

Rock, Paper, Scissors

A-E Strand(s): Probability and Statistics

Topic/Expectation

D.B.2 Discrete graphs

- a. Construct and interpret decision trees.

PS.A.1 Simple probability

- b. Compare probabilities of two or more events and recognize when certain events are equally likely.

Other Topic/Expectation(s)

PS.A.2 Relative frequency and probability

- d. Compare theoretical probabilities with the results of simple experiments (e.g., tossing dice, flipping coins, spinning spinners).

Rationale

This task encourages students to use tree diagrams to determine the probabilities used to decide if two versions of a popular student game are fair.

Instructional Task

Activity 1:

1. Work in groups of three to play 20 rounds of the original Rock, Paper, Scissors game. There should be two players (A and B) and one recorder who records game wins in the table below. Remember that rock beats scissors, paper beats rock, and scissors beats paper. For more information on how to play the game, you may want to refer to www.rinkworks.com/games/rps.shtml.

Player	Tally	Total Points	% Wins
A			
B			

2. After completing 20 rounds of the game, work as a group to calculate the percentage of wins for each player. Use these results to discuss whether each player has an equally likely chance of winning a game of Rock, Paper, Scissors.
3. Draw a tree diagram showing every possible outcome for the game.

4. Based on your tree diagram, write fractional, decimal, and percent values representing the theoretical probability each player has of winning the game. Do your experimental results match your theoretical results?
5. Take a class average of the experimental results. Are the averaged experimental results a closer match to the theoretical results? Why or why not?
6. Based on all your results, is this a fair game? Justify your answer.

Activity 2:

1. Continue to work in your groups. This time, however, all three group members will play 20 rounds of Rock, Paper, Scissors according to these rules:

Player A gets one point if all signs are the same.

Player B gets one point if two signs are the same.

Player C gets one point if all of the signs are different.

Record your results in the table below.

Player	Tally	Total Points	% Wins
A			
B			
C			

2. After completing 20 rounds of the game, calculate the percentage of wins for each player. Use these results to discuss whether each player has an equally likely chance of winning this version of Rock, Paper, Scissors.
3. Draw a tree diagram showing every possible outcome for the game.
4. Based on your tree diagram, write fractional, decimal, and percent values representing the theoretical probability each player has of winning the game. Do your experimental results match your theoretical results?
5. Take a class average of the experimental results. Are the averaged experimental results a closer match to the theoretical results? Why or why not?
6. Based on all your results, is this a fair game? Justify your answer.

Discussion/Further Questions/Extensions

Activity 2 is an example of an unfair game because the probabilities that each player will earn a point for each round of the game are not equal. Ask students how the game's point system could be changed to make this a fair game. One possible way to do this would be to assign varying point values for each player depending on the probabilities calculated from the tree diagram. For instance, students may choose to award Player A 6 points for each win, Player B 1 point, and Player C 3 points. Ask students to make a mathematically convincing argument explaining why their new game is fair.

Sample Solution

Activity 1:

1. Work in groups of three to play 20 rounds of the original Rock, Paper, Scissors game. There should be two players (A and B) and one recorder who records game wins in the table below.

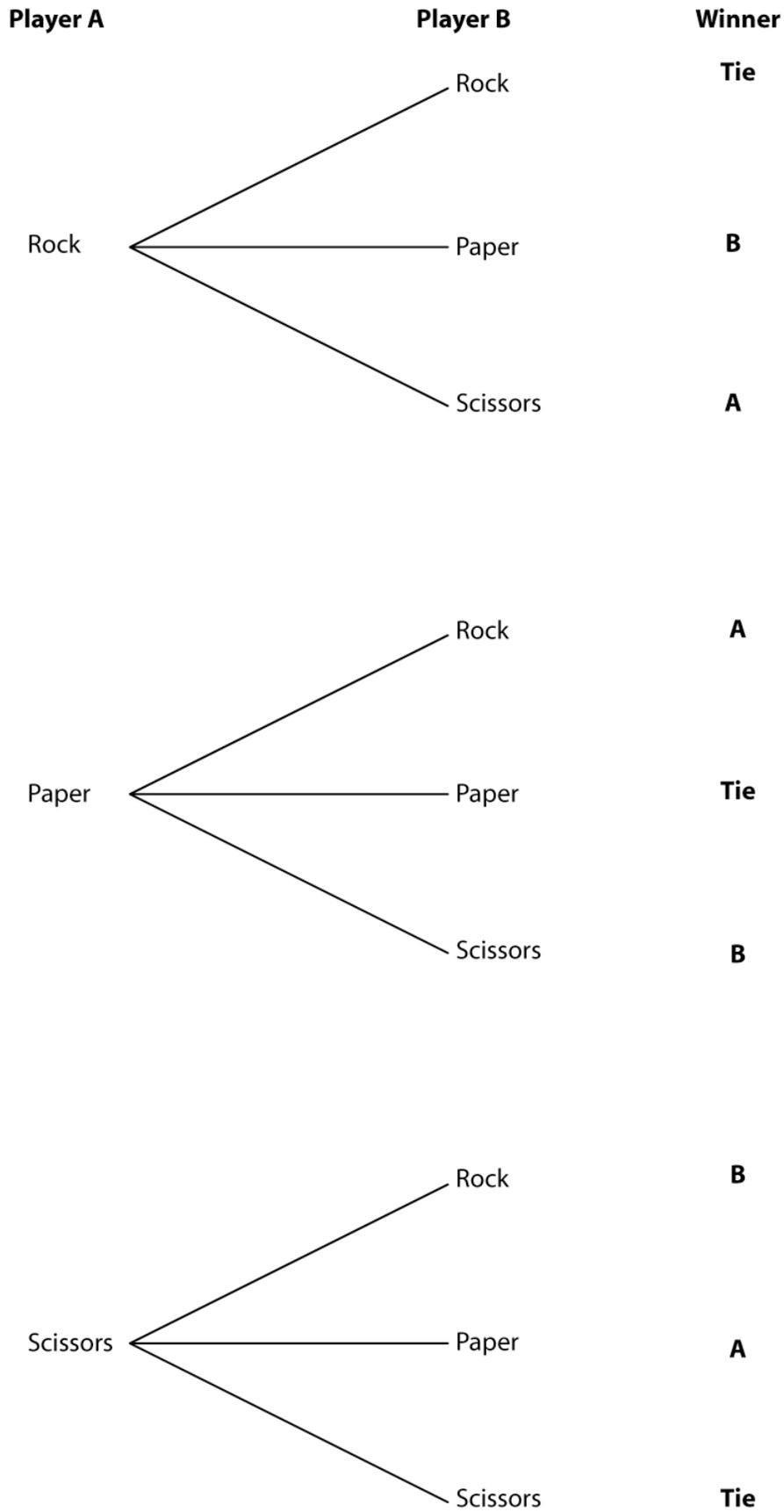
Answers will vary depending on each group's results.

2. After completing 20 rounds of the game, work as a group to calculate the percentage of wins for each player. Use these results to discuss whether each player has an equally likely chance of winning a game of Rock, Paper, Scissors.

Answers will vary depending on each group's results.

3. Draw a tree diagram showing every possible outcome for the game.

Rock, Paper, Scissors



4. Based on your tree diagram, write fractional, decimal, and percent values representing the theoretical probability each player has of winning the game. Do your experimental results match your theoretical results?

Theoretical Probabilities: The tree diagram shows that there are 9 possible outcome; with 3 ways for Player A to win, 3 ways for Player B to win, and 3 ways to tie, resulting in the following probabilities:

$$P(A \text{ wins}) = \frac{3}{9} = \frac{1}{3} = .\bar{3} = 33\frac{1}{3}\%$$

$$P(B \text{ wins}) = \frac{3}{9} = \frac{1}{3} = .\bar{3} = 33\frac{1}{3}\%$$

$$P(A \text{ and } B \text{ tie}) = \frac{3}{9} = \frac{1}{3} = .\bar{3} = 33\frac{1}{3}\%$$

Student comparisons between the theoretical and experimental probabilities will vary.

5. Take a class average of the experimental results. Are the averaged experimental results a closer match to the theoretical results? Why or why not?

Answers will vary. Students might note or hypothesize that increasing the number of trials (rounds of the game) will result in the experimental probability moving increasingly toward the theoretical probability.

6. Based on all your results, is this a fair game? Justify your answer.

It is a fair game since each player has an equal chance of winning, losing, or tying.

Activity 2:

1. Continue to work in your groups. This time, however, all three group members will play 20 rounds of Rock, Paper, Scissors according to these rules:

Player A gets one point if all signs are the same.

Player B gets one point if two signs are the same.

Player C gets one point if all of the signs are different.

Record your results in the table below.

Answers will vary depending on each group's results.

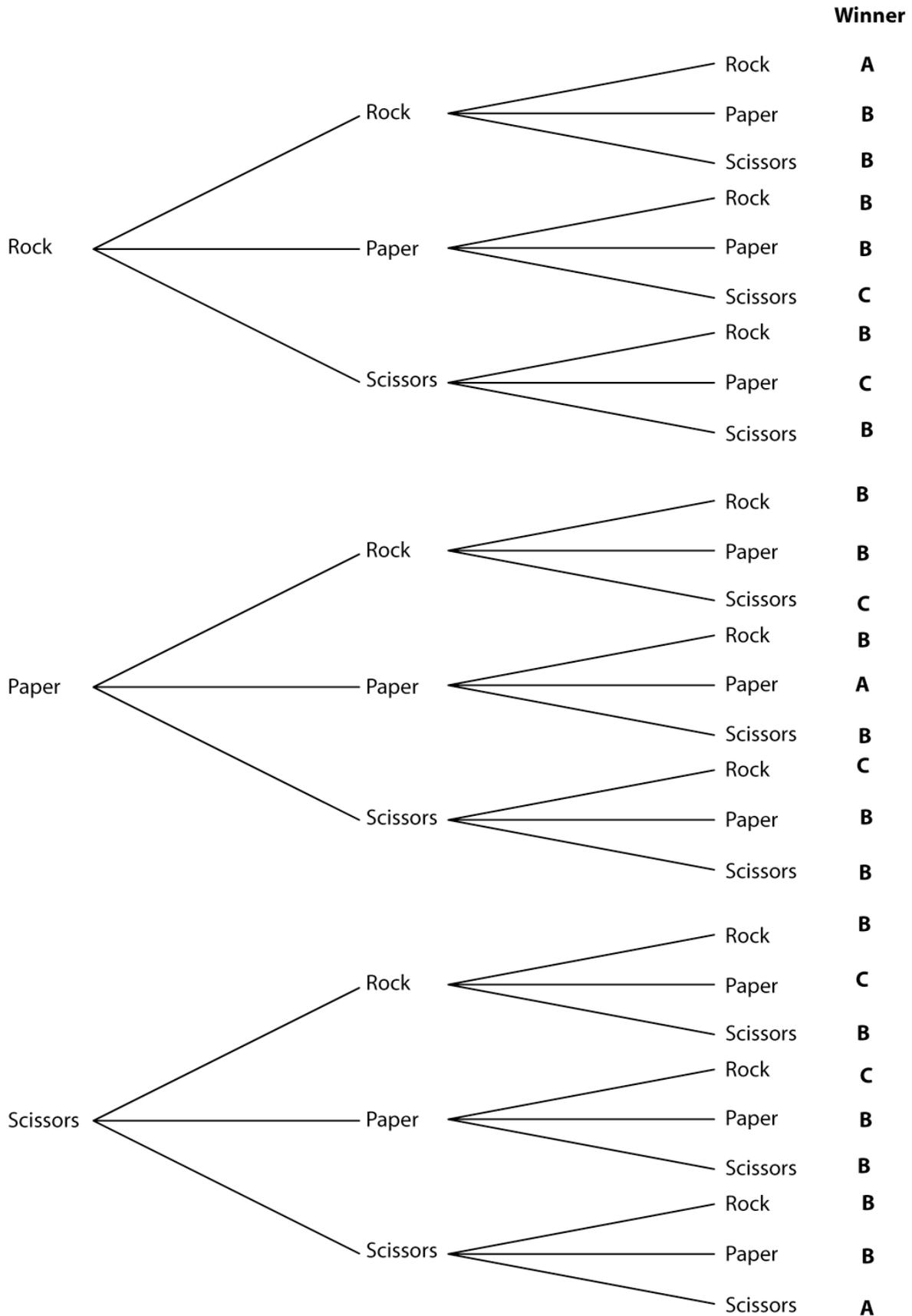
Rock, Paper, Scissors

2. After completing 20 rounds of the game, calculate the percentage of wins for each player. Use these results to discuss whether each player has an equally likely chance of winning this version of Rock, Paper, Scissors.

Answers will vary depending on each group's results.

3. Draw a tree diagram showing every possible outcome for the game.

Rock, Paper, Scissors



4. Based on your tree diagram, write fractional, decimal, and percent values representing the theoretical probability each player has of winning the game. Do your experimental results match your theoretical results?

From the tree diagram, there are 27 possible outcomes.

Player A wins if all 3 signs are the same (that is, all rocks, all scissors, or all paper), so A's chances of winning are $\frac{3}{27} = \frac{1}{9} = .\bar{1} = 11\frac{1}{9}\%$.

Player B wins if 2 are the same; B's chances of winning are $\frac{18}{27} = \frac{2}{3} = .\bar{6} = 66\frac{2}{3}\%$.

Player C wins if all are different; C's chances of winning are $\frac{6}{27} = \frac{2}{9} = .\bar{2} = 22\frac{2}{9}\%$.

Student comparisons between the theoretical and experimental probabilities will vary.

5. Take a class average of the experimental results. Are the averaged experimental results a closer match to the theoretical results? Why or why not?

Answers will vary. Students might note or hypothesize that increasing the number of trials (rounds of the game) will result in the experimental probability moving increasingly toward the theoretical probability.

6. Based on all your results, is this a fair game? Justify your answer.

This is not a fair game. Player B has a greater advantage since there are more chances of getting two signs that are the same.