a different pattern and hence a different line. The line would be slightly steeper than the original line in the first quadrant. In the original graph, there was a vertical change of \$0.10 for every peso; whereas, in this graph, there is a vertical change of \$1 for every 9 pesos. This rate can be converted to about \$0.11 for every peso.

  $$\frac{\$1}{9 \text{ pesos}} = \frac{\$1 \div 9}{9 \text{ pesos} \div 9} \approx \$0.11 \text{ for every 1 peso}$$

- How would this new exchange rate have affected the Marcos family?

  An exchange rate of 1 USD = 9 pesos would have meant the Marcos family would have had to spend more USDs on their souvenir items. With the original exchange rate, the Marcos family would have spent \$1 for something valued at 10 pesos. Now that same \$1 can only buy an item worth 9 pesos.

- Is the relationship between the number of pesos and the number of U.S. dollars proportional? Give reasons to support your answer.

  Yes. Examine the table of values showing the relationship between pesos and USDs. The ratio of USDs to pesos is 0.10 to 1 and can be expressed as $\frac{d}{p} = 0.10$ *where d represents the value in USDs and p, the value in pesos. There is a constant of proportionality of 0.10 or* $\frac{1}{10}$*. Each ordered pair (pesos, USDs) shows a multiplicative relationship between the value in USDs and the value in pesos. For example, the ordered pair (20, 2) shows that for 2 dollars, the number of pesos is 10 times 2, or 20. This same ordered pair shows that for 20 pesos the number of USDs is* $\frac{1}{10}$ *times 20, or 2.*

Texas Assessment of Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

Any pair of related values can be found using another related pair of values as shown in the table.

Value in USDs	0.10	0.50	1	2	3.5	7	10
Value in Mexican pesos	1	5	10	20	35	70	100

- Give two rules that describe the relationship between USDs and pesos in this problem.

  Let d represent the amount given in USDs and p represent the amount given in pesos. If the exchange rate is 1 USD = 10 pesos, then

$$d = \frac{1}{10}p \quad \text{or } p = 10d$$

What's in Your Wallet?
grade 8

Alfred won a radio contest where the prize was a trip around the world, visiting the countries of Canada, Great Britain, Germany, and Mexico. In addition to having all travel expenses paid for, Alfred was given $500 to spend in each country. However, Alfred was soon faced with the challenge that the prices in each country were given in the currency of that particular country—not USDs (U.S. dollars). Afraid that he would spend more than his allotted 500 USD, Alfred went online to find the current exchange rates and found the data below.

	USD United States Dollar	**EUR** Euro	**GBP** United Kingdom Pound	**JPY** Japan Yen	**CAD** Canada Dollar	**MXN** Mexican Peso
1 **USD** Buys	1.00000	0.973710	0.623950	120.396	1.55069	10.1910
Inverse	1.00000	1.02700	1.60269	0.00830782	0.64487	0.098126

1. Using the information from the table above, Alfred estimated about how much money he could spend based on the countries' currencies. What would be a reasonable estimate for each of the 4 countries?

   Canada (Canadian dollar): _____________

   Great Britain (British pound): ___________

   Germany (Euro): ________________

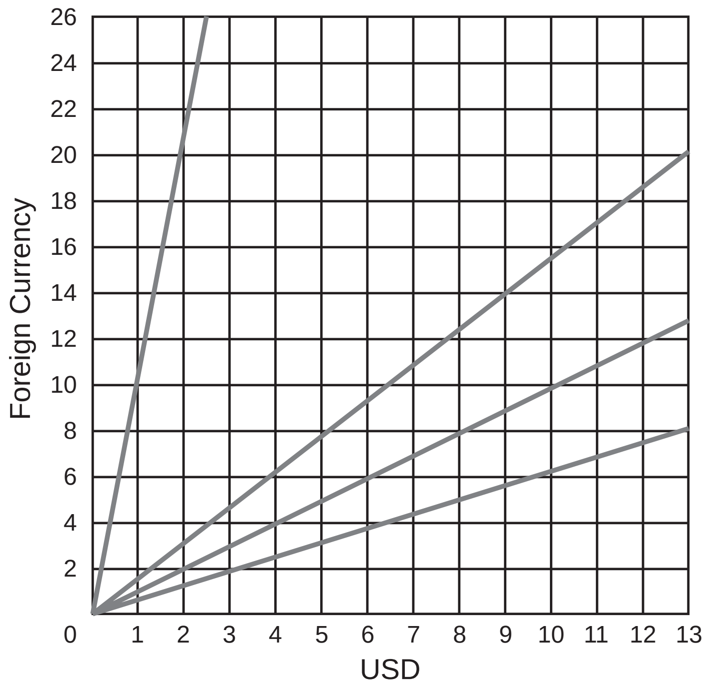
   Mexico (Mexican peso): _______________

2. Worried that an estimate might not be sufficient, Alfred made the following table for a more accurate exchange calculation.

USD	$1	$2	$3	$4	$5	$10	*n*
CAD							
GBP							
EUR							
MXN							

a. Complete the table to show the exchange amounts.

b. After further investigation online, Alfred saw the exchange rates displayed as a graph. Alfred thought this representation would be helpful on his trip; so he printed the graphs, only to find that some information did not print. Help Alfred determine which line represents each country's exchange rate by labeling each line appropriately. Explain how you made your determination.



c. If Alfred has $500 to spend in each country, how much money is this in foreign currency? Validate these amounts by using two strategies that produce the same conclusions.

Canada (Canadian dollar): ______________

Great Britain (British pound): ____________

Germany (Euro): ______________________

Mexico (Mexican peso): ________________

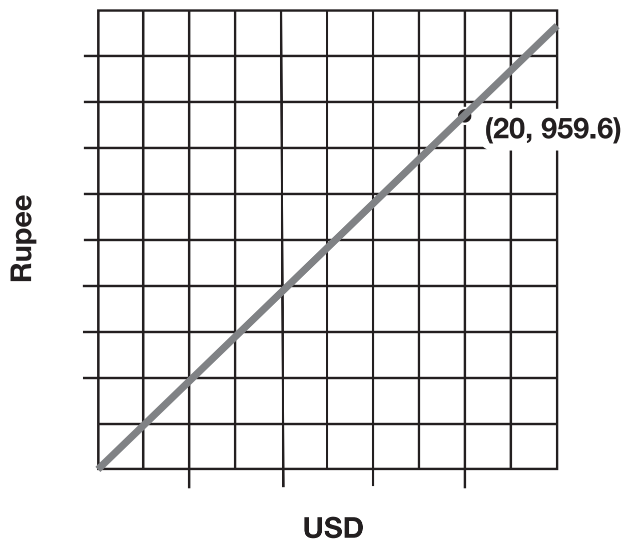
d. Which line would the ordered pair (55.50, 86.06) most likely belong to? Give evidence to validate your conclusion.

e. Sketch the line relating Australian dollars to U.S. dollars in the previous graph if 1 USD buys 1.78 Australian dollars.

f. What would be true if a country's exchange rate was shown on this graph as the line $y = x$?

3. On a one-night layover in Tokyo, Japan, Alfred spent 90 yen. What type of item would be a reasonable purchase for that amount of money? Explain your reasoning.

4. The graph below shows the exchange rate between the USD and the Indian rupee. Using the information provided, is it possible to determine how much an item would cost in rupees if it cost 5 USDs? Explain your reasoning.



Teacher Notes

Materials

Graphing calculator

Connections to Middle School TEKS

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(C) evaluate a solution for reasonableness

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$

Scaffolding Questions

- About how much of a Euro does 1 USD buy? 2 USDs? 3 USDs? Repeat question for each of the currencies.
- How could you use your graphing calculator to help you answer this question?
- What does the ordered pair (5, 7.75) mean on the graph in the context of the problem?
- What would that graph look like if the ratio between USDs and a foreign currency was 1 to 1? 1 to 2?
- What information do you have concerning the relationship between the yen and the USD?
- How could you describe this relationship in words? In symbols? With a graph?

Sample Solutions

1. One USD buys approximately $1\frac{1}{2}$ Canadian dollars. If Alfred had 500 USDs then he could spend 500 groups of $1\frac{1}{2}$ Canadian dollars. This can done mentally by breaking $1\frac{1}{2}$ into two parts, 1 and $\frac{1}{2}$. One whole group of 500 is 500; half a group of 500 is 250. Adding these parts together gives approximately 750 Canadian dollars for 500 USDs.

   The USD buys about 0.62 or 60% of the Great Britain pound. By expressing 60% as 50% + 10% , students can do the computation mentally: 50% of 500 = \$250 and 10% of 500 = \$50. Combining the parts together gives about 300 pounds for 500 USDs. Sixty percent of 500 is the same as 50% of 500 plus 10% of 500 because there is a proportional relationship between a number and the percentage of that number. As the percentage increases, the percentage of a given number also increases proportionally.

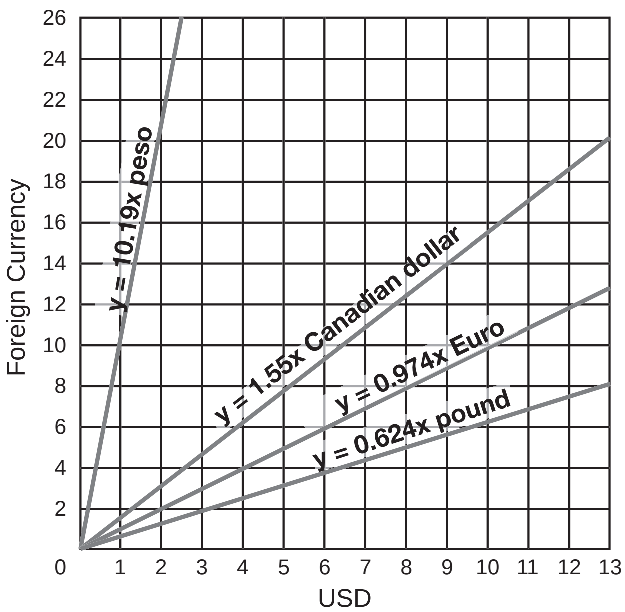
The Euro has about a 1 to 1 ratio with the USD. Therefore, 500 USDs would buy about 500 Euros.

The USD buys about 10 times as many pesos; therefore, 500 USDs would buy about 5,000 pesos.

2. a. Alfred's completed table shows an exchange rate among the currencies in the 5 countries he will visit.

USD	$1	$2	$3	$4	$5	$10	*n*
CAD	1.55	3.10	4.65	6.20	7.75	15.51	1.55069*n*
GBP	0.62395	1.25	1.87	2.50	3.12	6.24	0.62395*n*
EUR	0.97371	1.95	2.92	3.89	4.87	9.74	0.97371*n*
MXN	10.191	20.38	30.57	40.76	50.96	101.91	10.191*n*

b. To label the lines the student may look for the rates that appear in his table. The steepest graph contains the point (1, 10.191) on the line that represents the relationship between the dollar and the peso. The point (1, 1.55) is on the line that represents the relationship between the dollar and Canadian dollar. The point (1, 0.974) is on the line that represents the relationship between the dollar and the Euro. The point (1, 0.624) is on the line that represents the relationship between the dollar and the pound. The completed graph shows the relationship between the USD and other currencies.



(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

c. Using the rules found in the table, the exchanges for 500 USDs can be computed for Canada, Great Britain, Germany, and Mexico as shown below.

Canada (Canadian dollar):

1.55069*n* = 1.55069 x 500 ≈ 775.35 Canadian dollars

Great Britain (British pound):

0.62395*n* = 0.62395 x 500 ≈ 311.98 pounds

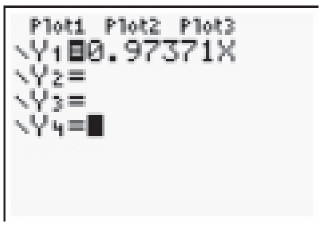
Germany (Euro):

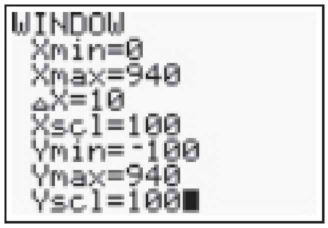
0.97371*n* = 0.97371 x 500 ≈ 486.86 Euros

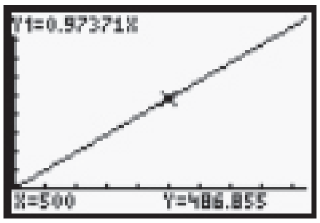
Mexico (Mexican peso):

10.191*n* = 10.191 x 500 ≈ 5,095.50 pesos

These amounts are reasonable when compared to estimates in problem 1. The answers can be validated by entering each equation into a graphing calculator and using the trace feature or table feature to find the amount of foreign currency equal to 500 USDs. For example, to compute the number of Euros, graph the rule *y* = 0.97371*x*.



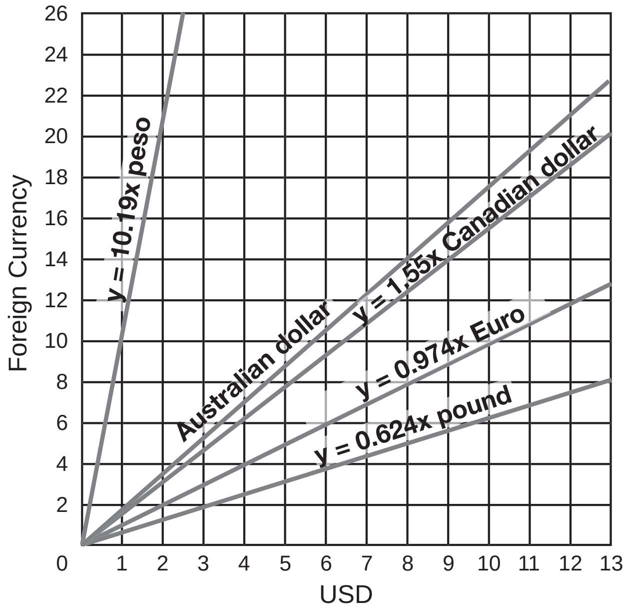




d. The ordered pair (55.50, 86.06) will most likely lie on the graph *y* = 1.55069*x*, because 86 is approximately $1\frac{1}{2}$ times the value of 55. To validate this conclusion, substitute the values for *x* and *y* in the equation *y* = 1.55069*x* as follows:

$$86.06 = (1.55069)(55.50)$$
$$86.06 = 86.06$$

e. The graph depicting the relationship between Australian dollars and U.S. dollars is shown below. The graph was created by plotting the points (0, 0) and (1, 1.78).



f. The line $y = x$ would represent a 1 to 1 ratio. In other words, 1 USD would buy 1 unit of foreign currency; 2 USDs would buy 2 units of foreign currency; 3 USDs would buy 3 units of foreign currency.

3. From the table, 1 USD can buy about 120 yen. Since 90 yen is $\frac{3}{4}$ of 120 yen, then 90 yen are equal to $\frac{3}{4}$ of a USD, or $0.75. For this price, it is reasonable to assume Alfred made a small purchase, like a soft drink, for example.

4. If 20 USD can buy 959.6 rupees, then $\frac{1}{4}$ of that amount, 5 USDs, should buy $\frac{1}{4}$ the number of rupees or 239.9 rupees. The ordered pair would be (5, 239.9).

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(8.16) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(B) validate his/her conclusions using mathematical properties and relationships

Texas Assessment of Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

Extension Questions

- Why do all the graphs meet at the origin?

  The relationships between the USD and the foreign currencies are proportional, and the graphs of the proportional relationships will contain the point (0, 0). In this case the ordered pair (0, 0) means 0 USD will buy 0 units of foreign currency.

- Exchange rates fluctuate daily. If the exchange rate changes so that 1 USD buys 9.9179 pesos, how would this affect its graph? What would this change mean in terms of the buying power of the USD?

  The slope of the line would be less steep because the exchange rate is less than it was before, causing the line to increase at a slower rate. For example, on December 20, 2002, 1 USD could purchase more than 10 pesos. However, at the new rate, the same 1 USD could purchase less than 10 pesos (9.9179 pesos).

- Find a currency converter online. What currency would produce a graph that would lie between the graphs of the Canadian dollar and the Mexican peso?

  Answers will vary depending on the market rate. However, students must find a currency with an exchange rate between 1.55 and 10.191 units per USD.

- Find a currency converter online. Find the currency that would produce the steepest line in relation to the USD. What does this rate mean in terms of the buying power of the USD?

  Answers will vary depending on the market rate. However, the greater the exchange rate the stronger the dollar, meaning you can buy more units of currency per 1 USD.

- Find the currency that would produce the least steep line in relation to the USD. What does this rate mean in terms of the buying power of the USD?

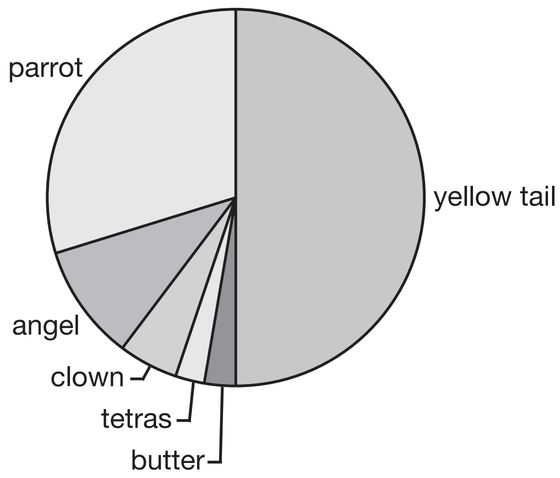
  Answers will vary depending on the market rate. However, the lesser the exchange rate the weaker the dollar, meaning you cannot buy as many units of currency per 1 USD.

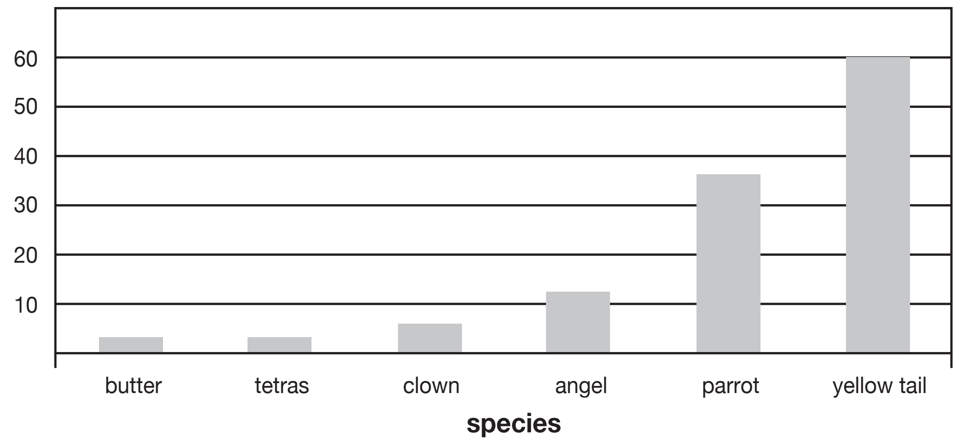
Gone Fishin’
grade 6

The Aqua Shoppe is installing a giant aquarium to house the tropical fish that they sell. The store sells angelfish, clown fish, parrot fish, butterfish, yellow tail damsels, and tetras. Based on demand, the manager requested the tank to be stocked with twice as many angelfish as clown fish; three times as many parrot fish as angelfish; $\frac{1}{2}$ as many butterfish as clown fish, and the same number of tetras as butterfish. Any remaining fish could be stocked as the sales clerk wished as long as every fish the store sold was represented in the tank and the tank contained its recommended quantity of 120 fish.

1. How many of each fish can the sales clerk order? Explain your reasoning.
2. Based on the sales order in question 1, what percentage of the fish in the aquarium will be
   a. Angelfish?
   b. Clown fish?
   c. Parrot fish?
   d. Butterfish?
   e. Yellow tail damsels?
   f. Tetras?
3. What is the maximum number of parrot fish that can be ordered for the tank? Explain.
4. What is the smallest number of parrot fish that can be ordered for the tank? Justify your answer.
5. What is the ratio of angelfish to butterfish? Use mathematical language to explain why this relationship exists.
6. If the sales clerk stocked the tank with 20% parrot fish, how many angelfish were ordered? Explain your answer.
7. When the manager of the Aqua Shoppe asked the sales clerk to leave the fish inventory on his desk, the clerk left the following graphical representations of the order.

Fish inventory





If possible, how could you help the manager determine the number of fish ordered from the information the sales clerk made available? If it is not possible, communicate your reasons.

8. If Aqua Shoppe got a new fish tank that contained 8 times as much water as the old tank, and was stocked with the same proportion of tropical fish, how many of each fish would be in the new tank if it contained 20% parrot fish? What would be the total number of fish in the aquarium? Explain your reasoning.

   a. Number of angelfish: ___________

   b. Number of clown fish: ___________

c. Number of parrot fish: __________

d. Number of butterfish: ___________

e. Number of tetras: ___________________

f. Number of yellow tail damsels: ________

g. Total number of fish: ____________

Materials

Grid paper, 10 inches by 12 inches

Multicolored cubes (ample amounts of 6 different colors to represent the 6 different kinds of fish)

Calculator (optional)

Connections to Middle School TEKS

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

> (C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

> (A) use ratios to describe proportional situations

Teacher Notes

Scaffolding Questions

- What are the conditions that the sales clerk must follow when ordering fish?
- According to the information, are there more angelfish or clown fish? How do you know?
- According to the information, are there more butterfish or clown fish? How do you know?
- If you know how many angelfish you have, how would you determine the number of parrot fish?
- What is the ratio of angelfish to clown fish? Parrot fish to angelfish? Butterfish to clown fish?
- How can you use ratios to find possible combinations of fish that could be ordered?
- How can you organize all the possible combinations of fish that could be ordered?
- How can you determine when you have found the correct combinations?
- Could there be more than one combination that meets the conditions the manager gave the sales clerk? How do you know?

Sample Solutions

1. There are 5 possible solutions to the problem. One option the clerk has is to order the following:

   8 angelfish (twice as many as clown)

   4 clown fish

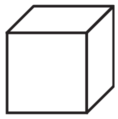
   24 parrot fish (three times as many as angel)

   2 butterfish (half as many as clown)

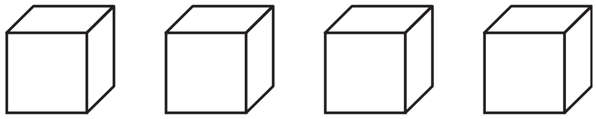
   2 tetras (same number as butter)

   80 yellow tail damsels

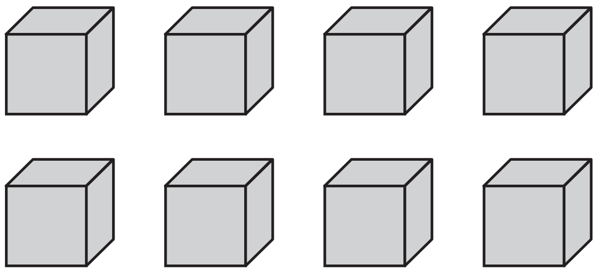
Multicolored cubes can be used to represent the different fish, along with a guess-and-check solution strategy to model this problem. For example: Let one cube



represent a clown fish. Then four clown fish can be represented by four cubes:



Twice as many angelfish can be represented by 8 cubes:



The ratio of angelfish to clown fish is 2 : 1

All possible solutions are given in the table in the table below.

Angelfish	Clown fish	Parrot fish	Butterfish	Tetras	Yellow tail damsels
4	2	12	1	1	100
8	4	24	2	2	80
12	6	36	3	3	60
16	8	48	4	4	40
20	10	60	5	5	20

2. Using the sales clerk's order, the tank will contain the following percentages of fish:

   8 ÷ 120 = 0.06667 ≈ 6.67% angelfish

(B) represent ratios and percents with concrete models, fractions, and decimals

(C) use ratios to make predictions in proportional situations

(6.10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to:

(A) draw and compare different graphical representations of the same data

(C) sketch circle graphs to display data

(D) solve problems by collecting, organizing, displaying, and interpreting data

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas

$4 \div 120 = 0.0333 \approx 3.33\%$ clown fish

$24 \div 120 = 0.2 = 20\%$ parrot fish

$2 \div 120 = 0.016667 \approx 1.67\%$ butterfish

$2 \div 120 = 0.016667 \approx 1.67\%$ tetras

$80 \div 120 = 0.6667 \approx 66.67\%$ yellow tail damsels

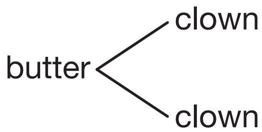
This process is used for all the possible combinations.

Angelfish	Clown fish	Parrot fish	Butterfish	Tetras	Yellow tail damsels
4÷120≈3.3%	2÷120≈1.7%	12÷120≈10%	1÷120≈0.8%	1÷120≈0.8%	100÷120≈83.3%
8÷120≈6.6%	4÷120≈3.3%	24÷120≈20%	2÷120≈1.7%	2÷120≈1.7%	80÷120≈66.7%
12÷120≈10%	6÷120≈5%	36÷120≈30%	3÷120≈2.5%	3÷120≈2.5%	60÷120≈50%
16÷120≈13.3%	8÷120≈6.6%	48÷120≈40%	4÷120≈3.3%	4÷120≈3.3%	40÷120≈33.3%
20÷120≈16.7%	10÷120≈8.3%	60÷120≈50%	5÷120≈4.2%	5÷120≈4.2%	20÷120≈16.7%

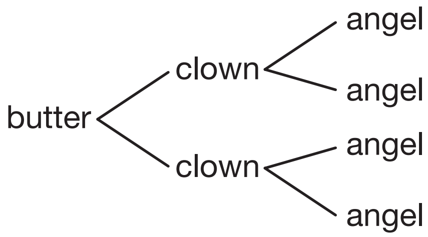
3. According to the table in number 1, the maximum number of parrot fish that can be ordered is 60. If 72 are ordered, then it would not be possible to order exactly 120 fish and follow the guidelines given by the manager.

4. The smallest number of parrot fish that can be ordered is 12. If fewer than 12 are ordered, then there would only be part of a butterfish according to the constraints of the manager. Ordering fewer than 12 parrot fish is not possible in this situation.

5. The ratio between angelfish and butterfish is 4 to 1. This relationship exists because the manager asked for "$\frac{1}{2}$ as many butterfish as clown fish." In other words, for every butterfish there are 2 clown fish.

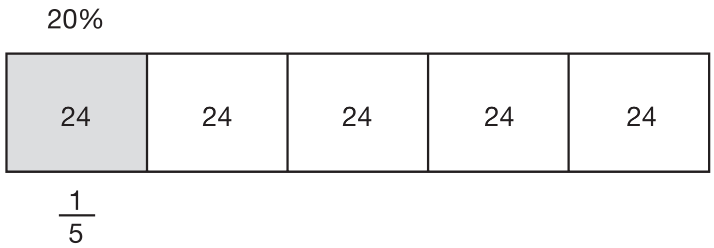


Furthermore, the manger asked for "twice as many angelfish as clown fish." This means that for every clown fish there are 2 angelfish.



Therefore, for every butterfish there will be 4 angelfish.

6. Saying that 20% of the tank consisted of parrot fish is the same as saying that $\frac{1}{5}$ of the tank consisted of parrot fish. The model below represents the Aqua Shoppe's fish tank containing 120 fish. The model has been divided into 5 equal parts.



Each fifth contains 24 fish; therefore, if 20%, or $\frac{1}{5}$ of the tank contains parrot fish, then the clerk ordered 24 parrot fish. There are 3 times as many parrot fish as there are angelfish; therefore, there is $\frac{1}{3}$ the number of angelfish as parrot fish. One-third of 24 is 8, so there are 8 angelfish.

7. Of the two graphs, the circle graph provides more information. From the circle graph provided by the sales clerk, the manager can see that about 50% or $\frac{1}{2}$ of the fish ordered were yellow tail damsels, and slightly more than 25% of the order were parrot fish. This is enough information for the manager to determine the exact number of each type of fish, assuming the clerk followed the manager's directions for ordering the fish. From the circle graph, it can easily be seen that of the 120 fish ordered, $\frac{1}{2}$ of them, or 60 fish were yellow tail damsels. All the possible options for ordering the fish are given in the table below.

using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(B) evaluate the effectiveness of different representations to communicate ideas

Texas Assessment of Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

Angelfish	Clown fish	Parrot fish	Butterfish	Tetras	Yellow tail damsels
4	2	12	1	1	100
8	4	24	2	2	80
12	6	36	3	3	60
16	8	48	4	4	40
20	10	60	5	5	20

Of these possible orders, only the highlighted option would provide a tank that was $\frac{1}{2}$ yellow tail damsels (60 out of 120 or $\frac{60}{120} = \frac{1}{2}$) and slightly more than 25% parrot fish ($\frac{36}{120} = \frac{18}{60} = \frac{9}{30} = \frac{3}{10} = \frac{30}{100} = 30\%$ parrot fish).

8. To determine the number of each species in the new tank, it will be necessary to know the number of each type of fish in the old tank. From question 6 we know that 20% or 24 of the 120 fish in the old tank are parrot fish. Therefore, according to the manager's conditions, Aqua Shoppe's tank contains the following numbers of fish.

    8 angelfish ($\frac{1}{3}$ as many as parrot fish and twice as many as clown fish)

    4 clown fish

    24 parrot fish (three times as many as angelfish)

    2 butterfish (half as many as clown fish)

    2 tetras (same number as butterfish)

    80 yellow tail damsels (120 total fish minus all the other fish combined)

    If the new tank holds 8 times as much water as the old tank, then it will hold 8 times the number of fish, or 8 x 120 = 960. Since both tanks have the same ratio of fish, the number of each species in the new tank will be 8 times the number of each species in the old tank. Therefore, the new tank will contain the following number of each species.

    64 angelfish (twice as many as clown fish)

    32 clown fish

    192 parrot fish (three times as many as angelfish)

    16 butterfish (half as many as clown fish)

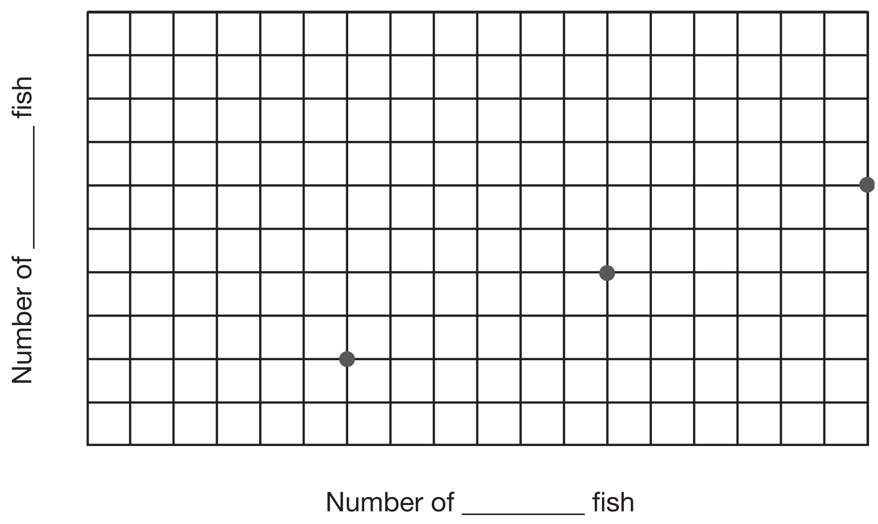
16 tetras (same number as butterfish)

640 yellow tail damsels (8 x 120 total fish minus all the other fish combined)

960 total fish (120 x 8)

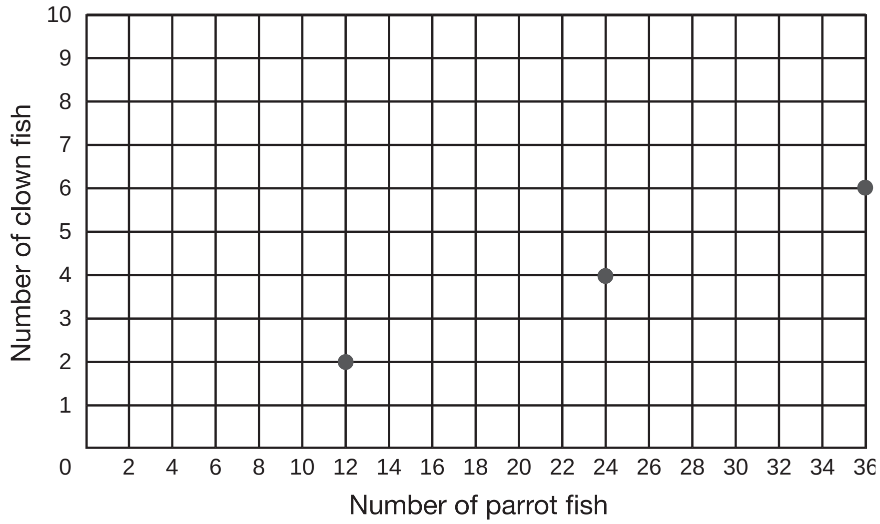
Extension Questions

- The following graph shows the relationship between two different types of fish in Aqua Shoppe's tank. What are the two fish represented in this graph? Give evidence for your conclusion.



Possible solution:

The scale of the graph could be defined as the following:



Identifying the scales as shown above illustrates the relationship between the number of parrot fish and the number of clown fish. These ordered pairs can be found in the table below that lists all the possible ordering options.

Angelfish	Clown fish	Parrot fish	Butterfish	Tetras	Yellow tail damsels
4	2	12	1	1	100
8	4	24	2	2	80
12	6	36	3	3	60
16	8	48	4	4	40
20	10	60	5	5	20

- If one exists, give the ratio of parrot fish to clown fish that is represented in the previous graph. Be sure to provide mathematical evidence that will validate your conclusion.

The pattern in the table shows that there are 6 times as many parrot fish as clown fish. Each ordered pair of the graph shows this same multiplicative relationship. For example, the ordered pair (12, 2) shows that the number of parrot fish (12) is 6 times the number of clown fish (2). Comparing the number of parrot fish to the number of clown fish for each row in the given table, the ratio of parrot fish to clown fish is 6 : 1. This means that for every clown fish there are 6 parrot fish. These equivalent ratios are shown below:

$$\frac{\text{parrot fish}}{\text{clown fish}} = \frac{6}{1} = \frac{12}{2} = \frac{24}{4} = \frac{36}{6} = \frac{48}{8} = \frac{60}{10} = \frac{6}{1}$$

- Write an equation relating the number of angelfish *a* to the number of parrot fish *p*.

Number of angelfish *a*	Process	Number of parrot fish *p*
1	3 x 1	3
2	3 x 2	6
4	3 x 4	12
8	3 x 8	24
a	3 x *a*	

From the table, the number of parrot fish is three times the number of angelfish. This relationship can be written as p = 3a.

- Suppose the tank was stocked with the following fish:

Angelfish	Clown fish	Parrot fish	Butterfish	Tetras	Yellow tail damsels
16	8	48	4	4	40

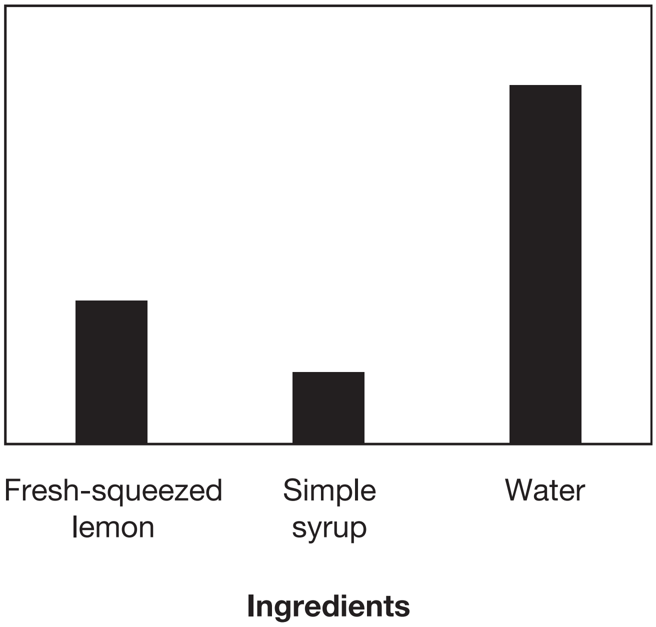
If the sales clerk reaches into the tank with a net and scoops out 1 fish, what is the probability (expressed as a percentage), that it will be a parrot fish?

$$\frac{48}{120} = \frac{24}{60} = \frac{12}{30} = \frac{6}{15} = \frac{2}{5} = \frac{40}{100} = 40\%$$

Secret Recipe
grade 6

The ingredients for Sal's famous Ole Timer's Lemonade are given in the graph below.

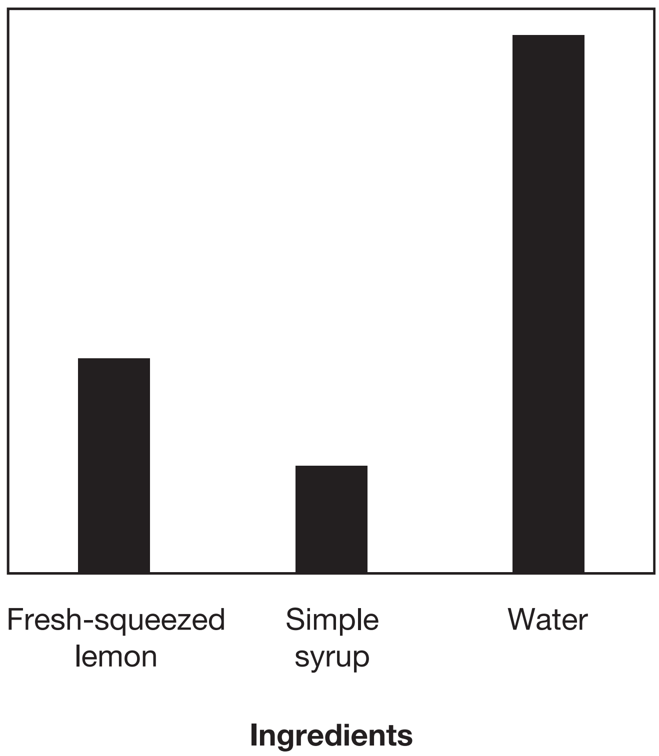
Sal's Ole Timer's Lemonade



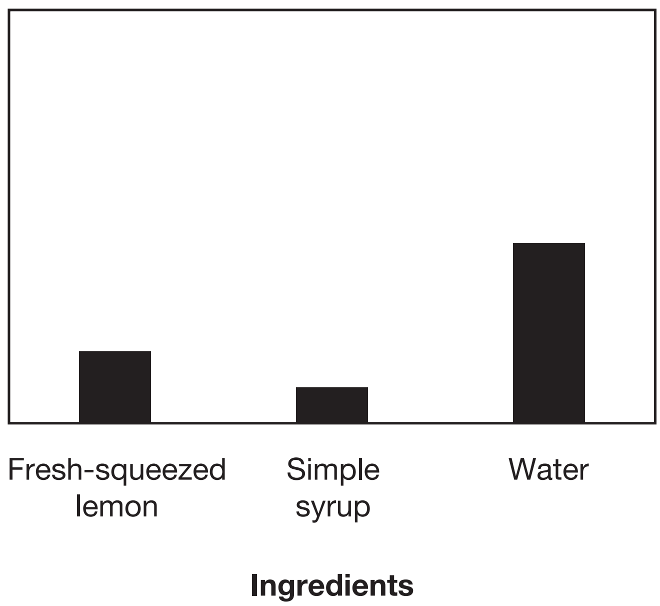
Sal's Ole Timer's lemonade is so popular many others have tried to imitate his recipe.

The graphs on the following pages represent attempts by competitors to copy Sal's lemonade recipe. All bar graphs are drawn to the same scale.

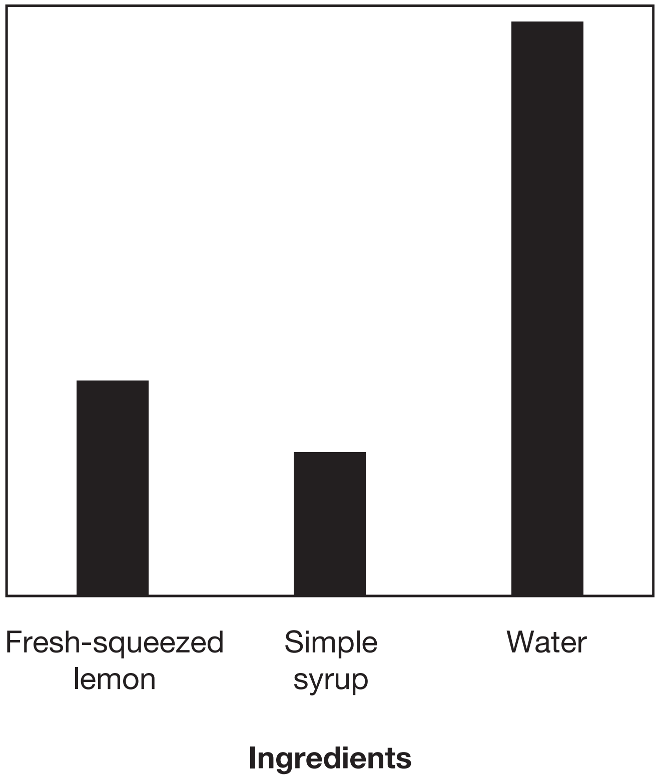
Sweet and Sour Lemonade



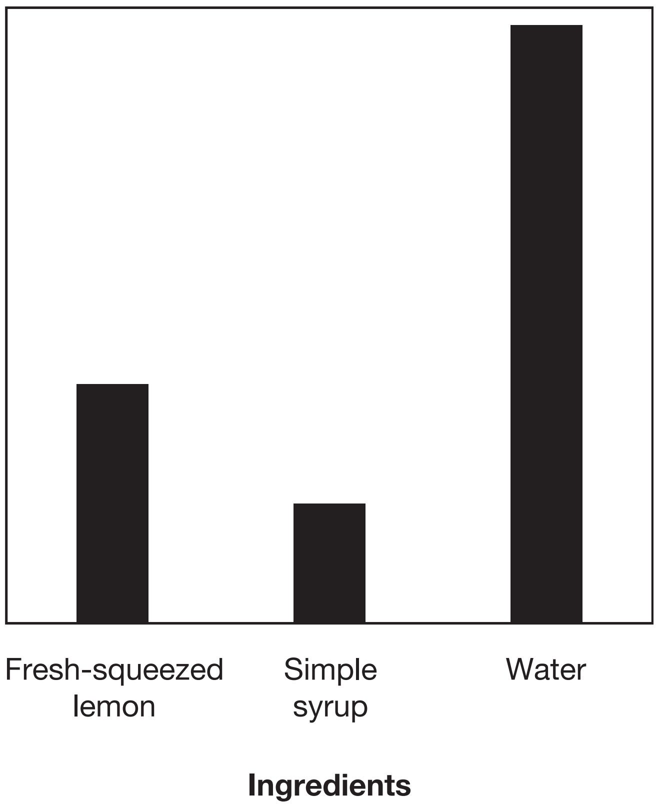
Tart and Tangy Lemonade



Yellow Birdie Lemonade



Lemon Lite Lemonade



1. Which recipe(s) would taste identical to Sal's Ole Timer's Lemonade? Give evidence for your conclusions.
2. In Sal's Old Timer's recipe, what is the ratio of lemon juice to simple syrup to water? What does this ratio mean in terms of ounces?
3. If Sal poured $1\frac{1}{2}$ cups of lemonade for himself, how much of his glass, expressed in ounces, would be lemon juice? Simple syrup? Water? Explain how you determined this.

Teacher Notes

Materials

Measuring tools to select from (ruler, acetate, centimeter graph paper)

Scissors

Transparent grid paper

Connections to Middle School TEKS

(6.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations

(B) represent ratios and percents with concrete models, fractions, and decimals

(6.8) Measurement. The student solves application problems involving estimation and measurement of length, area, time, temperature, capacity, weight, and angles. The student is expected to:

(A) estimate measurements and evaluate reasonableness of results

Scaffolding Questions

- What must be true in order for the different recipes to taste exactly the same?
- Describe how you would make one glass of lemonade. If you were to make two glasses, how would this affect the amount of ingredients needed?
- How would the bar graphs for this situation compare?
- How do the three ingredients in Sal's recipe relate to one another?
- How can you compare the other bar graphs to the Ole Timer's Lemonade bar graph?
- What must be true for a recipe represented by another bar graph to be the same as the Ole Timer's Lemonade recipe?

Sample Solutions

1. Sweet and Sour, Tart and Tangy, and Lemon Lite would taste identical to Sal's Ole Timer's Lemonade. These recipes have the same ratio of cups of fresh-squeezed lemon to cups of simple syrup to cups of water as Sal's Ole Timer's recipe. This can be determined by examining the relationship between the heights of the bar graphs. Since the number of cups is not shown on any of the bar graphs, the 3 ingredients and their ratios must be compared in a more concrete way. To verify conclusions about these ratios, transparent centimeter grid paper can be used to compare the heights of the bar graphs. The following table shows how the data can be organized for comparisons.

Estimated height of bar graph (given in cm)

Ingredient	Sal's Ole Timer's	Sweet and Sour	Tart and Tangy	Yellow Birdie	Lemon Lite
Fresh-squeezed lemon	2	3	1	3	3.3
Simple syrup	1	1.5	0.5	2	1.7
Water	5	7.5	2.5	8	8.3

From these estimated measurements, it can be shown that all three ingredients in the Sweet and Sour recipe are $1\frac{1}{2}$ times the ingredients in Sal's Ole Timer's Lemonade. By examining the columns in the table above for Sweet and Sour and Sal's Ole Timer's recipes you can see a relationship between the two types of lemonade.

Ingredient	Sal's Ole Timer's	Process Sweet & Sour	Sweet and Sour
Fresh-squeezed lemon	2	2(1.5)	3
Simple syrup	1	1(1.5)	1.5
Water	5	5(1.5)	7.5

There is a proportional relationship between the number of cups of ingredients in Sal's Ole Timer's recipe and the number of cups of corresponding ingredients in the Sweet and Sour recipe (1.5 represents the constant rate of change). This is the reason why the two recipes will taste the same.

(D) convert measures within the same measurement system (customary and metric) based on relationships between units

(6.11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(6.12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(6.13) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples

(B) validate his/her conclusions using mathematical properties and relationships

The Yellow Birdie recipe has 1.5 times the lemon juice, twice the amount of simple syrup, and 1.6 times the amount of water as Sal's recipe. The following table shows this comparison.

Ingredient	Sal's Ole Timer's	Process Yellow Birdie	Yellow Birdie
Fresh-squeezed lemon	2	2(1.5)	3
Simple syrup	1	1(2)	2
Water	5	5(1.6)	8

There is not a proportional relationship between the number of cups of ingredients in Sal's Ole Timer's recipe and the number of cups of corresponding ingredients in the Yellow Birdie recipe. The process column above shows that there is not a constant rate of change. This would cause the drinks to have a different taste.

The height of the bars representing the ingredients in Tart and Tangy are all half the height of bars representing corresponding ingredients in Sal's recipe. Finally, every bar height in the Lemon Lite graph is $1\frac{2}{3}$, or about 1.7 times the bar height of corresponding ingredients in Sal's. This relationship can be modeled by cutting out the bars in the Lemon Lite graph and folding them to show that each bar is $1\frac{2}{3}$ the height of corresponding bars in Sal's graph.

2. Using the simple syrup bar height for Sal's Ole Timer's as the unit, it can be shown that the fresh-squeezed lemon bar height is 2 units and the water bar height is 5 units. Therefore, the ratio of fresh-squeezed lemon to simple syrup to water is 2 : 1 : 5.

   In terms of ounces, this ratio means that for every ounce of simple syrup there are 2 ounces of fresh-squeezed lemon and 5 ounces of water.

3. $1\frac{1}{2}$ cups are equivalent to 12 ounces (1 cup = 8 oz and $\frac{1}{2}$ cup = 4 oz; therefore, 1 cup + $\frac{1}{2}$ cup = 8 oz + 4 oz, or 12 oz). Using the ratio 2 : 1 : 5 for 1 batch, the following

table can be developed. Two batches will have 2 times the number of ounces of each ingredient in one batch; $1\frac{1}{2}$ batches will have 1.5 times the number of ounces of each ingredient in one batch.

	Fresh-squeezed lemon (given in ounces)	Simple syrup (given in ounces)	Water (given in ounces)	Total
1 batch	2 oz	1 oz	5 oz	8 oz = 1 cup
2 batches	4 oz	2 oz	10 oz	16 oz = 2 cups
1.5 batches	3 oz	1.5 oz	7.5 oz	12 oz=$1\frac{1}{2}$cups

Extension Questions

- Consider the graphs of the recipes that taste the same. How can you tell which recipe produces more juice?

  It is not possible to tell which recipe produces more juice because the scales on the graphs are missing. For example, even though Tart and Tangy appears to have the smallest number of ingredients, the graph could be describing the ratio of ingredients needed to make enough lemonade to serve 200 people. The scale on that graph could be given in 100s. Likewise, the graph for Sal's Ole Timer's Lemonade could be describing the amount of ingredients needed to make 1 cup of the juice.

- Is it possible that the graphs for Sal's Ole Timer's Lemonade, Tart and Tangy, Lemon Lite, and Sweet and Sour could all be describing the same amount of juice? Explain your reasoning.

  Yes, because the scales on the graphs are unknown, the graphs could be describing the same amounts. The heights of the bar graphs give no information when examined individually; however, when compared in relation to the other ingredients, one can reason proportionally to determine which recipes have the same ratios, hence the same taste.

Texas Assessment of Knowledge and Skills:

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

R_x
grade 7

An infection known as strep throat was spreading throughout the small town of Allen. The waiting room of Dr. King's office was filled with children from the ages of 2 months to 9 years waiting to be seen. To fight the infection, Dr. King was prescribing penicillin to her patients.

On this particular morning, Dr. King will see 4 patients with strep throat and will prescribe penicillin. Information on each patient can be found in the table below.

Patient name	Age	Weight (pounds)	Height (inches)
Collin	7 yrs	70.4	50
Hayley	9 yrs	88	55
Scottie	2 months	11	22.5
Elena	20 months	35.2	31

1. Before seeing the doctor, the patients must be weighed on the digital scale that measures a person's weight to the nearest tenth of a pound. The amount of penicillin the doctor prescribes will be based on a person's weight in kilograms. (Note: 1 kilogram ≈ 2.2 pounds.)

   a. Describe an efficient strategy or construct a graphical representation that Dr. King could use to easily determine the weight of any of her patients in kilograms.

   b. Use your strategy or tool to find the weight of each of Dr. King's patients in kilograms.

2. Because Dr. King's patients vary in age from infants to pre-teens, she had to carefully calculate the dosage of penicillin for each individual based on the patient's weight. To calculate the number of milligrams of penicillin needed per day, the doctor used the formula $P = 40k$, where P represents the amount of penicillin measured in milligrams and k represents the number of kilograms the patient weighs.

a. Using words, describe the meaning of this formula to a patient using the situation of penicillin and kilograms.

b. Based on the patient's weight, how much penicillin should each patient receive per day?

3. It is common practice to take the medicine in smaller, equivalent doses 2 times a day. The doctor can order the penicillin in various concentrated strengths listed below.

   a. 125 milligrams of penicillin per teaspoon of medicine

   b. 200 milligrams of penicillin per teaspoon of medicine

   c. 250 milligrams or penicillin per teaspoon of medicine

   With these different options, doctors are better able to ensure that patients are taking as close to the exact amount of medicine needed as possible. For example, when given the choice of prescribing 2.26 teaspoons of one solution or 2.52 teaspoons of another solution, a doctor might prescribe the latter, knowing that it is more reasonable for a patient to measure 2.5 teaspoons of medicine compared to 2.26 teaspoons.

   For each patient, determine which solution the doctor should prescribe and write a reasonable prescription detailing how many teaspoons of which solution should be given twice a day so that the patient receives the recommended daily dosage of the drug. Provide an explanation for why the prescription is the most reasonable of the options available.

4. If a patient were prescribed 3 teaspoons of penicillin 2 times a day at the concentrated strength of 200 milligrams per teaspoon, what would the patient weigh in pounds? Explain how you determined this.

5. If another patient weighed half the amount of the patient described in question 4, what dosage should be prescribed and why?

Teacher Notes

Materials

Graphing calculator

Graph paper

Connection to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(G) determine the reasonableness of a solution to a problem

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

Scaffolding Questions

- If a person weighs 100 pounds, would that person's weight be expressed using more or fewer kilograms? Explain.
- What is the relationship between pounds and kilograms?
- Is the relationship between pounds and kilograms proportional? Explain.
- How could you use this relationship to help you determine the weight of a 100-pound patient in kilograms?
- Suppose a patient weighs 1 pound. How many milliliters of penicillin would he or she need a day?
- How could you use the previous information to determine the amount of penicillin needed for a patient weighing 10 pounds? 20 pounds? 100 pounds?
- If a patient needed 1,250 milligrams of penicillin a day, what are the various ways the doctor could prescribe the dosage using the different solutions?
- How would you determine which dosage is most reasonable?

Sample Solutions

1. a. Using the information 1 kg ≈ 2.2 lbs, a rate table can be constructed as shown below.

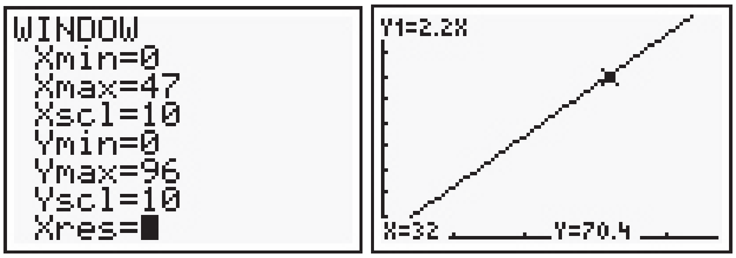
Number of kilograms	1	2	3	4	5	. . .	x
Number of pounds	2.2	4.4	6.6	8.8	11	. . .	$2.2x$

The table shows a pattern where the number of pounds increases by 2.2 pounds for each additional kilogram.

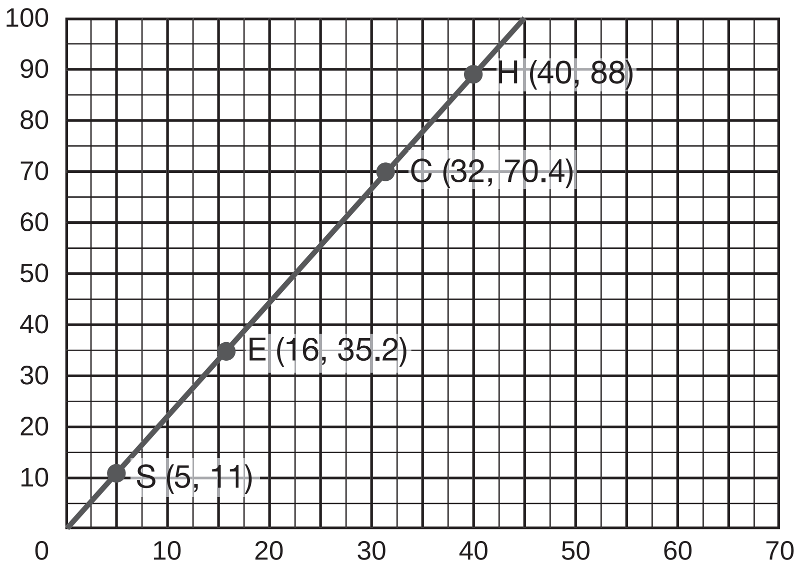
The rate is approximately

$$\frac{2.2 \text{ pounds}}{1 \text{ kilogram}}$$

This relationship can be shown in a rule $y = 2.2x$, where x represents the number of kilograms and y represents the number of pounds. Using the graphing calculator's trace feature or table feature, the number of kilograms for any given number of pounds can be found. For example, the point on the graph below shows the related pair 70.4 pounds and 32 kilograms.



b. The graph below shows each patient's weight in kilograms and pounds.



2. a. The amount of penicillin needed per day is 40 milligrams per kilogram of weight. In other words, for every kilogram a patient weighs, the patient will need to take 40 mg of penicillin.

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units

(7.5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to:

(B) formulate a possible problem situation when given a simple equation

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model

that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

Texas Assessment of Academic Skills:

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

b. To calculate the amount of penicillin needed per day, multiply the weight of a patient given in kilograms by 40 milligrams using dimensional analysis as shown below.

Collin: $\frac{40\text{ mg}}{1\text{ kg}} \times 32\text{ kg} = 1{,}280\text{ mg a day}$

Hayley: $\frac{40\text{ mg}}{1\text{ kg}} \times 40\text{ kg} = 1{,}600\text{ mg a day}$

Scottie: $\frac{40\text{ mg}}{1\text{ kg}} \times 5\text{ kg} = 200\text{ mg a day}$

Elena: $\frac{40\text{ mg}}{1\text{ kg}} \times 16\text{ kg} = 640\text{ mg a day}$

3.

Patient Collin	Daily dosage (mg)	Daily dosage (teaspoons)	Recommended amount per dose	Reasonable amount per dose
125 mg/tsp solution	1280	$\frac{1280}{125}$ = 10.24 tsp	$\frac{10.24}{2}$ = 5.12 tsp	5 tsp
200 mg/tsp solution	1280	$\frac{1280}{200}$ = 6.4 tsp	$\frac{6.4}{2}$ = 3.2 tsp	3 tsp
250 mg/tsp solution	1280	$\frac{1280}{250}$ = 5.12 tsp	$\frac{5.12}{2}$ = 2.56 tsp	2.5 tsp

Patient Collin should be prescribed to take $2\frac{1}{2}$ teaspoons twice a day of the 250 mg per teaspoon solution. This amount would ensure that he would receive almost all (5 teaspoons) of the recommended 5.12 teaspoons, which is not a reasonable amount to measure using teaspoons.

Patient Hayley	Daily dosage (mg)	Daily dosage (teaspoons)	Recommended amount per dose	Reasonable amount per dose
125 mg/tsp solution	1600	$\frac{1600}{125}$ = 12.8 tsp	$\frac{12.8}{2}$ = 6.4 tsp	6.5 tsp
200 mg/tsp solution	1600	$\frac{1600}{200}$ = 8 tsp	$\frac{8}{2}$ = 4 tsp	4 tsp
250 mg/tsp solution	1600	$\frac{1600}{250}$ = 6.4 tsp	$\frac{6.4}{2}$ = 3.2 tsp	3 tsp

Patient Hayley should be prescribed 4 teaspoons twice a day of the 200 milligrams of penicillin per teaspoon solution. This is the best option, as this prescription will ensure she takes exactly the amount of medicine recommended. Any other option would have Hayley taking slightly more or slightly less than the amount recommended.

Patient Scottie	Daily dosage (mg)	Daily dosage (teaspoons)	Recommended amount per dose	Reasonable amount per dose
125 mg/tsp solution	200	$\frac{200}{125} = 1.6$ tsp	$\frac{1.6}{2} = 0.8$ tsp	1 tsp
200 mg/tsp solution	200	$\frac{200}{200} = 1$ tsp	$\frac{1}{2} = 0.5$ tsp	0.5 tsp
250 mg/tsp solution	200	$\frac{200}{250} = 0.8$ tsp	$\frac{0.8}{2} = 0.4$ tsp	0.5 tsp

Patient Scottie should be prescribed 0.5 teaspoon twice a day of the 200 mg of penicillin per teaspoon solution. This would ensure the infant received exactly the amount of medicine needed based on his weight. The other options would mean the infant would consume more medicine than recommended.

Patient Elena	Daily dosage (mg)	Daily dosage (teaspoons)	Recommended amount per dose	Reasonable amount per dose
125 mg/tsp solution	640	$\frac{640}{125} = 5.12$ tsp	$\frac{5.12}{2} = 2.56$ tsp	2.5 tsp
200 mg/tsp solution	640	$\frac{640}{200} = 3.2$ tsp	$\frac{3.2}{2} = 1.6$ tsp	1.5 tsp
250 mg/tsp solution	640	$\frac{640}{250} = 2.56$ tsp	$\frac{2.56}{2} = 1.28$ tsp	1.5 tsp

Patient Elena should be prescribed 2.5 teaspoons twice a day of the 125 mg of penicillin per teaspoon solution. This option would give her the amount closest to her recommended amount of medicine.

4. The patient would be taking 6 teaspoons of 200 milligrams of penicillin a day for a total of 1,200 milligrams of penicillin. Penicillin is prescribed at a rate of 40 milligrams per kilogram. Therefore, the

patient must weigh 30 kilograms since $\frac{40 \text{ mg}}{1 \text{ kg}} \times 30 \text{ kg} = 1{,}200 \text{ mg}$. If a patient weighs 30 kilograms and each kilogram is equivalent to approximately 2.2 pounds, then the patient weighs $30 \text{ kg} \times \frac{2.2 \text{ pounds}}{1 \text{ kg}} \approx 66 \text{ pounds}$.

5. If the patient weighed half the amount of the patient above (33 pounds, or about 15 kilograms) then the patient should require half of the amount of penicillin (600 mg per day, or 1.5 teaspoons of the 200 mg per teaspoon solution twice a day).

Extension Questions

- If 1 kilogram equals approximately 2.2 pounds, then 1 pound equals how many kilograms?

$$\text{x } \frac{1}{2.2}$$

$$\frac{1 \text{ kg}}{2.2 \text{ lbs}} = \frac{?}{1 \text{ lb}} = \frac{^{1}/_{22} \text{ kg}}{1 \text{ lb}} \approx \text{0.45 kilograms per pound}$$

$$\text{x } \frac{1}{2.2}$$

The number $\frac{1}{2.2}$ *is called a scale factor and is used to "scale" the original ratio of 1 kg : 2.2 lbs.*

- Write an equation that could be used to find *k*, the number of kilograms for any given number of pounds, *n*.

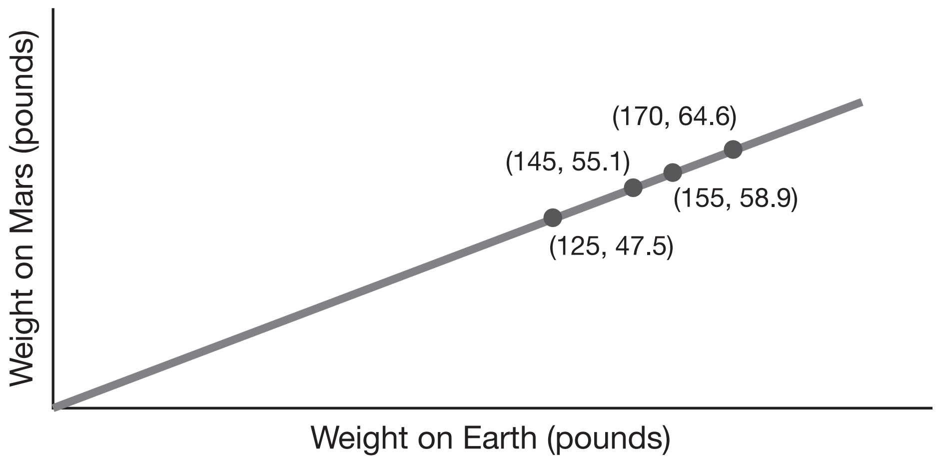
Pounds n	1	2	3	4	5	...	n
Kilograms k *(approximation)*	0.45	0.9	1.35	1.8	2.25	...	0.45n

The pattern in the table shows that the number of kilograms increases by 0.45 for each additional pound. This can be expressed using the formula $k = 0.45n$.

It's a Weighty Matter
grade 7

Did you know your weight is a factor of where you are standing? Every object in the universe with mass attracts every other object with mass. The amount of attraction depends on the size of the masses and how far apart they are. For everyday-sized objects, this gravitational pull is immeasurably tiny, but the pull between a very large object, like a planet, and another object, such as yourself, can be easily measured. How do we measure this gravitational pull? Simply stand on a scale! Scales measure the force of attraction between you and the planet. This force of attraction between you and the planet is called your weight.

This graph shows the relationship between weights on Earth and how they relate to corresponding weights on Mars.



1. What does the ordered pair (155, 58.9) mean in words in this problem?

2. Organize the data from the graph above using a table similar to the one below.

Name	Weight on Earth (pounds)	Weight on Mars (pounds)	Ratio $\frac{\text{MarsWeight}}{\text{EarthWeight}}$	Ratio *(Expressed as a decimal)*
Astronaut A				
Astronaut B				
Astronaut C				
Astronaut D				

3. Is this a proportional relationship? Provide evidence that will validate your conclusion.

4. Identify two different unit rates that describe this relationship.

5. Complete this table.

Earth weight (lbs)		1	2	2.5	**10**		50		100	*n*	
Mars weight (lbs)	1				**3.8**	17.1		32.3			*m*

6. Write a rule using words and symbols that describes the relationship between the weight on Earth and the corresponding weight on Mars.

7. Before lift off, it was noted that Astronaut C had lost $4\frac{1}{2}$ pounds. How will this affect his weight on Mars?

8. If a Martian were discovered and found to weigh 475 pounds on its home planet, how much would the Martian weigh on Earth? Explain how you determined the Martian's weight on Earth.

Materials

Graphing calculator

Connections to Middle School TEKS

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio

(7.3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving proportional relationships such

Teacher Notes

Scaffolding Questions

- What does the ratio *weight on Mars* : *weight on Earth* expressed as a decimal mean in this situation?
- What do the unit rates mean in this situation?
- Would you weigh more or less or Mars? Describe how your weight would change using a percentage (approximate).
- What would be the weight of an object on Mars if it weighed 1 pound on Earth?
- What would be the weight of an object on Earth if it weighed 1 pound on Mars?
- How would the unit rates be helpful when completing the tables in problems 2 and 4?
- How can some values in the tables help you think about other missing values in the table? For example, how could knowing 10 pounds on Earth is equivalent to 3.8 pounds on Mars be helpful in finding the weight on Mars related to 100 pounds on Earth? Or 50 pounds on Earth?
- If Astronaut C lost 1 Earth pound, how would that change his weight on Mars?

Sample Solutions

1. The ordered pair (155, 58.9) means an object that weighs 155 pounds on Earth would weigh 58.9 pounds on Mars.

2. Data from the graph can be represented in the following table. Also included in the table is the ratio of Mars weight to Earth weight expressed in both fractional and decimal forms.

Name	**Weight on Earth** (pounds)	**Weight on Mars** (pounds)	**Ratio** $\frac{\text{MarsWeight}}{\text{EarthWeight}}$	**Ratio** *(Expressed as a decimal)*
Astronaut A	125	47.5	$\frac{47.5}{125}$	$\frac{0.38}{1}$
Astronaut B	145	55.1	$\frac{55.1}{145}$	$\frac{0.38}{1}$
Astronaut C	155	58.9	$\frac{58.9}{155}$	$\frac{0.38}{1}$
Astronaut D	170	64.6	$\frac{64.6}{170}$	$\frac{0.38}{1}$

3. Yes, the relationship between the weight on Earth and the weight on Mars is proportional. The graph is a linear graph that contains the point (0, 0). The point (0, 0) means no weight on earth and no weight on Mars. However if you examine the ordered pairs in the table, there is a constant ratio

$$\frac{y}{x} = 0.38$$

The constant rate of proportionality is 0.38 pounds on Mars per pound on Earth.

as similarity, scaling, unit costs, and related measurement units

(7.4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(A) generate formulas involving conversions, perimeter, area, circumference, volume, and scaling

(7.7) Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane. The student is expected to:

(A) locate and name points on a coordinate plane using ordered pairs of integers

(7.13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(7.15) Underlying processes and mathematical tools. The student uses logical

4. The ratio 0.38 to 1 is a unit rate and means the weight on Mars is 0.38 pound on Mars for every 1 pound on Earth, or approximately $\frac{2}{5}$ of a pound on Mars for every 1 pound on Earth. Another unit rate would be the reciprocal of $\frac{0.38}{1}$, which is $\frac{1}{0.38}$. This unit rate is approximately 2.6 to 1, which means every 2.6 pounds on Earth would weigh 1 pound on Mars.

5. The completed table shows the relationship between Earth weight and corresponding Mars weight:

Earth Weight (lbs)	2.6	1	2	2.5	10	45	50	85	100	*n*	2.6*m*
Mars Weight (lbs)	1	0.38	0.76	0.95	3.8	17.1	19	32.3	38	0.38*n*	*m*

6. The table in problem 5 shows two generalizations:

The weight of an object on Mars is 0.38 times the weight of that object on Earth, and the weight of an object on Earth is 2.6 times its weight on Mars. Therefore, to find a weight on Mars, multiply 0.38 by the number of pounds on Earth. In other words—

Weight on Mars = 0.38 x weight on Earth
or $m = 0.38 \times n$

where *m* represents weight on Mars and *n* represents number of pounds on Earth.

To find a weight on Earth, multiply 2.6 times the weight of that object on Mars.

Weight on Earth = 2.6 x weight on Mars
or $n = 2.6 \times m$

where *n* represents weight in pounds on Earth and *m* , the weight in pounds on Mars.

7. Since the ratio of weight in pounds on Mars to weight in pounds on Earth is 0.38 to 1, the astronaut would weigh

0.38 fewer pounds on Mars for every pound he lost on Earth as shown in the table below. Number sense can be used to reason through this situation. Since there is a proportional relationship between weight in pounds on Mars and weight in pounds on Earth, the ratios $0.19 : \frac{1}{2}$ and 1.52 : 4 can be combined by adding the first entries and then finding the sum of the second entries to get $1.71 : 4\frac{1}{2}$ as shown in the table. This means that Astronaut C would weigh 1.71 pounds less on Mars for a weight loss of $4\frac{1}{2}$ pounds on Earth.

Weight loss on Earth *(lbs)*	**Weight loss on Mars** *(lbs)*
$\frac{1}{2}$	0.19
1	0.38
2	0.76
3	1.14
4	1.52
$4\frac{1}{2}$	1.52 + 0.19 = 1.71

8. Using the rule *n* = 2.6 x *m* where *m* represents 475 pounds on Mars, find the value of *n,* the corresponding weight on Earth.

   2.6 x 475 = 1,250 pounds on Earth

   Therefore, a Martian weighing 475 pounds on Mars will weigh 1,250 pounds on Earth.

reasoning to make conjectures and verify conclusions. The student is expected to:

(B) validate his/her conclusions using mathematical properties and relationships

Texas Assessment Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

Extension Questions

- Does it seem reasonable that the ordered pair (150, 80) would belong on this graph? Explain your reasoning.

  No, it is not reasonable for the ordered pair (150, 80) to be a pair of related weights when comparing Earth to Mars. Using the ratio 0.38 pounds on Mars to 1 pound on Earth means that a weight on Mars is approximately 40% the weight on Earth. This ordered pair is relating 80 pounds on Mars to 150 pounds on Earth and 80 is more than 50% of 150.

- Without using a calculator, estimate the missing value for the ordered pair (200, __).

  If an object weighs 200 pounds on Earth, it should weigh about $\frac{2}{5}$ *of that weight on Mars. Using mental math,* $\frac{2}{5}$ *of 100 is 40, so* $\frac{2}{5}$ *of 200 is 80. An object on Earth weighing 200 pounds would weigh slightly less than 80 pounds on Mars; therefore, the ordered pair could be (200, 78).*

- Without using a calculator, estimate the missing value for the ordered pair (__, 400).

  The ratio 0.38 pounds on Mars to 1 pound on Earth can be expressed as 0.40 : 1 or $\frac{2}{5}$*: 1 and shows that the weight on Mars is* $\frac{2}{5}$ *the weight on Earth. Therefore, the 400 pounds on Mars is about* $\frac{2}{5}$ *of the weight on Earth. Using mental math, if 400 is* $\frac{2}{5}$ *of Earth's weight, then* $\frac{1}{5}$ *would be 200. If* $\frac{1}{5}$ *is 200, then* $\frac{5}{5}$ *or 100% is 1,000. If an object weighed 400 pounds on Mars, then its weight would be approximately 1,000 pounds on Earth.*

- What would an ounce of chocolate weigh on Mars?

  The ratio of weight on Mars to weight on Earth is 0.38 to 1. This ratio will hold regardless of the units being measured. An ounce of chocolate on Earth would weigh 0.38 ounces on Mars.

Java Joe's
grade 8

Customers at Java Joe's can make their coffee to their liking. Java Joe's is famous for their vanilla latte, which is made from steamed milk, coffee, and vanilla syrup. At one table the coffee mugs contained the following amounts:

Sara's Mug	**Luke's Mug**	**Ella's Mug**	**Ricky's Mug**
2 oz milk	3 oz milk	4 oz milk	1 oz milk
3 oz coffee	5 oz coffee	8 oz coffee	2 oz coffee
$\frac{3}{4}$ oz vanilla syrup	$1\frac{1}{2}$ oz vanilla syrup	2 oz vanilla syrup	$\frac{1}{2}$ oz vanilla syrup

1. Whose coffee tasted the most vanilla-y? Justify your answer using language, appropriate units, and graphical, numerical, physical, or algebraic models.
2. Whose coffee tasted the same? Provide evidence for your answer.
3. If Ella had originally ordered a large 20-ounce coffee, describe the amount of each ingredient that was in her coffee drink. Explain how you found the amounts.
4. Select a graphical representation that shows the relationship among the ingredients in each mug. What conclusions can you draw based on your representation?

Materials

Graphing calculator

Grid paper

Angle rulers or protractors

Connections to Middle School TEKS

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(A) select and use appropriate operations to solve problems and justify the selections

(8.3) Patterns, relationships, and algebraic thinking.

Teacher Notes

Scaffolding Questions

- What is the ratio of the number of ounces of vanilla syrup to the number of ounces of milk and coffee combined for each coffee drink?
- What do these ratios tell you about the vanilla-y flavor of each drink?
- How can you determine which of the drinks is the least vanilla-y? The most vanilla-y?
- How can vanilla latte drinks of different sizes have the same vanilla-y taste?

Sample Solutions

1. Luke's coffee was the most vanilla-y. This can be shown by examining the ratio of parts of vanilla syrup to parts of milk and coffee combined in the following table.

Sara's Mug	Luke's Mug	Ella's Mug	Ricky's Mug
vanilla : milk & coffee	vanilla : milk & coffee	vanilla : milk & coffee	vanilla : milk & coffee
0.75 oz : 5 oz	1.5 oz : 8 oz	2 oz : 12 oz	0.5 oz : 3 oz
1.5 oz : 10 oz		1 oz : 6 oz	1 oz : 6 oz
		1.5 oz : 9 oz	1.5 oz : 9 oz

By scaling the ratios up or down, the combined total ounces of milk and coffee that compares to 1.5 ounces of vanilla can be found for each mug. The mug with the least amount of milk and coffee to the 1.5 ounces of vanilla is the most vanilla-y. Therefore, Luke's coffee is the most vanilla-y because he has 1.5 ounces of vanilla syrup in only 8 oz of his coffee drink, causing the flavor to be more concentrated.

2. Ella and Ricky's coffee drink tasted the same. Both mugs have the same ratio of ingredients, but in different amounts. Both mugs contain 1 ounce of milk to 2 ounces of coffee to $\frac{1}{2}$ ounce of vanilla. This can be illustrated using the following model that scales up the ounces of ingredients in Ricky's coffee drink by a factor of 4 to

show the ounces of ingredients are equivalent to the ounces of ingredients in Ella's drink.

Scale Factor	1	2	3	4
Ounces of milk	■	■ ■	■ ■ ■	■ ■ ■ ■
Ounces of coffee	■ ■	■ ■ ■ ■	■ ■ ■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■
Ounces of vanilla syrup	◧	■	■ ◧	■ ■
Ratio milk to coffee to vanilla	$1 : 2 : \frac{1}{2}$	$2 : 4 : 1$	$3 : 6 : 1\frac{1}{2}$	$4 : 8 : 2$

3. Currently, Ella's mug contains 14 ounces. Of those 14 ounces, $\frac{4}{14}$ are milk, $\frac{8}{14}$ are coffee, and $\frac{2}{14}$ are vanilla syrup. To determine how many ounces of each ingredient she started out with initially, a ratio could be written. Then a scale factor could be determined that will scale the14-ounce drink to a 20-ounce drink. The scale factor method is used to solve each proportion as shown below. To scale from 14 ounces to 20 ounces, a scale factor of $\frac{20}{14}$ or $\frac{10}{7}$ can be used. For example

Milk: $$\frac{4 \text{ oz of milk} \times \frac{10}{7}}{14 \text{ oz of latte} \times \frac{10}{7}} = \frac{\frac{40}{7} \text{ oz of milk}}{20 \text{ oz of latte}} = \frac{5\frac{5}{7} \text{ oz of milk}}{20 \text{ oz of latte}}$$

The equivalent ratio $5\frac{5}{7}$ oz : 20 oz shows that there were $5\frac{5}{7}$ oz of milk in the 20 oz mug of vanilla latte that Ella was served initially.

Coffee: $$\frac{8 \text{ oz of coffee} \times \frac{10}{7}}{14 \text{ oz of latte} \times \frac{10}{7}} = \frac{\frac{80}{7} \text{ oz of coffee}}{20 \text{ oz of latte}} = \frac{11\frac{3}{7} \text{ oz of coffee}}{20 \text{ oz of latte}}$$

The ratio 8 : 14 shows that there are currently 8 ounces of coffee to 14 ounces of Ella's vanilla latte drink. By multiplying both amounts by the same scale factor ($\frac{20}{14}$ or $\frac{10}{7}$), the original number of ounces of coffee in a

The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.12) Probability and statistics. The student uses statistical procedures to describe data. The student is expected to:

(C) construct circle graphs, bar graphs, and histograms, with and without technology

(8.13) Probability and statistics. The student evaluates predictions and conclusions based on statistical data. The student is expected to:

(B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(8.14) Underlying processes and mathematical tools. The student applies Grade

8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

20-ounce drink of vanilla latte is determined to be $11\frac{3}{7}$ ounces.

Vanilla syrup: $$\frac{2 \text{ oz of vanilla} \times \frac{10}{7}}{14 \text{ oz of latte} \times \frac{10}{7}} = \frac{\frac{20}{7} \text{ oz of vanilla}}{20 \text{ oz of latte}} = \frac{2\frac{6}{7} \text{ oz of vanilla}}{20 \text{ oz of latte}}$$

Using the scale factor method to solve the proportion above, there were $2\frac{6}{7}$ oz of vanilla syrup in Ella's 20 oz vanilla latte drink originally.

To verify, add the amount of milk, coffee, and vanilla syrup to show that the sum is 20 ounces.

$$5\frac{5}{7} \text{ oz} + 11\frac{3}{7} \text{ oz} + 2\frac{6}{7} \text{ oz} = 20 \text{ oz}$$

4. Circle graphs may be created to show the relationship among the ingredients in each mug. To create the circle graph we must find the fractional part of the total for each of the ingredients.

The ratio of ounces of milk to ounces of coffee to ounces of vanilla syrup is $2 : 3 : \frac{3}{4}$ for Sara's mug of vanilla latte. This ratio can be expressed as 8 : 12 : 3 when scaled up using a scale factor of 4. By comparing the number of ounces of each ingredient to total number of ounces, a part-to-whole relationship can be found. For example, the fractions $\frac{8}{23}$, $\frac{12}{23}$, and $\frac{3}{23}$ can be determined, where 23 equals 8 + 12 + 3. The sum of the fractions $\frac{8}{23}$, $\frac{12}{23}$, and $\frac{3}{23}$ equals 1 and represents the whole circle in the circle graph below. Estimation can be used to determine what fraction of the circle is represented by each ingredient. The fraction $\frac{8}{23}$ is close to $\frac{8}{24}$, which is equivalent to $\frac{1}{3}$. The fraction $\frac{12}{23}$ is close to $\frac{12}{24}$ or $\frac{1}{2}$, and $\frac{3}{23}$ is close to $\frac{3}{24}$, or $\frac{1}{8}$. Since a circle has 360 degrees, $\frac{1}{3}$ of 360 degrees is 120 degrees, the degrees in the central angle representing milk; $\frac{1}{2}$ of 360 degrees is 180 degrees, the degrees in the central angle for coffee; and, $\frac{1}{8}$ of 360 degrees is 45 degrees, the measure of the central angle representing vanilla syrup. Using this information, the circle graph depicting the ratio of ingredients in Sara's mug can be constructed. The other circle graphs can be constructed in a similar manner using the ratio of ingredients for milk, coffee, and vanilla syrup provided in the given problem.

Luke: 3 : 5 : 1.5

6 :10 : 3

6 + 10 + 3 = 19

$\frac{6}{19}$, $\frac{10}{19}$, $\frac{3}{19}$

Ella: 4 : 8 : 2

4 + 8 + 2 = 14

$\frac{4}{14}$, $\frac{8}{14}$, $\frac{2}{14}$

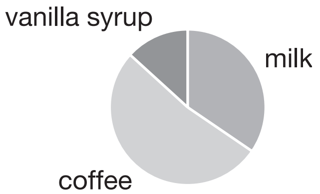
Ricky: 1 : 2 : 0.5

2 : 4 : 1

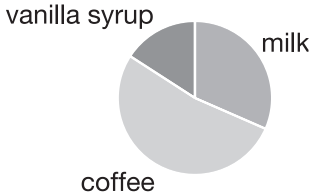
2 + 4 + 1 = 7

$\frac{2}{7}$, $\frac{4}{7}$, $\frac{1}{7}$

Sara's mug



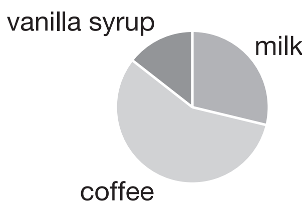
Luke's mug



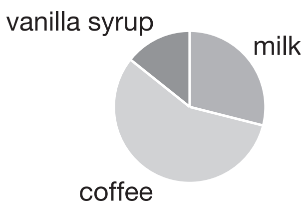
Texas Assessment of Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.

Ella's mug



Ricky's mug



From the circle graphs, one can verify that Luke's coffee drink is the most vanilla-y because his drink's vanilla syrup portion of the graph is larger in area than any other drink's. Therefore, Luke's drink has the highest percentage of vanilla syrup than any other drink. It is also evident from the graphs that Ella and Ricky's drinks have the same ratio of ingredients. A comparison of the measures of the central angles for corresponding ingredients validates that their drinks taste the same.

Extension Questions

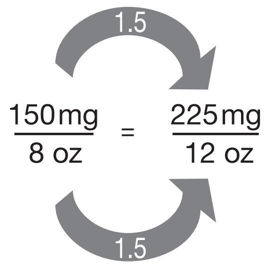
- Assume all four customers ordered the large 20-ounce drink and finished the entire mug of coffee. Who consumed the most caffeine?

  The circle graphs show that Ella and Ricky had the highest percentage of coffee in their drinks. Approximately 58% of each of their drinks was coffee. Therefore, they consumed the most caffeine.

- On average, coffee contains about 150 mg of caffeine in an 8-ounce of coffee. Approximately how many milligrams of caffeine did Ella consume if she drank all 20 ounces?

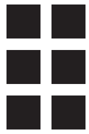
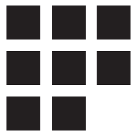
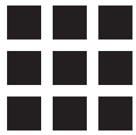
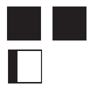
  From problem 3, Ella had between 11 and 12 ounces of coffee in her drink (58% of 20 = 11.6). Using the proportion comparing milligrams of caffeine to ounces of coffee, it can be shown that for a little less than 12 ounces of coffee, Ella consumed slightly less than 225 mg of caffeine.

  The scale factor from 8 oz to 12 oz is 1.5. Multiply 150 mg by the same scale factor to get 225 mg of caffeine.



- Ella thinks her coffee is too strong and prefers the taste of Sara's coffee. What amount of ingredients could Ella add to her drink to get the same taste as Sara's?

  The ratio in Sara's mug between the number of ounces of milk and the number of ounces of coffee is 2 : 3. This ratio states that for every 2 ounces of milk there are 3 ounces of coffee. In order for Ella's drink to taste the same, she will need the same ratio of ounces of milk to ounces of coffee. To accomplish this, Ella must add 2 more ounces of milk and 1 more ounce of coffee to her drink. Refer to the table below:

	Sara's coffee	Ella's existing coffee	Ella's coffee adjusted
Ounces of milk			
Ounces of coffee			
Ounces of vanilla syrup			
Ratio onces of coffee to ounces of milk to ounces of vanilla	$2 : 3 : \frac{3}{4}$	4 : 8 : 2	$6 : 9 : 2\frac{1}{4}$

Notice that for every 2 parts milk and 3 parts coffee there is $\frac{3}{4}$ part vanilla. If Ella's coffee has 3 groups of 3 parts coffee and 3 groups of 2 parts milk, then she will also need 3 groups of $\frac{3}{4}$ part vanilla, or $\frac{9}{4} = 2\frac{1}{4}$ ounces of vanilla. Since Ella already has 2 ounces of vanilla in her mug she will only need to add $\frac{1}{4}$ of an ounce of vanilla. This will give Ella a total of $17\frac{1}{4}$ ounces of her coffee drink.

Student Work Sample

This student's work shows the use of multiple ways to find solutions.

The work exemplifies many of the criteria on the solution guide, especially the following:

- Recognizes and applies proportional relationships
- Develops and carries out a plan for solving a problem that includes understand the problem, select a strategy, solve the problem, and check
- Solves problems involving proportional relationships using solution method(s) including equivalent ratios, scale factors, and equations
- Evaluates the reasonableness or significance of the solution in the context of the problem
- Demonstrates an understanding of mathematical concepts, processes, and skills
- Uses multiple representations (such as concrete models, tables, graphs, symbols, and verbal descriptions) and makes connections among them
- Communicates clear, detailed, and organized solution strategy
- Uses appropriate terminology, notation, and tools

Java Joe's

Table #1	Sarah	Luke	Ello	Ricky
Milk	2oz	3 oz	4oz	1 oz
Coffee	3oz	5 oz	8oz	2 oz
Vanilla Syrup	$\frac{3}{4}$ oz	$1\frac{1}{2}$ oz	2 oz	$\frac{1}{2}$ oz
coffee/milk to syrup	.15	.1875	$.16\frac{2}{3}$	$.16\frac{2}{3}$

2) same

1) Lukes coffee tast the most vanilla-y because he has the highest percent of Vanilla.

2) Ella and Ricky's coffee taste the same. percent is the same above. 16.67%

3) Milk

$$\frac{14}{20} = \frac{4}{X}$$

$$20 \cdot 4 = 14X$$

$$\frac{80}{14} = \frac{14X}{14}$$

$$5.7 = X$$

Coffee

$$\frac{14}{20} = \frac{8}{X}$$

$$20 \cdot 8 = 14X$$

$$\frac{160}{14} = \frac{14X}{14}$$

$$11.4 = X$$

V. syrup

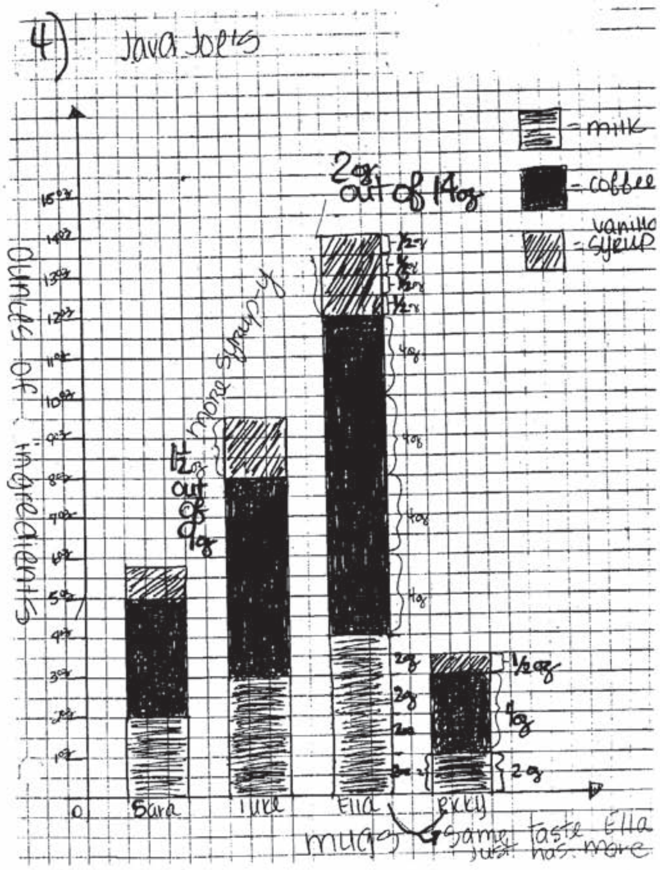
$$\frac{14}{20} = \frac{2}{X}$$

$$2 \cdot 20 = 14X$$

$$\frac{40}{14} = \frac{14X}{14}$$

$$2.9 = X$$

$$5.7 + 11.4 + 2.9 = 20\text{oz.}$$



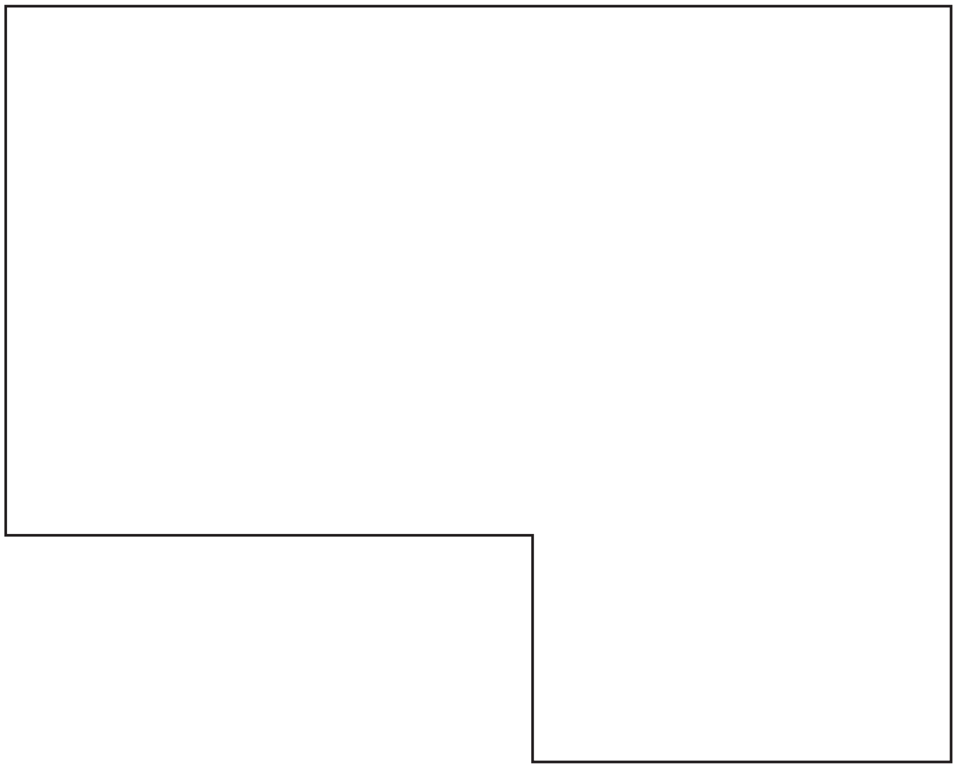
How Green is Green?
grade 8

Three primary elements—nitrogen, phosphorus, and potassium—need to be added to your lawn in the form of fertilizer to keep your lawn healthy and appealing. Nitrogen is an important element for grass, as it promotes growth and helps grass get its green color.

Commercial fertilizers that are sold in stores contain all three of these elements. You will commonly see the amounts of these elements displayed on the bags of fertilizer in numbers such as 20-10-6 or 30-15-10. The first number gives the percentage of nitrogen present; the second, the percentage of phosphorus; and the third, the percentage of potassium. The three numbers represent the total weight of each element per bag. For example, a 100-pound bag of 30-20-10 has a ratio of 3 : 2 : 1, where 30% of the 100-pound bag, or 30 pounds is nitrogen, 20% or 20 pounds is phosphorus, and 10 % or 10 pounds is potassium. The remaining 40%, or 40 pounds are other ingredients often used to help distribute the product.

The Lawn and Garden Store recommends any of the following bags of fertilizer (note the percentages of nitrogen-phosphorus-potassium, as well as the price and size of the bags given on the label). The experts at Lawn and Garden Store recommend applying   pound of nitrogen per 1,000 square feet of yard 5 times a year.

The yard is drawn to scale in the model below, where 1 cm represents 5 ft.



1. Which bag of fertilizer is the best choice if you plan to purchase enough to last for an entire year of applications? In your answer, be sure to communicate the mathematical ideas and logical reasoning you used to validate your choice.
2. Complete the rate table below that shows the relationship between the size of the lawn *n* and the amount of nitrogen application recommended.

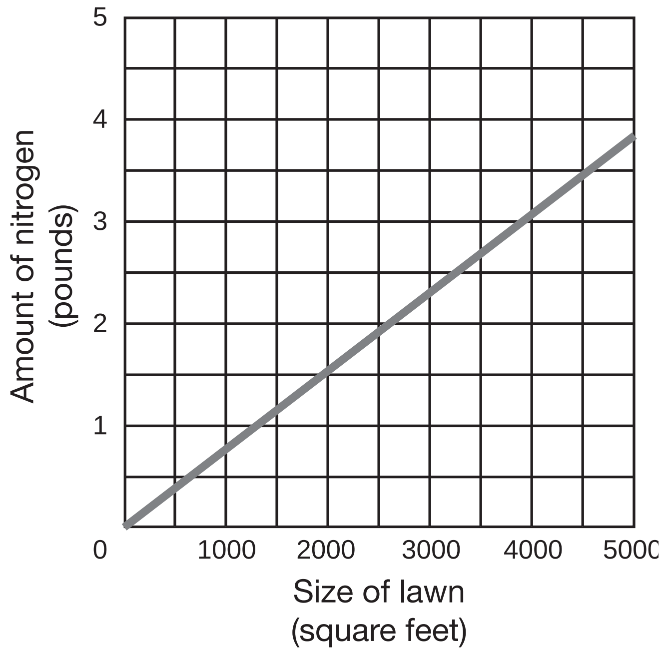
Lawn (square feet)	1,000	2,000	3,000	3,500	4,200	*n*
Process						
Amount of nitrogen applied						

3. How much nitrogen (measured in pounds) is needed per square foot of lawn? Explain how you determined this.

4. The graph below illustrates the relationship between the amount of nitrogen needed based on the number of square feet of lawn.

   a. What is the equation of this line? How did you determine this?

   b. Identify one point on the line and explain its significance.



Materials

Calculator

Centimeter ruler

Connections to Middle School TEKS

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(A) select and use appropriate operations to solve problems and justify the selections

(B) add, subtract, multiply, and divide rational numbers in problem situations

Teacher Notes

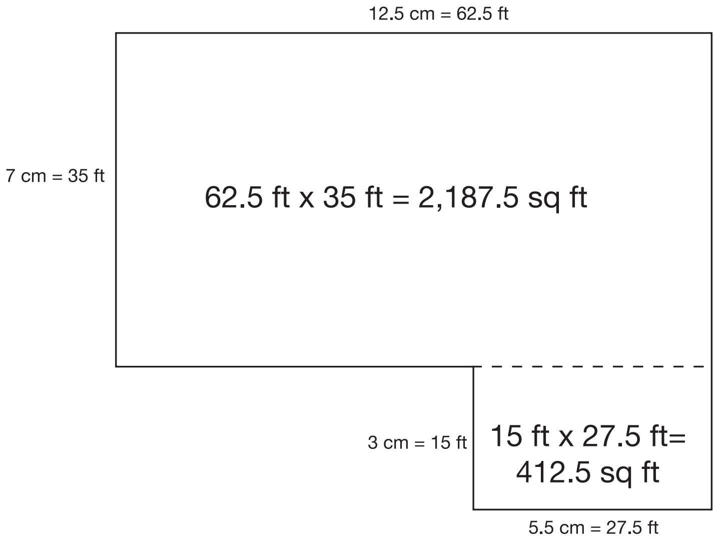
Scaffolding Questions

- What is the length of the yard in centimeters? In feet?
- What is the area of the yard as square feet? How did you find this?
- About how much nitrogen needs to be applied to this yard for one application?
- About how much nitrogen will be applied to this yard over the course of a year?
- What part of the StartUp Rite bag contains nitrogen? What about the Greeny Gro and Grow Best bags?
- How many pounds of nitrogen are in the StartUp Rite bag? In the Greeny Gro bag? In the Grow Best bag? How do you know?
- How many bags of StartUp Rite will it be necessary to purchase if you buy enough fertilizer to last the entire year? Of Greeny Gro? Grow Best? Explain.
- Which bag is the most cost effective? Explain.
- How do you determine how much fertilizer will be left over?

Sample Solutions

Note: There are many possible solutions based upon a student's reasoning. For example, a student may base his or her choice on price, while another student may consider the factor of transporting a large number of bags and their weight.

1. To find the amount of nitrogen needed for one application, the area of the yard can be found. Each dimension on the diagram is measured and converted to feet by multiplying by 5 feet per centimeter. For example, $12.5 \text{ cm} \times \frac{5 \text{ ft}}{1 \text{ cm}} = 62.5 \text{ ft}$.



The total area of the yard is 2,600 square feet (2187.5 + 412.5).

To find the amount of nitrogen needed for 2,600 square feet, the following proportion can be written.

$$\frac{\frac{3}{4}\text{ lb of nitrogen}}{1{,}000\text{ square feet}} = \frac{x}{2{,}600\text{ square feet}}$$

This proportion can be solved using the scale factor method. A scale factor of 2.6 is found by scaling up 1,000 to 2,600.Then multiply $\frac{3}{4}$ of a pound of nitrogen by 2.6 to get 1.95 pounds of nitrogen.

$$\frac{\frac{3}{4}\text{ lb of nitrogen} \times 2.6}{1{,}000\text{ square feet x } 2.6} = \frac{1.95\text{ lbs of nitrogen}}{2{,}600\text{ square feet}}$$

Therefore, the yard will need about 2 pounds of nitrogen applied per application. Since it is recommended to apply fertilizer 5 times a year, the yard will need a total of nearly 10 pounds (1.95 x 5 = 9.75 lbs) of nitrogen.

The StartUp Rite bag contains 15%, or 6 pounds of nitrogen. The Greeny Gro contains 29%, or 14.5 pounds of nitrogen; and the Grow Best contains 6%, or 0.9 pound of nitrogen per bag. There are several options available for purchasing enough nitrogen for the year. These are presented in the following table along with other deciding factors.

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$

(8.3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates

(8.4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations

(8.8) Measurement. The student uses procedures to determine measures of solids. The student is expected to:

(C) estimate answers and use formulas to solve application problems involving surface area and volume

(8.14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

(A) identify and apply mathematics to everyday experiences,

Fertilizer	Cost per bag	Amount of nitrogen per bag	Number of bags needed	Amount of unused nitrogen	Total cost	Total weight of purchase
StartUp Rite	$28.79	6 lbs	2	≈ 2 lbs	$57.58	80 lbs
Greeny Gro	$59.99	14.5 lbs	1	$\approx 4\frac{1}{2}$ lbs	$59.99	50 lbs
Grow Best	$4.99	0.9 lbs	11	0.15 lbs	$54.89	165 lbs

Given that all of these brands came recommended, 2 bags of StartUp Rite seems to be a good choice. The price is relatively near the cheapest option of Grow Best, and it would be much easier to carry and store two 40-pound bags compared to 11 bags totaling 165 pounds.

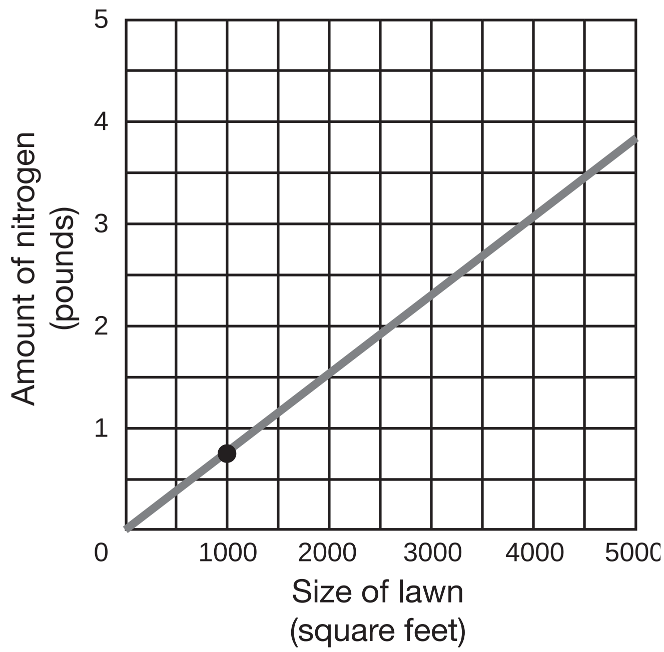
2. The following completed table shows the relationship between the size of the lawn and the amount of nitrogen applied.

Lawn (sq ft)	1,000	2,000	3,000	3,500	4,200	n
Process	$0.75 \times \frac{1{,}000}{1{,}000}$	$0.75 \times \frac{2{,}000}{1{,}000}$	$0.75 \times \frac{3{,}000}{1{,}000}$	$0.75 \times \frac{3{,}500}{1{,}000}$	$0.75 \times \frac{4{,}200}{1{,}000}$	$0.75 \times \frac{n}{1{,}000}$
Amount of nitrogen applied (lbs)	0.75	1.5	2.25	2.625	3.15	$\frac{0.75n}{1{,}000}$

3. The expression $\frac{0.75n}{1{,}000}$ can be used to find the amount of nitrogen to apply, where n represents the number of square feet. The amount of nitrogen necessary for 1 square foot can be calculated as follows.

$$\frac{0.75 \text{ lb nitrogen}}{1{,}000 \text{ square feet}} \times 1 \text{ square foot} = 0.00075 \text{ lb nitrogen}$$

4. Using the expression we found in problem 3, the equation of the line shown in the graph below is $y = 0.00075x$.



A possible point that could be identified is (1,000, 0.75). This point shows that 1,000 square feet of lawn needs 0.75 pounds of nitrogen.

Extension Questions

- If you could purchase a combination of brands to meet your yearly fertilizer needs, what combination would be the most affordable?

  If you purchased 1 bag of StartUp Rite and 4 bags of Grow Best you would have just enough nitrogen needed for a year's application.

  Weight: 6 lbs + 4(0.9 lbs) = 9.6 lbs

  Cost: $28.79 + 4($4.99) = $48.75

  You would save almost $9 with this combination purchase vs. purchasing two bags of the StartUp Rite and about $11 vs. purchasing 1 bag of Greeny Gro. Purchasing a combination of brands vs. only one brand can save you money!

to activities in and outside of school, with other disciplines, and with other mathematical topics

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(8.15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models

(8.16) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(B) validate his/her conclusions using mathematical properties and relationships

Fertilizer	**Cost per bag**	**Amount of nitrogen per bag**	**Number of bags needed**	**Amount of unused nitrogen**	**Total cost**	**Total weight of purchase**
StartUp Rite	$28.79	6 lbs	2	≈ 2 lbs	$57.58	80 lbs
Greeny Gro	$59.99	14.5 lbs	1	≈ $4\frac{1}{2}$ lbs	$59.99	50 lbs
Grow Best	$4.99	0.9 lbs	11	0.15 lbs	$54.89	165 lbs

- How would the equation and graph differ if each described the amount of nitrogen needed per year in relation to the size of the yard?

  Since the recommended application of fertilizer is 5 times a year, the equation would become $y = 5(0.00075x)$ or $y = 0.00375x$, *because the amount needed per square foot per year would be 5 times the amount recommended in a single application. The graph would be steeper given that the unit rate (amount per square foot per year) is 5 times greater compared to that of a single application.*

- Suppose the scale of the model of the yard had been 1 cm = 10 ft. How many pounds of nitrogen would be needed for a year's application?

  If the scale were to change from 1 cm = 5 ft to 1 cm = 10 ft, then all of the linear lengths of the lawn would double. Increasing the length and width by a scale factor of 2 causes the area to grow by the square of the scale factor; hence, the area of the lawn is now 4 times greater (22) than the area of the original lawn. If the area were 4 times larger, it would need 4 times as much nitrogen during the year. Instead of the previously estimated 10 pounds of nitrogen per year, the new yard would need approximately 40 pounds of nitrogen per year.

Texas Assessment of Knowledge and Skills

Objective 6: The student will demonstrate an understanding of the mathematical process and tools used in problem solving.