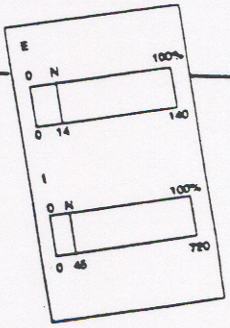
Percent Bars



Institute Notes

Concept: Connect the characteristics of proportional relationships to percent situations.

TEKS Focus: **6.3**—The student solves problems involving proportional relationships.
7.3—The student solves problems involving proportional relationships.
8.3—The student identifies proportional relationships in problem situations and solves problems.

Also:
Grade 6:
1B, 2C, 2D, 4A, 4B, 5, 11A-D, 12A
Grade 7:
1B, 2D, 2F, 2G, 4A, 4B, 13A-D, 14A
Grade 8:
1B, 2A, 2C, 2D, 4, 5A, 14A-D, 15A

Overview: Participants will use the Cuisenaire rods and centimeter rulers to develop a percent bar to model the proportional relationships in percent problems. In Activity 1, participants investigate finding a variety of percents of a given quantity (holding the size of the "whole" constant while varying the percent amount) and display their results in table, graph, and equation form. In Activity 2, participants will investigate finding a given percent of a variety of quantities (holding the percent constant while varying the size of the "whole") and display their results in table, graph, and equation form. Participants will then compare the results, making verbal statements about the proportional relationships that are involved in percent situations. In Activity 3, participants will use the percent bar model and the proportional relationships identified in Activities 1 and 2 to solve percent problems.

Materials:
Meter stick
Centimeter rulers
Cuisenaire rods
Overhead Cuisenaire rods
Graphing calculators

Procedure:

1. Have participants work in groups of four. Each group member should have either a meter stick or or an orange, blue, brown, or purple rod. Ask the following questions:
   - What measurement in centimeters is equivalent to one meter?
     100 centimeters

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- What fraction is equivalent to 1%?
  $\frac{1}{100}$ [or 0.01]
- What fraction of a meter is one centimeter?
  $\frac{1}{100}$ [or 0.01]
- So, what length represents 1% of a meter?
  $\frac{1}{100}$ of a meter, or 1 cm
- What percent of a meter is 10 centimeters?
  10%
- What percent of a meter is 20 centimeters?
  20%
- Using the meter stick, show 35% of a meter.
  any interval that is 35 centimeters long

2. Ask, "Which of the Cuisenaire rods would you use to represent 100%? Why?"
3. Explain that, since not all participants have meter sticks (and they do not fit onto a piece of notebook paper!), they will use the orange rod to represent 100% of a meter stick and the beige (white) rods to represent parts of a meter stick. Give each pair of participants a set of Cuisenaire rods. The participants should use the rods to help answer questions as the presenter models on the overhead.

- How many beige rods are equal in length to an orange rod?
  10 beige rods
- So a beige rod is what percent of an orange rod?
  10%
- How many beige rods represent 50% of the orange rod? Explain.
  5 beige rods, because they are half of the orange rod, or because each beige rod is 10% of the orange rod
- How many beige rods represent 70% of the orange rod? Explain.
  7 beige rods, because each beige rod is 10% of the orange rod
- How many beige rods represent 25% of the orange rod? Explain.
  $2\frac{1}{2}$ beige rods, because each beige rod is 10% of the orange rod, and $\frac{1}{2}$ of a beige rod is 5% of an orange rod

Math Notes:
Every participant has an object in their hand—meter stick or Cuisenaire rod. *Everyone is involved!* After this observation, groups will conclude that the orange rod would be their choice since it measures 10 cm or 100 mm.

The beige rod represents a "benchmark" of 10%. We can use this benchmark to find how many beige rods it takes to represent a given percent of the orange rod.

- Lay your centimeter ruler next to the orange rod and use it to describe the length of a rod that would be 48% of the orange rod. Describe to your partner the process you used.
  The orange rod is 10 cm in length. 10 cm is equivalent to 100 mm. Since 48% is 48/100, 48% of 100 mm is 48 mm. Then 48% of the orange rod would be 48 mm long.

4. Ask participants to determine what percent each of the other Cuisenaire rods is of the orange rod.
   Red is 20% of orange; light green is 30%; purple is 40%; yellow is 50%; green is 60%; black is 70%; brown is 80%; blue is 90%. The orange rod is 100% of itself.
5. Distribute the activity sheet for Activity 1 and allow participants to work in pairs to answer the questions. Use the *Reason and Communicate* and *Math Notes* sections to lead a discussion of Activity 1 with the whole group.
   The answer sheet for Activity 1 points out some of the methods which may be used in solving the percent problems.
6. Distribute the activity sheet for Activity 2 and allow participants to work in pairs to answer the questions. Use the *Reason and Communicate* and *Math Notes* sections to lead a discussion of Activity 2 with the whole group.
   The answer sheet for Activity 2 points out some of the methods which may be used in solving the percent problems.

Debriefing:

1. After Activities 1 and 2 have been completed, have participants compare the results and verbalize the proportional relationships that hold when the size of the whole is kept constant and the percent changes (Activity 1) and when the percent is kept constant and the size of the whole changes (Activity 2). Lead participants to recognize how the knowledge of these proportional relationships can help students set up equations for solving percent problems and recognize reasonable answers. For example, have participants count off 1 - 6, go to the corresponding numbered table, and work in a small group on the corresponding problem. Then, post solutions on the walls

Math Notes:
In Activity 1, the proportional relationship exhibited in the percent bar models, the tables, and the graphs is that the percent of a quantity increases proportionally as the percent increases. In other words, 50% of a quantity is 5 times 10% of that quantity, or 25% of a quantity is 1/2 of 50% of that quantity, or 70% of a quantity is 7 times as large as 10% of that quantity. Knowing this proportional relationship allows students to

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and have participants walk around the room to discuss the solutions.

2. Have participants work individually or in small groups on the problems in Activity 3 and share their methods for solving the problems.
3. It is important that a variety of approaches be discussed by the group for each activity. Circulate while the participants work and select individuals or groups with different approaches to present their solutions to the whole group.

Extension: Ask the participants to find a reference to percent in a newspaper or magazine and to write a related problem which they solve using the percent bar model.

Notes:

use easily identified benchmarks such as 1%, 10%, and 50% of a quantity to estimate other percentages of that quantity.

In Activity 2, the proportional relationship exhibited in the percent bar models, the tables, and the graphs is that a given percent of a quantity increases proportionally as that quantity increases. In other words, 25% of 100 is twice as big as 25% of 50 (or 25% of 50 is half of 25% of 100). Knowing this proportional relationship allows students to use easily recognized percentages (such as 25% of 100 = 25) to estimate percents of related amounts (25% of 50 is half of 25% of 100, so 25% of 50 is half of 25 or 12.5).

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Activity 1

6\. a For each point, y represents the number of kilograms that is x percent of 60 kilograms. Example: the ordered pair (20, 12) means that 20% of 60 kg is 12 kg. (60, 36) means that 60% of 60 kg is 36 kg. In general, (x, y) means that x% of 60 kg is y.

b The points lie on a straight line which passes through the origin.

7\. a and b. To determine the equation of the line one must find the constant of proportionality. The rate of change from the graph using the points (0,0) and (20, 12) is 12 kilograms up for 20 percent points over. $\frac{12}{20}$ or $\frac{3}{5}$

If the points (0,0) and (100,60) are used, the rate of change is $\frac{60}{100}$ or $\frac{3}{5}$.

The equation describing y as x percent of 60 is: $y = \frac{60}{100}x$ or $y = \frac{3}{5}x$.

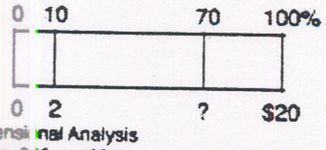
Another method: Use the constant ratio from your table $y = \frac{3}{5}$ to get $y = \frac{3}{5}x$.

c. You could use this equation to find other percents of 60 kilograms. For example, 18% of 60 kilograms = $y = \left[\frac{3}{5}\right] x\ 18 = 10.8$ kilograms.

If you kept the equation in the form $y = \left[\frac{60}{100}\right]x$, you could change the 60 to find percents of other quantities. For example, 18% of 85 kilograms would be $y = \left[\frac{85}{100}\right] x\ 18 = 15.3$ kilograms.

8\. The unit rates are 0.6 kg for each 1 percent (60 kilograms for every 100 percent) or $\frac{5}{3}$ percent or about 1.67 percent per 1 kilogram (100 percent for every 60 kilograms).

9\.



Benchmarks: 100% = \$20: 10% = \$2; 70% = \$14; 5% equals \$1 (using a scale factor of 7)

Dimensional Analysis

Rates: $\frac{\$20}{100\%}$ or $\frac{100\%}{\$20}$

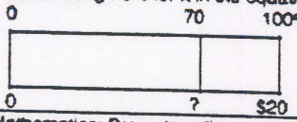
Unit Rates: $\frac{5\%}{\$1}$ or $\frac{\$0.20}{1\%}$

Gain Dollars

Lose Percent (from *Lost and Gained*)

Since we want to gain "dollars" and lose "percent," we multiply $\frac{\$0.20}{1\%} \cdot \frac{70\%}{1} = \$14$

Equation Method: From the percent bar or table, we have $\frac{y}{x} = \frac{1}{5}$, where $\frac{1}{5}$ is the constant ratio or constant of proportionality. Now, $y = \frac{1}{5}x$ means "The number of dollars (y) is one-fifth of the percent (x)." Substituting 70% for x in the equation, we have $y = \frac{1}{5}(70)$ or $y = \$14$.



Percent	Dollars	Dollars / Percent
100	20	$\frac{20}{100}$
50	10	$\frac{10}{50}$
25	2.50	$\frac{2.5}{25}$
80	16	$\frac{16}{80}$

Reason and Communicate:

How is the method of solution described in Problem 9 related to the traditional proportion equation and use of cross products?

The proportion which might be written is

$$\frac{N}{20} = \frac{70}{100}$$

$$100N = 20(70)$$

$$N = \frac{20(70)}{100} = 14$$

The expression $\left(\frac{20}{100}\right) \times 70$ *is equivalent to the fraction for N determined above.*

How is the solution related to the solution which uses the decimal equivalent of percent?

The traditional solution might be "What is 70% of 20?"

$N = 0.70\ (20) = 14$

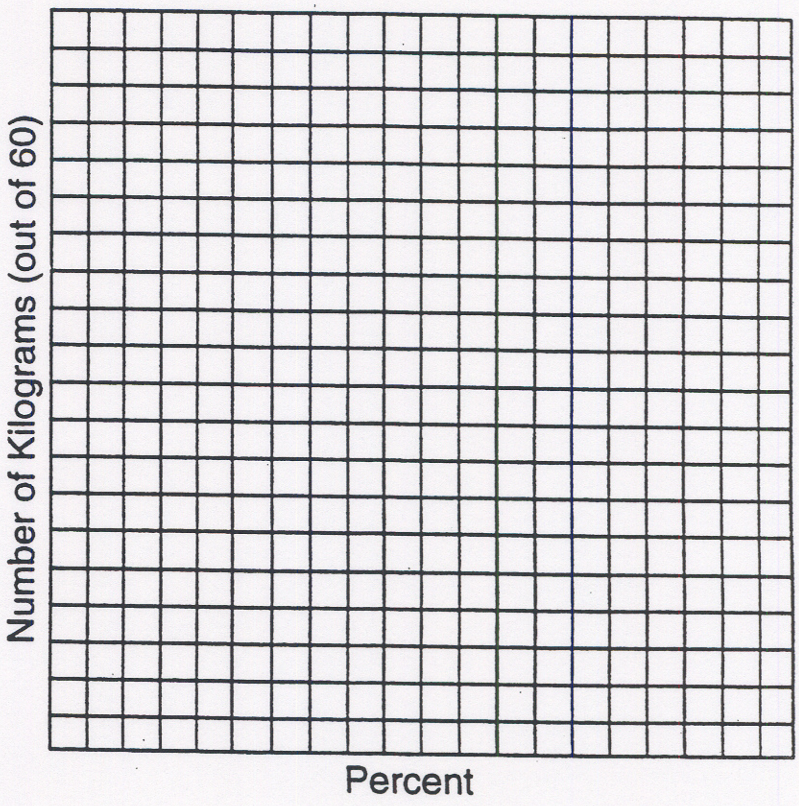
$0.70 = \frac{70}{100}$

$N = \frac{70}{100}(20)$

This is equivalent to the equation used in the solution to Problem 9.

Percent Bars

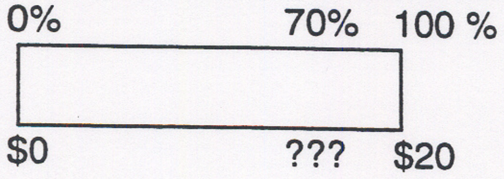
Activity 1 continued



5. Graph the data from your table.
6. a. What do the points on your graph represent?
   b. Explain how you know that a proportional relationship exists between the size of the percent and the number of kilograms out of 60 that represent that percent.
7. a. Write the equation that describes this proportional relationship.
   b. Explain how you determined the equation.
   c. Describe how you might be able to use this equation in other situations.

8. What unit rates appear in this proportional relationship?

9. Suppose you wish to find 70% of $20 using this percent bar.



a. First, set up a table:

Dollars	____	??
Percent	100	70

b. Then, determine a unit rate of dollars per percent:

$$\frac{\$\_\_\_\_\_\_\_\_}{100\ \%} = \frac{\$\_\_\_\_\_\_\_\_}{1\ \%}$$

c. Use dimensional analysis (from *Lost and Gained*) to multiply

$________ x ________% = $________

Percent Bars

Activity 2

Percents:	0%	50%	100%
Lengths:	0 dm	0.5 dm	1 dm
	0 cm	5 cm	10 cm
	0 mm	50 mm	100 mm

1. Measure this bar using your centimeter ruler. Label each measurement to show 50%. How do you know that 50% of the bar is equivalent to 0.5 = 1/2 dm? 5 cm? 50 mm? Because they are equivalent ratios: 50/100 = 0.5/1 = 5/10 = 50/100

2. Label the following percent bar to show 20% of the different "wholes" that are shown. Explain your reasoning.

0%	20%	100%
0 kg	12 kg	60 kg
0 kg	24 kg	120 kg
0 kg	8 kg	40 kg

3. Use the percent bar and your experience in Problems 1 and 2 to estimate the number of kilograms that represents 20% of each total. Record your estimates in the table below.

Number of kg in 100% (x)	0	20	30	40	60	100	150	180
Number of kg in 20% (y)	0	4	6	8	12	20	30	36

Percent Bars

Activity 2

Percents:	0%	50%	100%
Lengths:	0 dm	___ dm	___ dm
	0 cm	___ cm	___ cm
	0 mm	___ mm	___ mm

1. Measure this bar using your centimeter ruler. Label each measurement to show 50%. How do you know that 50% of the bar is equivalent to $0.5 = \frac{1}{2}$ dm? 5 cm? 50 mm?

2. Label the following percent bar to show 20% of the different "wholes" that are shown. Explain your reasoning.

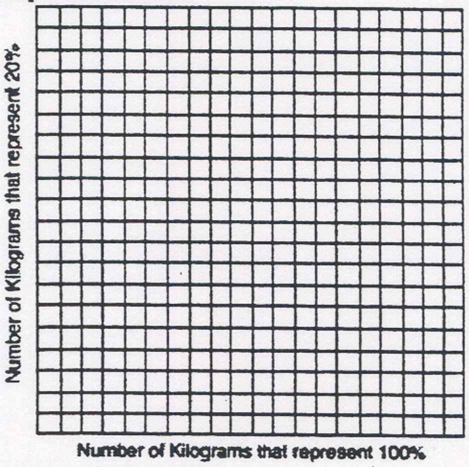
0%	20%	100%
0 kg	___ kg	60 kg
0 kg	___ kg	120 kg
0 kg	___ kg	40 kg

3. Use the percent bar and your experience in Problems 1 and 2 to estimate the number of kilograms that represents 20% of each total. Record your estimates in the table below.

Number of kg in 100%	0	20	30	40	60	100	150	180
Number of kg in 20%								

Percent Bars

Activity 2 continued



4. Graph the data from your table.

5. a. For each point, *y* is 20% of *x*.

   b. The graph forms a straight line through the origin.

6. a. and b. To determine the equation of the line one must find the constant of proportionality. The rate of change from the graph using the points (0,0) and (20, 4) is 4 kilograms up for 20 kilograms to the right. So the constant of proportionality is 4/20 = 1/5.

   The equation describing y as 20% percent of *x* is $y = \frac{1}{5}x$.

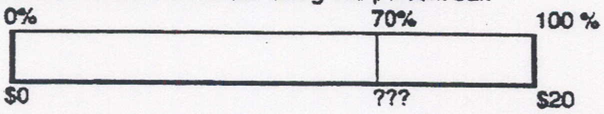
   If the rate of change is found using the points (0,0) and (100, 20), the constant of proportionality is seen as 20/100 and an equivalent equation can be written as $y = \frac{20}{100}x$. This form is more visibly related to the 20% that the graph is representing.

   c. You could use this equation to find 20% of any number of kilograms. For example, 20% of 18 kilograms = $y = \frac{1}{5} \times 18 = 3.6$ kilograms. Making the connection between $y = \frac{20}{100}x$ and the graph of 20%, the equation could be changed to find 15% of any number: $y = \frac{15}{100}x$. Or any given percent (p) of any number: $y = \frac{p}{100}x$.

7. a. For every point on this graph, $\frac{y}{x} = \frac{1}{5}$. For example, 4:20, 6:30, 8:40...

   b. $\frac{1}{5}$ It represents the slope of the line.

8. Suppose you wish to find 70% of $20 using this percent bar.



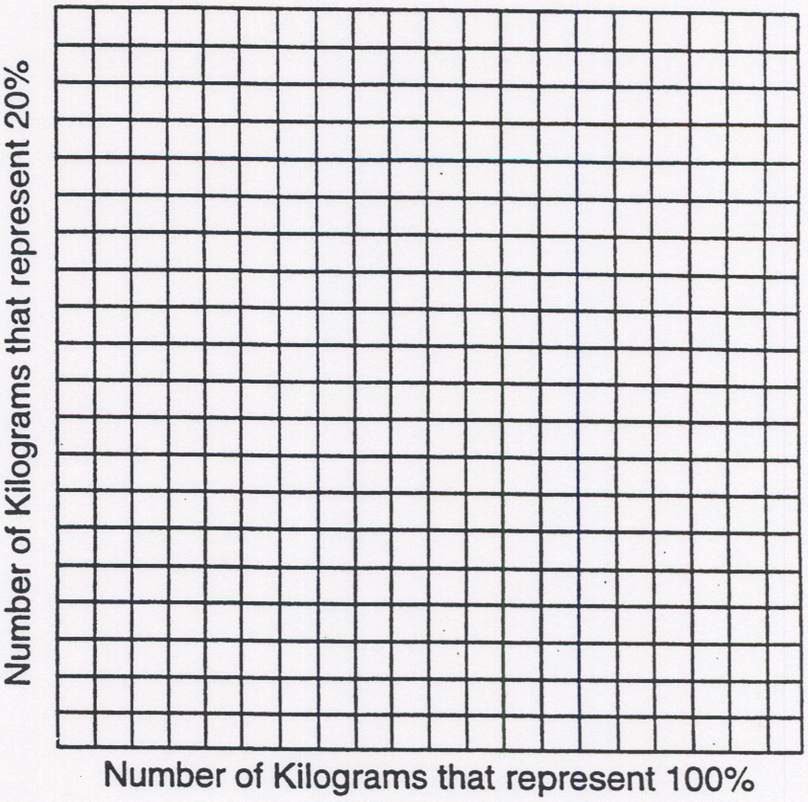
   a. You can use the constant of proportionality, $\frac{70}{100} = 0.7$ to answer this question.

   b. Set up the equation: y = <u>0.7 x $20.</u>

   c. So, 70% of $20 is <u>$14.</u>

Percent Bars

Activity 2 continued



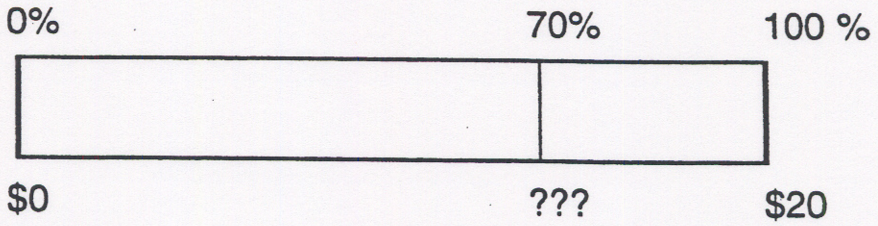
4. Graph the data from your table.

5. a. What do the points on your graph represent?
   b. Explain how you know that a proportional relationship exists between the number of kilograms that represent 20% and the number of kilograms that represent 100%.

6. a. Write the equation that describes this proportional relationship.
   b. Explain how you determined the equation.
   c. Describe how you might be able to use this equation in other situations.

7. a. What equivalent ratios appear in this proportional relationship?
   b. What constant of proportionality appears in this relationship? What does this constant of proportionality represent in the graph?

8. Suppose you wish to find 70% of $20 using this percent bar.



   a. You can use the constant of proportionality, __________, to answer this question.

   b. Set up the equation: $y =$ (constant of proportionality) x ___________.

   c. So, 70% of $20 is ___________.

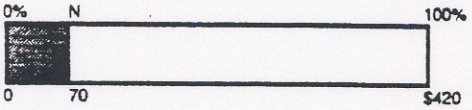
Percent Bars

Percent Bars

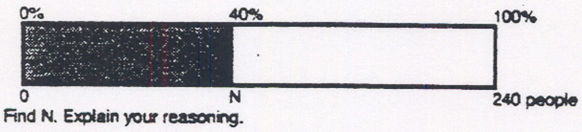
Activity 3

Solve each problem using a) Benchmarks, b) Dimensional Analysis, and c) Equation Method.

1. The percent bar has been marked to answer the question, "What percent of 420 is 70?" Find the value for N. Explain your reasoning.



2. a. What question is this percent bar a model of?



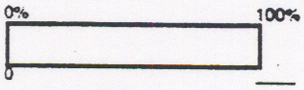
b. Find N. Explain your reasoning.

3. a. What question is this percent bar a model for?

b. Find N. Explain your reasoning.

4. A certain tablet contains 120% of the recommended daily requirement of a necessary mineral. If the recommended daily requirement is 60 mg, how much of the mineral does the tablet contain?

a. Use the percent bar model to help solve the problem by showing 120% of this bar.



b. Solve the problem.

Answers and Math Notes:

1. The problem is "70 is what percent of 420?" and may be solved by various methods. This solution shows the use of dimensional analysis to change the units. The rates of change are 100 percent for every 420 units or 420 units for every 100 percent. You wish to gain an answer in percent, so the percent must be in the numerator.

$\frac{100 \text{ percent}}{420 \text{ units}}(70 \text{ units}) = 16\frac{2}{3}$ percent

A second approach could be to find the constant of proportionality in the equation for finding a given percent of any quantity, y = p(x), replacing y and x with the appropriate values and solving for p:

70 = [p/100](420); p/100 = 70/420 = 7/42 = 1/6; p = (0.1666...)100 = 16.666...%.

2. The question is, "What is 40% of 240?" and may be solved by using a table and unit rate.

people	240	??
percent	100	40

The unit rate is 2.4 people/1 percent.

Using dimensional analysis,

(240people /100%) x 40% = $\frac{(240)(40)}{100}$ = 96 people

Reason and Communicate:

Before using paper and pencil to solve each problem, ask participants to use the proportional relationships they have investigated in Activities 1 and 2 to mentally estimate or calculate the solutions to the problems and explain their thinking. (For example, in Problem 1, 210 is 50% of 420, and 70 is 1/3 of 210. Then 1/3 of 50% is a little more than 15%. In Problem 2, 10% of 240 is 24, so 40% of 24 is 4 times as much, or nearly 100. In Problem 3, you need 10% more to make 100%, and 10% of 360 is 36, so 100% is around 400. And in Problem 4, 10% of 60 is 6, so 20% of 60 is 12, and 120% of 60 is 60 + 12, or 72.

This could also be described with the equation

$y = \frac{240}{100}x \qquad y = \frac{240}{100} \bullet 40 = 96$

A second method would involve the use of the constant of proportionality in the equation describing the relationship between a quantity and 40% of that quantity: $y = \frac{[40]}{100}x$.

So y = 0.40(240) = 96 people.

3. The problem is "360 is 90% of what number of liters?"

One approach to the solution would be to use the table to determine the unit rate.

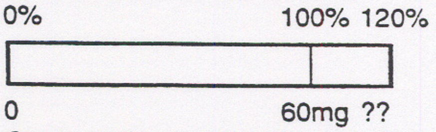
Liters	360	N
Percent	90	100

The unit rate is 4 liters per 1 percent (360 liters for every 90 percent). Then, using dimensional analysis,

[4 liters/ 1 %] x 100% = 400 liters.

Another approach would be to use the relationship 360 liters = 90/100 x N.

4. a. To show 120 % of this bar:



One possible method for solution is to find the unit rate and its related equation. The rate is 60 mg for every 100 percent.

$y = \frac{60}{100}x \qquad y = \frac{60}{100} \bullet 120 = 72$

So the tablet conains 72 mg of the mineral.

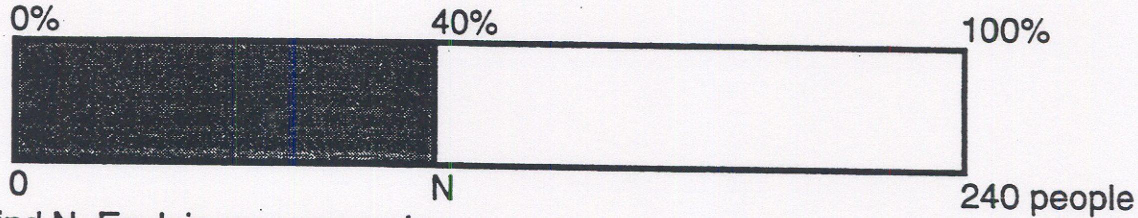
Percent Bars

Activity 3

Solve each problem using a) Benchmarks, b) Dimensional Analysis, and c) Equation Method.

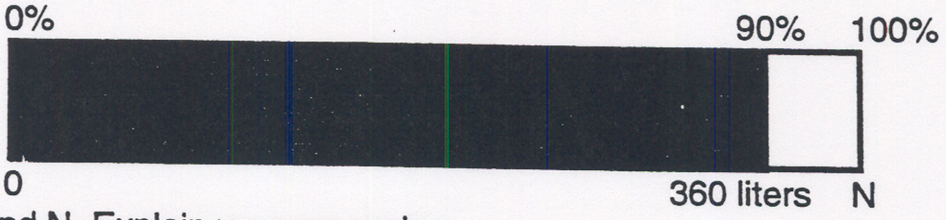
1. The percent bar has been marked to answer the question, "What percent of 420 is 70?" Find the value for N. Explain your reasoning.

2. a. What question is this percent bar a model of?



b. Find N. Explain your reasoning.

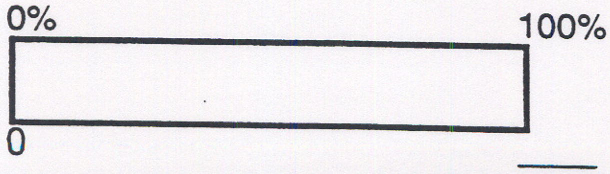
3. a. What question is this percent bar a model for?



b. Find N. Explain your reasoning.

4. A certain tablet contains 120% of the recommended daily requirement of a necessary mineral. If the recommended daily requirement is 60 mg, how much of the mineral does the tablet contain?

a. Use the percent bar model to help solve the problem by showing 120% of this bar.



b. Solve the problem.

Unit IX: Reflect and Apply

Percent Bar Matching Cards

7. Ken earned $200 in July for mowing lawns. His expenses were $42. What percent of his earnings were the expenses?	**1.** The tax on $140 item is $14. What percent of the cost is the tax?
3. Carmen found a stereo, which regularly sells for $200. It was discounted by 42%. How much was the discount?	**2.** 45% of the freshmen said that they watched more than four hours of television each day. If the freshman class has 720 students, how many watch more than four hours of television?
12. Mr. Wilson has saved 20% of the cost of a new television. If he has saved $90, what is the cost of the new television?	**9.** Last year there were 140 students taking algebra. This represents 14% of the students. How many students are there?

Unit IX: Reflect and Apply

Percent Bar Matching Cards

6. Melanie will have to pay 20 % of the money she earns to income tax and other deductions. If she earns $90, how much of this must she pay for income tax and other deductions?

10. Jeff sold 45 boxes of candy for the school fund raising project. If 720 boxes were sold, what percent of the boxes did Jeff sell?

11. In the park, there are 75 trees. 42 of the trees are elm trees. What percent of the trees in the park are elm trees?

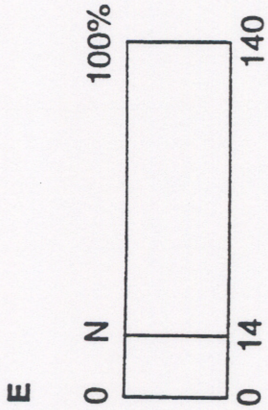
4. Marcus made 10 baskets out of 48 attempts. What percent of the attempts were the baskets?

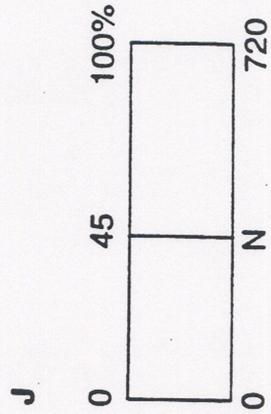
5. Last week Matt sold 42 magazines. This week he sold 75 magazines. What percent of 42 is 75?

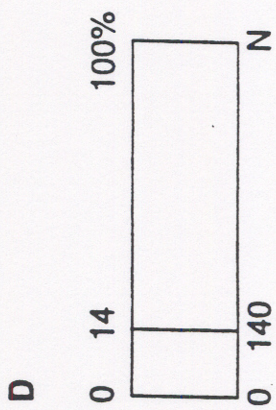
8. Helena will save 10% of her tip money each day. If she made $48 in tips, how much should she save?

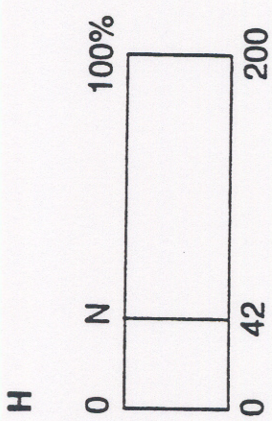
Unit IX: Reflect and Apply

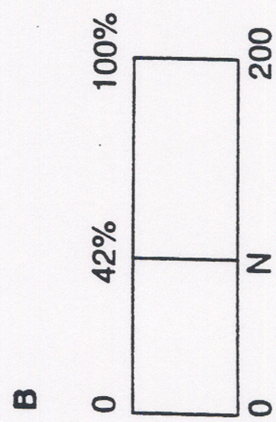
Percent Bar Matching Cards

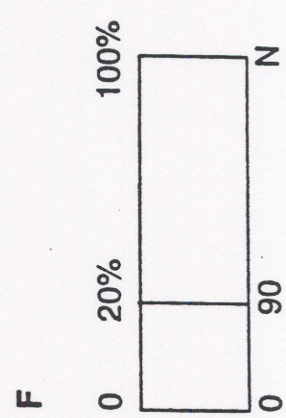












Unit IX: Reflect and Apply

Percent Bar Matching Cards