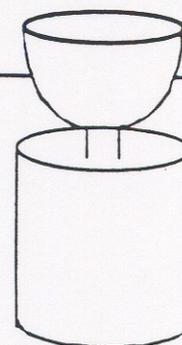# Slithering Solids

Institute Notes

Concept: Investigate using ratios to represent rates of change; use fractions and decimals to describe these rates of change as unit rates, and compare the unit rates to solve problems.

TEKS Focus: **6.3**—The student solves problems involving proportional relationships.

7.3—The student is expected to estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

8.3—The student identifies proportional relationships in problem situations and solves problems.

Also:
Grade 6
6B, 6B
Grade 7
7B, 7D
Grade 8
8B, 8D

Overview: Participants will collect data based on dry ingredients being emptied through a funnel into a cylindrical container. They will record the amount (in scoops) of dry ingredient, the time (in seconds) for the funnel to empty, and the resulting height (in cm) of the column of dry ingredient in the container. Three different sets of ratios will be formed from the data to describe three different rates (seconds per scoop, cm per scoop, and cm per second). The three sets of ratios will be graphed, and the results analyzed according to the properties of proportional relationships.

Materials: Clear cylinder
1" graph paper (optional)
Stop watches
Cm ruler
4 oz. or smaller scoops
Paper plate or paper towels
Funnel (store-bought or made by cutting top from plastic bottle)
Markers or Peel-and-stick dots
Graphing calculators (optional)
Dry ingredients (salt, rice)
5 x 5" cardboard

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Procedure:

1. Divide participants into groups of four.
2. Distribute experiment instructions and the materials. Pose the following discussion questions before beginning the experiment:

   How will ratios play a part in describing the results of this experiment?

   What do you expect the graph of the set of ordered pairs (number of scoops, number of seconds) to look like? Why?

   If two different groups have the same ingredient but different funnels, how do you expect the graphs to be different? Why?
3. Ask participants to follow the directions, work on the experiment, construct the three graphs, and answer the questions.
4. If large grid paper and peel-and-stick dots are available, have each group graph their results on the large paper and post the graphs. If different groups have different ingredients or different funnels, ask them to guess which group made which graph.

Debriefing:

Conclude the activity with a whole group discussion of the ratios that can be used to describe the rates represented in the data collected. Use the *Reason and Communicate* questions to have participants analyze the three graphs for properties of proportional relationships.

Describe how to change this experiment to affect the rates of change (answers vary.)

Describe what you would change in this experiment so that it represents a non-proportional relationship. *Start with some of the solid material already in the container.*

The following sample data is used to present a discussion of the results:

# of scoops	1	2	3	4	5	6	7	8
Time in seconds	0.41	0.84	1.63	1.91	2.07	2.63	3.07	3.57
Height in cm	0.5	1.1	1.5	2.1	2.6	3.1	3.7	3.42

Math Notes:

There are several functional relationships that can be explored in this experiment. Since the amount of the ingredient poured into the funnel determines the number of seconds, one can say that the number of seconds is a function of the number of scoops. Also, the height of the ingredients in the cylinder is dependent upon the number of scoops or the time which has elapsed. (If there was a constant supply of ingredient, it could also be said that the number of scoops that flowed through the funnel is dependent upon the time.)

Each set of points plotted on the three graphs will lie approximately on a line. In each set of ratios, the rates of change represented may not be exactly constant in the experiment, but they are very close. It is important to make a connection between the set of points, the line used to represent the set of points, and a rate of change that can be used to make predictions based on that set of data. By drawing a line through each point and the origin to represent what the data would look like with each constant rate of change, you can lead the participants to identify the one line that they think best represents the data (or lead them to the use of the graphing calculator's regression capabilities to find the line of best fit). The connection can then be made between the unit rate of change that the points on that line represent (e.g., the constant number of seconds per each scoop) and

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How can we use ratios to describe the results of the experiment? The ratios 0.41 sec/1 scoop, 0.84 sec/2 scoops, 1.63 sec/3 scoops, and so forth describe the relationship of number of seconds to number of scoops (for Graph 1). Similarly, ratios can be made from the data to describe the relationships in Graph 2 (number of centimeters to number of scoops) and Graph 3 (number of centimeters to number of seconds).

How can we compare the ratios, e.g., do 3 scoops flow through the funnel at the same rate (number of seconds per 1 scoop) as 2 scoops?
Using division to find decimal equivalents for each fraction, each of these ratios can be described with an equivalent unit rate: 0.41 sec/1 scoop. 0.84 sec/2 scoops = 0.41 sec/scoop, 1.63 sec/3 scoops is about 0.54 sec/scoop, and so forth.

What should Graph 1 look like if all the rates (of seconds per scoop) represented by our data were the same?
If all the ratios were equivalent, that would mean that the relationship between the time to drain and the number of scoops is a proportional relationship, and the points would form a straight line throught the origin. For example, we can find all the points that have the charateristics $\frac{y}{x}$ = 0.4 sec/scoop if we draw a line connecting (0,4) and (0,0).

What does Graph 1 actually look like?
…points scattered around, but near the line that represents y/x = 0.4 seconds per scoop

If you had to pick one point from our data to draw a line through the origin to represent the whole set of data, which point would you pick? Why? What would this line represent?
It would represent the set of points that all have the same unit rate as the data point you picked.

How can you find the line that best represents the set of points on Graph 1?
You can use the regression function on a graphing calculator. For example, for this data, Graph 1 is best represented by the line y = 0.423x + 0.104.

the slope of that line (the constant change in y for each unit change in 2). This single line through the origin, then, represents a proportional situation (e.g., the number of seconds is proportional to the number of scoops) in which the constant of proportionality, the unit rate, can be used in the form $y = kx$ to make predictions.

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Is the relationship of time to the number of scoops a proportional relationship? Why or why not?

Not exactly, because the data do not lie exactly on a straight line through the origin. However, the regression line (the line of best fit) goes pretty close to the origin, meaning that the data are fairly well represented by a proportional relationship described by the unit rate of 0.423 seconds per scoop.

How can you use the unit rate you decide upon to describe the relationship to make predictions?

For example, if you decide upon the unit rate of 0.4 seconds per scoop to describe a proportional relationship that is close to the data in Graph 1, then you can use the $y = kx$ representation of proportionality to determine that the number of seconds, y, is equal to the unit rate times the number of scoops, or $y = (0.4$ seconds per scoop) (15 scoops) = 6 seconds. The same questions should be asked about Graph 2 (cm per scoop) and Graph 3 (cm per second).

Extensions: Try the experiment again using odd-shaped bottles instead of cylinders and predict which graphs you think will be different from the cylinder graphs and how you think they will be different.

Develop your own experiment that models equivalent ratios describing a constant rate of change.
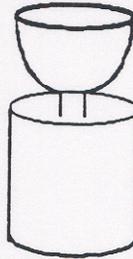
Assessment: Journal Entry—Describe other situations in which a ratio describing a rate of change would be an important component in an activity.

Notes:

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Activity 1

Set-up: A group member will need to be assigned to each of the following jobs: Measurer, Funnel Holder, Time Keeper, Recorder

Procedure:
1. Place the funnel over the cylinder with your finger, cardboard, or a ruler under the spout.
2. Measure 1 cup of the dry ingredient and pour it into the funnel.
3. Slip your finger or cardboard off the spout. Time the number of seconds from the time you release your finger until the funnel is emptied.
4. Shake the cylinder to level the dry ingredients in the cylinder. Measure the height of the cylinder from the table to the top of the column of dry ingredient.
5. Record the data in the table on the data page.
6. Empty the cylinder by pouring the dry ingredient into another container. You may use this cup of dry ingredients again as part of the next measurement using 2 cups.
7. Repeat the steps 1–6 for the other measurements shown in the table.
8. Graph the sets of ordered pairs described on Graph 1, Graph 2, and Graph 3 using the graph grid or a graphing calculator.

Data

Number of Cups	Time to Nearest Hundredth of a Second	Height to Nearest Tenth of a Centimeter
1		
2		
3		
4		
5		
6		
7		
8		
9		

Activity-5

Reason and Communicate:
How can we use ratios to describe the results of this experiment?

What do you expect each graph of the sets of ordered pairs (e.g., number of cups, time in seconds) to look like? Why?

If two different groups have the same ingredient but different funnels, how do you expect the graphs to be different? Why?

If two groups have the same funnels, but different ingredients, how do you expect the graphs to be different? Why?

How can we compare the ratios?

What should the graph look like if all the ratios (rates) represented by our data were the same?

What does the graph actually look like?

If you had to pick one point from our data to draw a line through the origin to represent the whole set of data, which one would you pick? Why? What would this line represent?
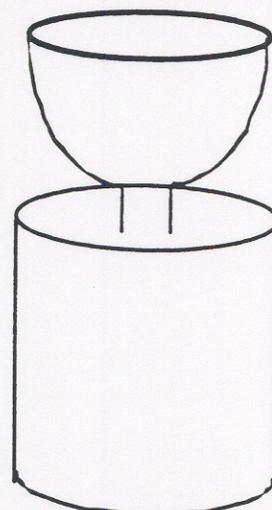
How can you find the line that best represents the set of points on the graph?

Is the relationship a proportional one? Why or why not?

How can you use the unit rate you decide upon to describe the relationship to make predictions?

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Activity 1

Set-up: A group member will need to be assigned to each of the following jobs: Measurer, Funnel Holder, Time Keeper, Recorder

Procedure:
1. Place the funnel over the cylinder with your finger, cardboard, or a ruler under the spout.
2. Measure 1 scoop of the dry ingredient and pour it into the funnel.
3. Slip your finger or cardboard off the spout. Time the number of seconds from the time you release your finger until the funnel is emptied.
4. Shake the cylinder to level the dry ingredient in the cylinder. Measure the height of the cylinder from the table to the top of the column of dry ingredient.
5. Record the data in the table on the data page.
6. Empty the cylinder by pouring the dry ingredient into another container. You may use this scoop of dry ingredient again as part of the next measurement using 2 cups.
7. Repeat the steps 1-6 for the other measurements shown in the table.
8. Graph the sets of ordered pairs described on Graph 1, Graph 2, and Graph 3 using the graph grid or a graphing calculator.

Data

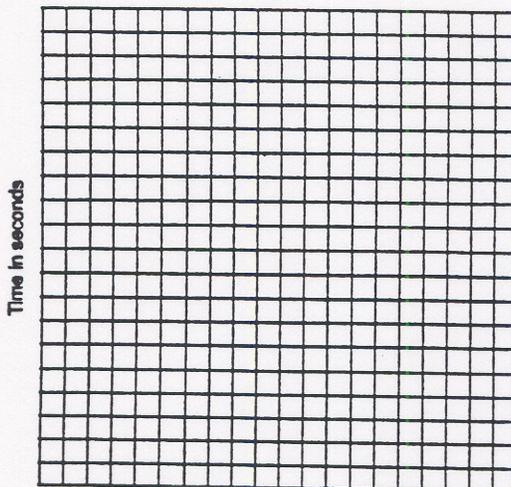| Number of Scoops	Time to Nearest Hundredth of a Second	Height to Nearest Tenth of a Centimeter
1		
2		
3		
4		
5		
6		
7		
8		
9		

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Activity 2

Make a graph of the ordered pairs (number of scoops, time in seconds) using 1"
graph paper or the grid below.

Graph 1

[Grid graph with axis labels: vertical axis "Time in seconds", horizontal axis "Number of scoops"]

a. Use ratios to describe the relationship between the number of scoops and the
    time in seconds to empty the funnel.
b. Use a ratio to describe the rate in scoops per second at which the dry ingredi-
    ent is being emptied from the funnel.
c. Use a unit rate to predict how long (in seconds) it would take for 15 scoops to
    be emptied from the funnel.
d. Suppose the spout of the funnel were smaller in diameter. What effect would
    you expect this to have on the graph? Explain.

Reason and Communicate:
How can we compare the ratios?

What should the graph look like if all
the ratios (rates) represented by our
data were the same?

What does the graph actually look
like?

If you had to pick one point from our
data to draw a line through the origin
to represent the whole set of data,
which one would you pick? Why?
What would this line represent?

How can you find the line that best
represents the set of points on the
graph?

Is the relationship a proportional one?
Why or why not?

How can you use the unit rate you
decide upon to describe the relation-
ship to make predictions?

If you had poured the ingredient into
a container which was in the shape of
a cone, would this graph be any dif-
ferent? How? Why?

Answers:

a. Answers will vary. But, all should compare $y = \frac{\text{time in}}{x}$

seconds : number of scoops or $x = \frac{\text{number of scoops : time}}{y}$

in seconds.

b. This ratio is a unit rate and is written $\frac{\text{number of scoops}}{1 \text{ second}}$.

c. (unit rate) $\frac{\text{time in 1 second}}{\text{number of scoops}} \cdot 15$ scoops = time in seconds for 15
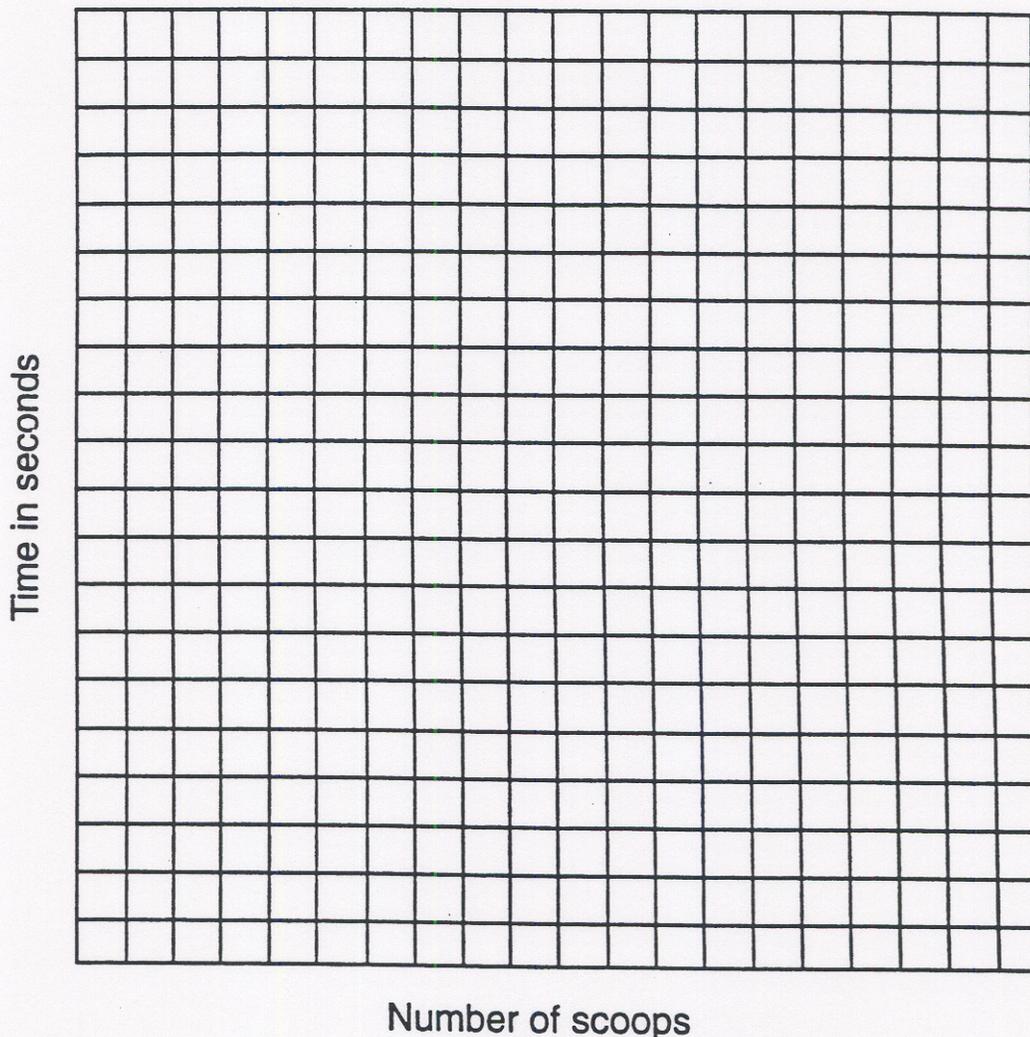
scoops to be emptied from the funnel

d. If the spout of the funnel were smaller in diameter, it would take more
time for the funnel to empty. This would make the graph steeper.

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Activity 2

Make a graph of the ordered pairs (number of scoops, time in seconds) using 1" graph paper or the grid below.

Graph 1

Time in seconds

Number of scoops

a. Use ratios to describe the relationship between the number of scoops and the time in seconds to empty the funnel.

b. Use a ratio to describe the rate in scoops per second at which the dry ingredient is being emptied from the funnel.

c. Use a unit rate to predict how long (in seconds) it would take for 15 scoops to be emptied from the funnel.

d. Suppose the spout of the funnel were smaller in diameter. What effect would you expect this to have on the graph? Explain.
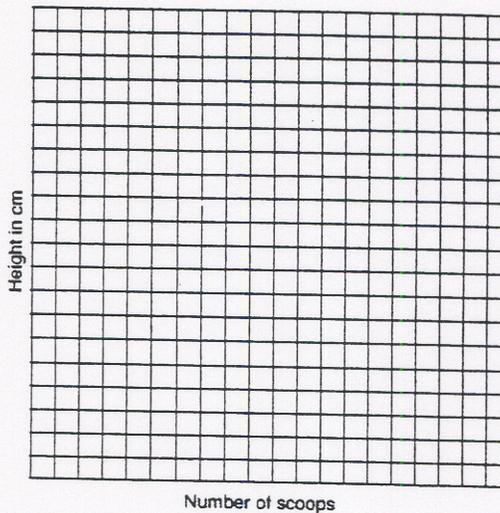
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Activity 3

Make a graph of the ordered pairs (number of scoops, height in centimeters) using 1" graph paper or the grid below.

Graph 2

Height in cm

Number of scoops

a. Use ratios to describe the relationship between the number of scoops and the height in centimeters.
b. Use a ratio to describe the rate in cm per scoop at which the height is changing.
c. Use the unit rate you chose in part "b" to predict how high the column of ingredients would be for 15 scoops.
d. Suppose the opening of the funnel were made twice as large as the original funnel. What effect would you expect this to have on the graph? Explain.

TEXTEAMS Rethinking Middle School Mathematics: Proportionality   Activity-111

Reason and Communicate:
How can we compare the ratios?

What should the graph look like if all the ratios (rates) represented by our data were the same?

What does the graph actually look like?

If you had to pick one point from our data to draw a line through the origin to represent the whole set of data, which one would you pick? Why? What would this line represent?

How can you find the line that best represents the set of points on the graph?

Is the relationship a proportional one? Why or why not?

How can you use the unit rate you decide upon to describe the relationship to make predictions?

If you had poured the ingredient into a container which was in the shape of a cone, would this graph be any different? How? Why?

If you had used a funnel with a larger diameter, would this graph be different? How? Why?

Answers:

a. Answers will vary. All ratios, however, should compare $\frac{y}{x}$ or

height in cm : number of scoops.

b. This ratio is a unit rate and is written as $\frac{\text{number of cm}}{1 \text{ scoop}}$.

c. $\frac{\text{number of cm}}{1 \text{ scoop}}$ • $\frac{15 \text{ scoops}}{}$ = number of cm in the height of the column of ingredient
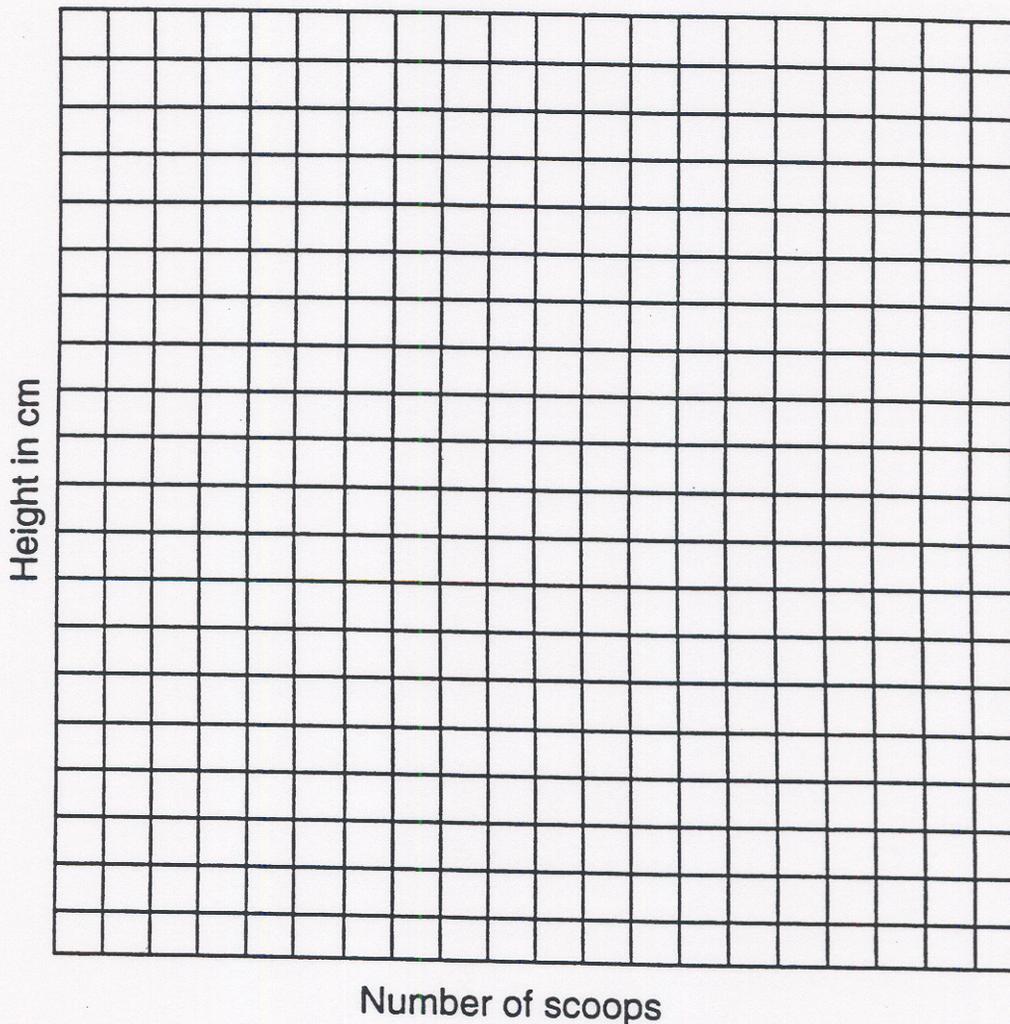
d. The ingredient would empty faster from the funnel but the height per scoop of ingredient would not be affected. The graph would not be changed.

Slithering Solids

Activity 3

Make a graph of the ordered pairs (number of scoops, height in centimeters) using 1" graph paper or the grid below.

Graph 2

[Graph with vertical axis labeled "Height in cm" and horizontal axis labeled "Number of scoops"]

a. Use ratios to describe the relationship between the number of scoops and the height in centimeters.

b. Use a ratio to describe the rate in cm per scoop at which the height is changing.

c. Use the unit rate you chose in part "b" to predict how high the column of ingredients would be for 15 scoops.

d. Suppose the opening of the funnel were made twice as large as the original funnel. What effect would you expect this to have on the graph? Explain.
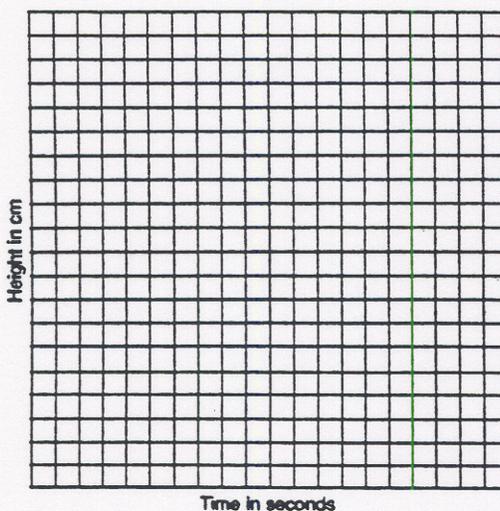
Slithering Solids

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Activity 4

Make a graph of the ordered pairs (time in seconds, height in centimeters) using 1" graph paper or the grid below.

Graph 3

Height in cm

Time in seconds

a) Use ratios to describe the relationship between the time in seconds and the height in centimeters.
b) Use a ratio to describe the rate in cm per second at which the height is changing.
c) Use the unit rate you chose in part "b" to predict how high the column of ingredients would be after 15 seconds (assuming a constant supply of ingredients).
d) Suppose the opening of the funnel was twice as large as the original funnel. What effect would this have on the graph? Explain.

TEXTEAMS Rethinking Middle School Mathematics: Proportionality   Activity-12

Reason and Communicate:
How can we compare the ratios?

What should the graph look like if all the ratios (rates) represented by our data were the same?

What does the graph actually look like?

If you had to pick one point from our data to draw a line through the origin to represent the whole set of data, which one would you pick? Why? What would this line represent?

How can you find the line that best represents the set of points on the graph?

Is the relationship a proportional one? Why or why not?

How can you use the unit rate you decide upon to describe the relationship to make predictions?

Answers:

a. Ratios will vary and will be of the form $\frac{y}{x}$ or $\frac{x}{y}$.

b. This is a unit rate and is written $\frac{\text{number of cm}}{1 \text{ second}}$.

c. $\frac{\text{number of cm}}{1 \text{ second}} \cdot 15$ scoops = number of cm in the height of the column of ingredient
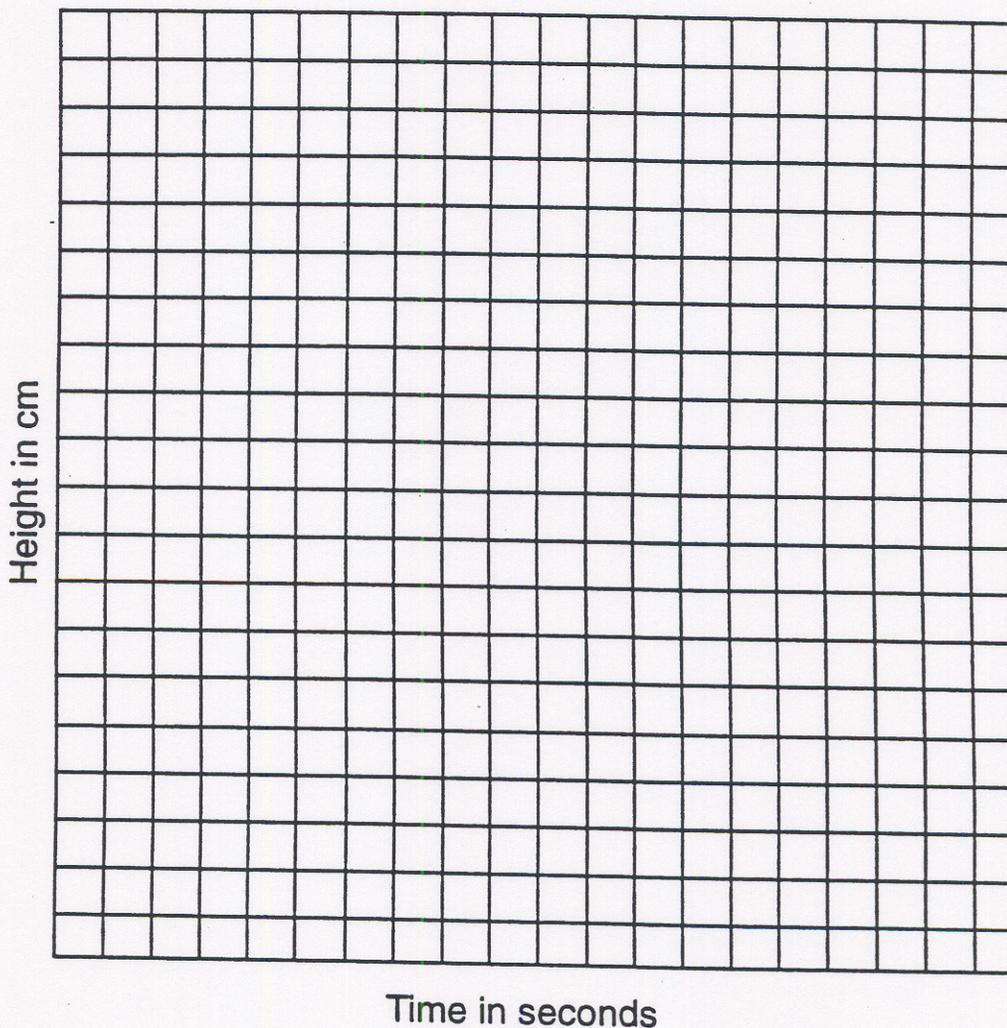
d. The ingredient would flow faster from the funnel, e.g., it would take more time for the height to rise. The graph would be steeper than the original.

Slithering Solids

Activity 4

Make a graph of the ordered pairs (time in seconds, height in centimeters) using 1" graph paper or the grid below.

Graph 3

Height in cm

Time in seconds

a.   Use ratios to describe the relationship between the time in seconds and the height in centimeters.

b.   Use a ratio to describe the rate in cm per second at which the height is changing.

c.   Use the unit rate you chose in part "b" to predict how high the column of ingredients would be after 15 seconds (assuming a constant supply of ingredients).

d.   Suppose the opening of the funnel was twice as large as the original funnel. What effect would this have on the graph? Explain.