

Relevant Ratios

Institute Notes

Concept: Compare and contrast ratios used in real-world contexts.

TEKS Focus: 6.3—The student solves problems involving proportional relationships.

7.3—The student is expected to estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

8.3—The student identifies proportional relationships in problem situations and solves problems.

Overview: Day 2 will begin with small-group discussions of the examples of ratios that participants have collected from newspapers, magazines, TV commercials or broadcasts, or other real-life experiences as homework assigned on Day 1. The presenter will lead a whole-group discussion of all examples, having participants examine the meanings and characteristics of these ratios to lay the groundwork for the use of ratios to describe constants of proportionality, such as unit rates. Interpretation of the ratios in the context in which they are used and equivalent representations using fractions, decimals, and percents (where appropriate) will be examined. This introductory activity focuses on the many ways that ratios are a part of our everyday lives.

Materials: One pad of easel paper or white butcher paper
Markers
Masking tape

Procedure: 1. Have participants work in groups of 4 to 6, sharing their examples of ratios from the real world. Record their examples on Transparency 1. Have participants describe how each ratio is used. (For example, to describe the power of an engine, or to describe the reliability of a product, or to describe the strength of a mixture.) Depending on the scope of the examples from participants, it may be necessary to share some examples of your own of ratios from the real world. (See Transparencies 2-4.)

Also:

Grade 6

1B, 11A, 12A

Grade 7

1B, 2D, 13A, 14A

Grade 8

1A, 14A, 15A

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2. Have participants sort the ratios into two or three groups, and explain how they think the ratios in each group are alike and how they are different from the other groups.
3. Have participants work in groups of 4 to discuss part (a) of the examples on Activity Sheet 1 of uses of ratios in the real world, referring to the characteristics of ratios discussed previously. Have participants discuss solutions now to the problems presented in part (b) of each example, or return to this Activity 1 at the end of Day 2 and ask participants to apply the ideas of equivalent ratios and unit rates to solve the problems presented in each part (b).

Assessment: Use the following prompts for journal entries:

A ratio is _____ . A ratio can be used to
_____ .

Notes:

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Transparency 1

TEXTEAMS Rethinking Middle School Mathematics: Proportionality

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whole ratio involves the comparison of like measures. Sometimes the part-whole ratio is not obvious to students because of what seems to be the comparison of different units: 4 boys to 25 students. In this case, we are really comparing students (who are boys) to students (total).

Participants may identify ratios that represent a comparison between two parts of a set, or a part-part ratio. Suppose the ratio of ground-level seats to stadium-level seats in a movie theater is 2 to 3. Since the comparison is between two parts of a whole (total number of seats), it is a part-part ratio. A part-part ratio, like a part-whole ratio, involves the comparison of like quantities or measures (quantities described with the same unit).

Some ratios (often called *rates*) are comparisons of two quantities measured in different units, e.g. 120 miles per 4 gallons of gasoline, 3 lb. of apples for \$2.49, 17 miles per gallon, 1960 sq.ft. per 10 gallons of paint, etc. Not all ratios can be easily categorized with these commonly-used descriptions. (See the examples on Transparencies 2-4 and in Activity 1.)

Math Notes:

Most results of medical tests such as for blood sugar level are ratios; many statistical measurements such as means are ratios; many scientific constants such as air pressure in pounds per square inch are expressed as ratios; prices in grocery stores are often expressed as ratios such as 5 lb of potatoes for \$2.49 or 3 cans of soup for \$1.00; recommended cold inflation pressure for tires on a car is expressed as 30 psi; the EPA estimates for a new car are given on the window sticker as 17 mpg city and 24 mpg highway; gear ratios of cars are given as decimals such as 3.35 for first gear in a 1993 Ford Mustang Cobra with a compression ratio of 9:1. The list can go on and on with numerous examples. What is important in each of these examples is their meaning according to the context of the situation and how they are used.

In sorting the ratios, participants may identify ratios that represent a comparison between a part of a set and the whole set, or a part-whole ratio. In the statement, "8 light bulbs out of 100 tested were defective," the ratio of defective bulbs to bulbs tested can be written as a fraction $\frac{8}{100}$ and repre-

sents a part-whole comparison. In a part-whole ratio, we describe the part and the whole using the same unit (e.g., bulbs). In other words, a part-

However, every ratio is a comparison between two quantities, with the quantities sometimes measured in the same units and sometimes in different units.

Also, a ratio can be expressed in fraction form (as long as the number that ends up in the denominator is not equal to zero). Some part-whole ratios, depending upon the context, can be represented with decimals and percents.

Relevant Ratios

Transparency 1

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Transparency 2—Ratios in Medicine

Dosage: weight

Heart rate =
number of beats/minute

Respiration =
number of breaths/minute

IV therapy =
number of cc/hour

TEXTEAMS Redefining Middle School Mathematics: Proportionality

Transparency-6

Math Notes:

All of the examples given here are rates because they are comparisons of two measurements using different units.

Dosage: weight describes the amount of medicine to prescribe for the desirable effect.

Heart rate and respiration describe the speed of a person's heartbeat and breathing.

The **IV therapy** ratio determines the dosage of medicine given to a person over time.

Participants may mention blood pressure as another ratio in medicine, since it is reported with two numbers—systolic pressure (the pressure while the heart is beating) and diastolic pressure (the pressure while the heart is at rest). However, the two numbers in blood pressure are not compared in a multiplicative sense. The interpretation of blood pressure is not “so much systolic pressure per so much diastolic pressure.” Instead, the interpretation is that a systolic pressure over 140 or a diastolic pressure over 90 is considered high. The values are considered fairly independently; therefore the two values in blood pressure are not used as a ratio is used.

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Transparency 2—Ratios in Medicine

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Transparency 3—Ratios in the Automobile Industry

Compression ratio of an engine =
expanded/compressed volume

Transmission ratio = crankshaft
speed/driveshaft speed in rev/min

Aspect ratio = height of a tire
(rim to road)/width of tire
(in percent)

TEXTEAMS Rethinking Middle School Mathematics: Proportionality

Transparency-6

Math Notes:

The **compression ratio** of an engine is the relationship between the combined cylinder and chamber volumes of the cylinder when the piston is at the bottom of its stroke and when the piston is at the top of its stroke. An example of the compression ratio of an engine is 440 cu. cm to 75 cu. cm., or a little less than 6:1. This ratio compares two measurements, expressed in the same units, but it does not fit into the part-part or part-whole categories that are often used to describe ratios. It is a relationship used to describe the power of an engine. The higher the compression ratio is, the more the air/fuel mixture will be compressed, and thus the more powerful the explosion will be.

The **transmission ratio** is an example of a rate (a comparison of measures with different units) where the measurement units are revolutions and minutes. It is used to compare the efficiency of different gears.

The **aspect ratio** is expressed as a percentage and compares different parts of the tire measured with the same units. If the raised numerals and letters on a tire are P225/60R16, the 225 represents the cross-section width of the tire in mm, and the 60 represents the aspect ratio. So $(\text{height in mm})/225 \text{ mm} = 60$. (The "16" represents the diameter of the wheel in inches). The lower the aspect ratio, the wider the tire.

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Transparency 3—Ratios in the Automobile Industry

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Transmission ratio = crankshaft
speed/driveshaft speed in rev/min

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(rim to road)/width of tire
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Transparency 4—Ratios in Science

Density = mass/volume

Density of gold = 19.3 grams/cu. cm

Specific gravity of a solid or liquid =
density of the solid or liquid: density of water

Specific gravity of a gas =
density of the gas: density of air

Specific gravity of lead = 11.34 (at 20° C)

Magnification of a mirror or lens =
image height: object height

Solutions given as percents
(such as a 35% acid solution)

Solutions given as mg/liter
(such as 20,000 mg/liter saline solution)

Speed of sound = 331 m/sec

Escape velocity for Venus = 10.3 km/sec

Escape velocity for Earth = 11.2 km/sec

TEXTEAMS Rethinking Middle School Mathematics: Proportionality

Transparency-10

Math Notes:

Density is a comparison of measurements with different units to produce a rate of weight (or mass) per volume, such as pounds per cubic inch. The higher the density, the heavier the solid (for the same volume).

Specific gravity of a solid or liquid is the ratio of the mass of that solid or liquid to the mass of an equal volume of distilled water (in other words, the ratios of their densities). Specific gravity of a gas is the ratio of the density of that gas to the density of air. Specific gravity is used to compare densities (where the standard being compared to is water, whose specific gravity is defined as 1). Specific gravity is also called relative density.

Magnification is a comparison of two measurements using the same units to describe the strength of a telescope or other magnifying device. It could be considered a part-whole ratio, if we think of the ratio as describing the image height as "what part" of the object height.

The percent description of a solution is an example of a part-whole ratio used to describe the strength of the solution.

The description of a saline solution is the comparison of two measures with different units (like a rate) used to describe the strength of the solution.

Speed of sound and escape velocity are rates that compare measures with different units.

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Transparency 4—Ratios in Science

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density of the solid or liquid: density of water

Specific gravity of a gas =
density of the gas: density of air

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Solutions given as percents
(such as a 35% acid solution)

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Activity 1

Identify a ratio for part (a) to describe each of the following situations, and solve the problems in part (b).

- The "aspect ratio" of a television screen is the ratio of length to width.
 - If a television screen measures 22" in length and 16.5" in width, find its aspect ratio.
 - Is this a "standard television screen" if a standard screen has an aspect ratio of 4:3?
- If the ratio of length to width of a rectangle is the "golden ratio" (approximately 1.618), it is called a golden rectangle.
 - Find the dimensions of a rectangle that is approximately "golden," and use a ratio to verify your selection.
 - If a rectangle has a length of 24 cm, what must its width be for it to be a golden rectangle?
- 1 punt in Ireland has been valued for exchange in U.S. dollars at \$1.54.
 - Write a ratio describing the exchange rate.
 - What is the exchange in U.S. dollars for 5 punts?
- In carpentry, the slope of a staircase is the ratio of riser height to tread length.
 - If a carpenter has measured the riser height as 8" and tread length as 12", what is the slope of the staircase?
 - How could you make the staircase more steep? less steep?
- There are 510 seats in the school auditorium. At the awards assembly, one-third of the seats will be occupied by adults and the rest by students.
 - What is the ratio of adult seats to student seats?
 - How many adult seats and student seats are there in the auditorium?
- If you roll a fair die, what is the probability that you will roll an odd number?
 - If you program a computer to select a random number from one through six 1000 times, how many times would you expect it to select an odd number?
- A 12 oz box of rice sells for \$1.99 and a 16 oz box of the same rice sells for \$2.79.
 - Use a ratio to describe each box's price per weight.
 - Which box of rice is the better buy? Explain.

TEXTEAMS Reinventing Middle School Mathematics: Proportionality

Activity 12

- The theoretical probability of an event E is determined by the ratio of number of favorable outcomes: number of possible outcomes. So, the theoretical probability of rolling an odd number on a fair die is 3:6 or 1:2.
 - Selecting a number from 1 through 6 is like rolling a die. In 1000 rolls you would expect 1 out of 2, or 500 selections, to be odd numbers (although in an actual experiment, it may not turn out exactly that way.)
- The 12 oz box of rice sells for \$1.99/12 oz or about \$0.1658/oz, and the 16oz box of the same rice sells for \$2.79/16oz or about \$0.1744/oz.
 - The 12 oz box of rice is the better buy, by about a penny an ounce. (But, notice that if the equivalent ratios had been rounded to the nearest penny, they would have looked the same, at \$0.17/oz.)

Answers:

1. a. The aspect ratio for the television given is 22:16.5 and can be expressed as a decimal 1.333 where "3" repeats or as a fraction $\frac{4}{3}$.

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b. This television screen is a standard screen.

2. a. Some simple examples are 3 x 5, 5 x 8, 8 x 13, etc.

b. In order to produce a golden ratio in a rectangle whose length is 24", the width must be approximately 14.833".

3. a. \$1.54 per punt or \$1.54

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b. The exchange for 5 punts in U.S. dollars will be \$7.70.

4. a. The slope of this staircase is the ratio of 8": 12" or 2:3;

b. The staircase can be made more steep by increasing the riser height or decreasing the tread length. It can be made less steep by decreasing the riser height or increasing the tread length.

5. a. The ratio of adult seats to student seats is 1:2. This can easily be represented by drawing a rectangle and dividing it into 3 equal sections. Let one section represent the number of adult seats and the other two sections, the number of student seats. From this diagram, it can be seen that the ratio of adult seats to student seats is 1 to 2.

b. If 510 is separated into 3 equal parts, each part is 170. So there are 170 adult seats and 170 (2) = 340 student seats.

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Identify a ratio for part (a) to describe each of the following situations, and solve the problems in part (b).

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 - a. If a television screen measures 22” in length and 16.5” in width, find its aspect ratio.
 - b. Is this a “standard television screen” if a standard screen has an aspect ratio of 4:3?
2. If the ratio of length to width of a rectangle is the “*golden ratio*” (approximately 1.618), it is called a golden rectangle.
 - a. Find the dimensions of a rectangle that is approximately “golden,” and use a ratio to verify your selection.
 - b. If a rectangle has a length of 24 cm, what must its width be for it to be a golden rectangle?
3. 1 punt in Ireland has been valued for exchange in U.S. dollars at \$1.54.
 - a. Write a ratio describing the exchange rate.
 - b. What is the exchange in U.S. dollars for 5 punts?
4. In carpentry, the slope of a staircase is the ratio of riser height to tread length.
 - a. If a carpenter has measured the riser height as 8” and tread length as 12”, what is the slope of the staircase?
 - b. How could you make the staircase more steep? less steep?
5. There are 510 seats in the school auditorium. At the awards assembly, one-third of the seats will be occupied by adults and the rest by students.
 - a. What is the ratio of adult seats to student seats?
 - b. How many adult seats and student seats are there in the auditorium?
6.
 - a. If you roll a fair die, what is the probability that you will roll an odd number?
 - b. If you program a computer to select a random number from one through six 1000 times, how many times would you expect it to select an odd number?
7. A 12 oz box of rice sells for \$1.99 and a 16 oz box of the same rice sells for \$2.79.
 - a. Use a ratio to describe each box’s price per weight.
 - b. Which box of rice is the better buy? Explain.