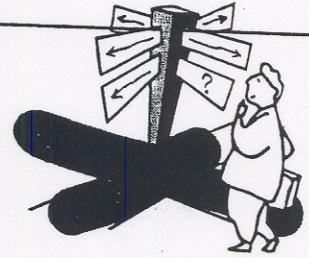# What's the Point?

Institute Notes

Concept: Answer the question, "What is a rational number?" by exploring the relationship between rational numbers and division. Look for patterns in and make connections between the different forms of quotients (integer quotient and remainder, fraction, and decimal).

TEKS Focus: **6.1B**—The student is expected to generate equivalent forms of rational numbers including whole numbers, fractions, and decimals.

7.1B—The student is expected to convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator.

8.1B—The student is expected to select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships.

Overview: Participants will collect data by dividing whole numbers and representing each quotient in three forms: integer quotient and remainder, fraction, and decimal. Participants will then organize the quotients to look for patterns within and between the different forms. From the patterns they will make generalizations and conjectures to present to the whole group to discuss.

Materials: Blank transparencies, Transparency pens, Easel or butcher paper, Markers, Colored counters, Calculators with fraction capabilities (e.g. TI-15)

Procedure: 1. Present the following scenario:
"The students in Miss Faith's math class are divided into groups. Each group is able to earn points through a variety of activities. At the end of a certain time period, the points are divided evenly between the members of the group. What are the advantages and disadvantages of recording this division in integer quotient and remainder form? In fraction form? In decimal form?"

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2. Have participants explore this question by considering different pairs of dividends and divisors. For example, if a group of 5 students earned 12 points, each student could get 2 points with 2 points left undistributed, $2\frac{2}{5}$ points, or 2.4 points. Use concrete materials to model the pictures first. Then draw a picture of each model. Record this on the Transparency as shown below. Have participants draw the pictures that they used.

Dividend ÷ Divisor	Integer w/R	Fraction	Decimal
12 ÷ 5	2 R2	$\frac{12}{5}=2\frac{2}{5}$	2.4

3. Ask, "What if the group of 5 participants earned 11 points instead? How would the quotients change? What if they earned only 2 points? What would the data in the table be?" Fill in the Transparency with the data from these examples. Discuss how organizing data makes it easier to find a pattern.

4. Hand out the Activity sheet. Participants will explore division by keeping the divisor (number of people) constant and increasing the dividend (number of points to be shared) by one each time. For example, participants using 5 as a divisor should record in their tables first 0 ÷ 5, then 1 ÷ 5, 2 ÷ 5, 3 ÷ 5, and so on. Assign each group (or let each group choose) a divisor with which to explore (e.g. 3, 4, 7, 9, etc.). A suggested grouping is to give them two divisors such as 2 and 9, 3 and 8, 4 and 7, and 5 and 6. Encourage them to use calculators when necessary to create a large set of data quickly.

5. After each group has explored with one or more divisors, have them record their observations of patterns or conjectures on the Activity sheet. Also, give groups butcher or easel paper on which to draw. Encourage them to use the calculator to validate the continuation of their patterns or to test their conjectures with larger dividends or other divisors.

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6. Have each group record its patterns and conjectures on easel or butcher paper and post them on the wall. Groups can then take a gallery tour to read and discuss the conjectures of the other groups.

7. Have each group of participants choose one or more of their patterns or conjectures to record on an overhead transparency to share with the whole group for discussion.

8. Give groups time to try one another's patterns on their own set of dividends and divisors to see which conjectures can be generally applied and which are specific to certain divisors.

Extensions:

1. Have participants form a conjecture about which pairs of dividends and divisors give terminating decimal representations and which do not. (If the simplified form of the fraction has a denominator with prime factors of only 2 or 5, then it will have a terminating decimal representation.)

2. Have participants try to find a combination of whole number dividends and divisors, other than 0, that do not produce a decimal quotient that is either terminating or repeating. (This is not possible since every pair of whole numbers in which the divisor is not 0 gives a rational number quotient. A decimal that is non-terminating and non-repeating cannot be described by a quotient of two whole numbers.)

Assessment: Use the following prompts for journal entries:

1. What pattern did you find most interesting in the different forms of the quotients?

2. Describe a relationship between the integer quotient with remainder, the fraction, and the decimal forms of $a \div b$.

3. What is a rational number?

4. How might exploring these patterns in different forms of quotients help participants and their students in later work with fractions and decimals?

Notes:

What's the Point?
Transparency

The students in Miss Faith's math class are divided into groups. Each group is able to earn points through a variety of activities. At the end of a certain time period, the points are divided evenly between the members of the group. What are the advantages and disadvantages of recording this division in integer quotient and remainder form? In fraction form? In decimal form?

What's the Point?

Dividend	Divisor	Picture (quotient with remainder)	Integer Quotient with Remainder	Picture (fraction quotient)	Fraction Quotient
12	5		2 R2		$\frac{12}{5}$ or $2\frac{2}{5}$
11	5				
2	5				

Reason and Communicate:

- In the problem about sharing points, what does the dividend represent? *The dividend represents the number of points.*

- What does the numerator of the fraction represent in the division problem? *The dividend*

- In the problem about sharing points, what does the divisor represent? *The divisor represents the number of students sharing the points.*

- What does the denominator of the fraction represent in the division problem? *The divisor*

- In the problem about sharing points, what does the integer quotient represent? *The number of whole points each person gets*

- What does the remainder represent? *The number of whole points left over that no one gets*

- What does the fraction quotient represent? *The amount, in wholes and parts, of points that each person gets with no points left over*

- What does the decimal quotient represent? *It represents the same thing as the fraction quotient; however, the division indicated by the fraction bar has been performed. If the division is done on a calculator, the whole decimal quotient may not appear on the display or may be made up of an infinite set of digits.*

Math Notes:

Terminating decimals come about from divisors that are powers of 2, powers of 5, or multiples of powers of 2 and powers of 5.

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Dividend	Divisor	Picture (quotient with remainder)	Integer Quotient with Remainder	Picture (fraction quotient)	Fraction Quotient	Decimal Quotient
12	5		2 R2		$\frac{12}{5}$ or $2\frac{2}{5}$	2.4
11	5					
2	5					

What's the Point?

Dividend	Divisor	Picture (quotient with remainder)	Integer Quotient with remainder	Picture (fraction quotient)	Fraction Quotient & D...
0					
1					
2					
3					
4					
5					
6					
7					
8					

Activity

Reason and Communicate:

· Is the decimal quotient shown on the calculator always equal to the fraction quotient? *No, sometimes the calculator cannot show the entire decimal quotient. It may be too long for the display or it may be a decimal that does not terminate.*

· How can you tell if the decimal quotient is not equal to the fraction quotient? *The patterns don't quite work. You can show with paper and pencil division that the decimal doesn't terminate.*

· How was using a table to explore the data helpful? *It displays the connections between the various representations (integer quotient and remainder, fraction, and decimal) and allows the patterns within each representation to emerge.*

Answers:
Answers will vary according to the divisor choosen.

Math Notes:
Encourage groups to try several different divisors with different properties (e.g. an even number, a multiple of 5, a prime number, etc.).
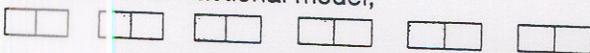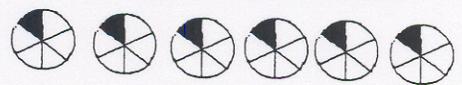
It is important for participants to organize the data in such a way that the dividends increase by 1 and the divisors remain the same in order to bring out the patterns.

The use of a calculator with fraction capabilities in this activity is helpful in that a lot of data can be created in a short amount of time and groups can experiment with a wide variety of dividends and divisors.

The pictures can be drawn in several ways. For the situation 3 points and 6 people, the integer division picture might look like:

* * *   0 points per person with 3 left over.

For the decimal or fractional model,

each person has $\frac{1}{2}$ point.

Math Notes, cont:
Another approach is to distribute one point amoung 6 people as follows:

Each person gets $\frac{1}{6}$ a point. Two points would look like this:
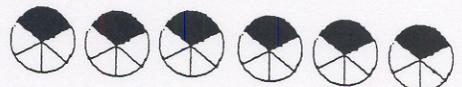
Each person gets $\frac{1}{3}$ a point. Three points would look like this:

Each person gets $\frac{1}{2}$ a point.

What's the Point?

Activity

Dividend	Divisor	Picture (quotient with remainder)	Integer Quotient with Remainder	Picture (fraction quotient)	Fraction Quotient	Decimal Quotient
0						
1						
2						
3						
4						
5						
6						
7						
8						

What's the Point?

We noticed that:

We think this happens because:

We would choose the _____ form for recording the division of the points between the team members because:

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Math Notes:
Encourage groups to find as many patterns as possible; point out their similarities and differences and how they are caused by the properties of rational numbers.

Reason and Communicate:
· What happens when you add 1/4 + 1/4 in decimal form? *You get 2/4=0.5 in decimal form.*
· What happens when you add 1/7 + 1/7 in decimal form? *You get 0.28571, which is not 2/7 in decimal form.*
· Why do you think that happens? *It happens because 0.1428571 is not exactly equal to 1/7. It is an approximation because 1/7 has a repeating decimal representation.*
· Select a pattern that some other group found with their divisor. Does the same pattern appear in the data for your group's divisor? Why or why not?
· What divisors have similar sets of patterns? *e.g. 2, 4, 8; 3 and 9.*
· Why do you think their patterns are similar? *Divisors whose prime factorization include powers of 2 and 5 only result in terminating decimals. All others will be repeating decimals.*

Answers:
Important patterns that groups should notice:
• The integer quotients remain the same while the remainder increases by one until the remainder is one less than the divisor. Then the quotient increases by one and the remainder starts again at zero.
• In the unsimplified fraction form of the quotient, the dividend is the numerator and the divisor is the denominator.
• The numerators in the unsimplified fraction quotients increase by one.
• In the unsimplified mixed number form, the numerator increases by one until it is one less than the divisor. Then the whole number increases by one and the remainder starts over at zero.
• There is a constant difference between consecutive decimal quotients; that constant difference is equal (or approximately equal, for non-terminating decimals) to $\frac{1}{n}$. For example, with a divisor of six, the difference between consecutive decimal quotients is about 0.16 which is approximately equal to the difference between consecutive fraction quotients, $\frac{1}{6}$.

Answers, cont:
• The remainder in the integer quotient matches the numerator in the mixed number form of the fraction quotient, before simplifying.
• With a divisor of 9, the pattern indicates that $\frac{9}{9}$ should be $0.\bar{9}$.

However, $\frac{9}{9}=1$. This is one way to show that $0.\bar{9}=1$. Other ways include multiplying both sides of $\frac{1}{3}=0.\bar{3}$ by 3. Then,

$1=3(\frac{1}{3})=3(0.\bar{3})=0.\bar{9}$. Also, try to find a decimal that is between $0.\bar{9}$ and 1. There isn't one; therefore, the two numbers are equal.

What's the Point?
Activity, cont.

We noticed that:

We think this happens because:

We would choose the _____ form for recording the division of
the points between the team members because: