2.2 Airlines

Overview: Participants graph real data on a scatter plot and find trend lines. They use transformations to explain aspects of the trend lines. Participants find a real application for horizontal shifts.

Objective:

Algebra II TEKS

(b.1) The student uses properties and attributes of functions and applies functions to problem situation.

(b.1.B) In solving problems, the student collects data and records results, organizes the data, makes scatter plots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

Precalculus TEKS

(c.3) The student uses functions and their properties to model and solve real-life problems.

(c.3.C) The student is expected to use properties of functions to analyze and solve problems and make predictions.

Terms: Scatter plot, trend line, independent variable, dependent variable

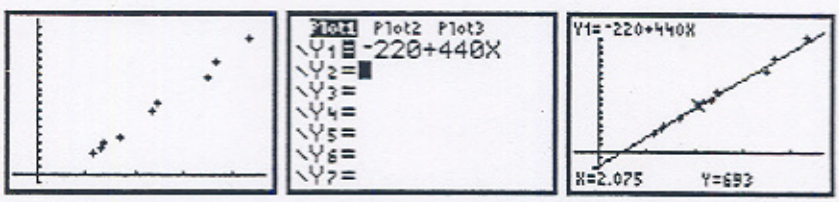
Materials: Graphing calculator

Procedures:

Activity 1: Trend Lines

Instruct participants to enter the data into their graphing calculators and create a scatter plot.

1. Refrain from using regression to find a trend line. Encourage participant to simply guess and check a trend line by estimating the y-intercept and the slope, graphing that estimated $y = b + mx$ and then adjusting the function rule until it is a reasonable trend line. This line is not a "line of best fit." It is a trend line.



2.

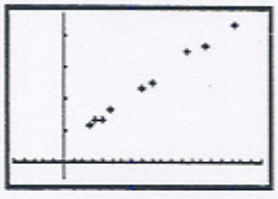
- What are the units of the slope? *Miles per hour.*
- What does the slope mean? *In our example above, the slope means that on average, the planes are flying about* 440 *miles per hour.*
- What is the unit for the y-intercept? *Miles.*

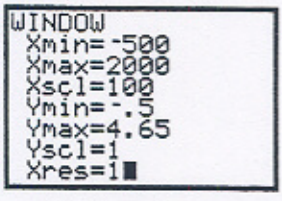
- Why is the y-intercept negative? *It does not make sense that at time = 0, the airplane has traveled a negative distance. But consider the x-intercept. This is the time after which the airplane starts to fly and gain distance. Therefore, the negative y-intercept allows for the time spent not gaining distance, i.e., time sitting on the tarmac, time spent circling in the air waiting for permission to land, etc.*

3. Using our trend line above, $y = -220 + 440x$, we write this as a transformation of $y = x$ as $y = 440(x - 0.5)$. This form shows that the time spent *not gaining any distance* is about a half of an hour. What was your wait time last time you flew?

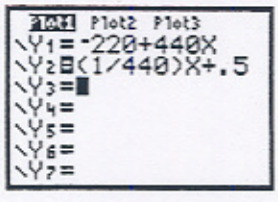
Activity 2: Another Trend Line

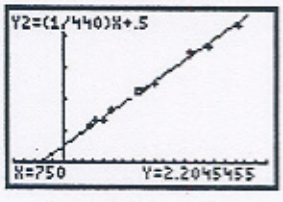
1. To set up the scatter plot of the inverse relation, simply reverse the x- and y-variables in the scatter plot set-up. Also exchange the x- and y- window values.





2. In our example, the trend line with distance as the independent variable is $y = \left(\frac{1}{440}\right)x + \frac{1}{2}$.





3. The two trend lines are inverses of each other.
   - Now what are the units of slope and the y-intercept? *The units for slope are hours per mile. The units for the y-intercept are hours.*
   - Which variable in the problem situation is dependent? Which is independent? Does it make more sense to say that the distance you fly depends on the time it takes to get there (Activity 1) or to say that the time it takes you to fly somewhere depends on the distance that place is from where you are (Activity 2)?
   - With distance as the independent variable, what does the y-intercept tell you? *The y-intercept of 0.5 hours means that to fly nowhere it will take you about one half of an hour. To fly anywhere, you will have to add in about a half of an hour for wait time.*

Summary: The big idea here is that we can use real data to see an application of the horizontal translation of a function. We also review the slope y-intercept form of a line by estimating trend lines and we review the notion of independence and dependence. It is an introduction to our next Activity, which is on inverse functions.

Activity 1: Trend Lines

The table below shows the United flying time and mileage from Chicago to indicated cities.

City	Time	Distance
St. Louis	1:10	258
Phoenix	3:41	1440
Salt Lake City	3:29	1249
Dallas	2:20	802
Los Angeles	4:20	1745
Denver	2:27	901
Minneapolis	1:20	334
Kansas City	1:21	403
Memphis	1:41	491

1. Make a scatter plot and find a trend line. Use time for the independent variable.

2. What is the slope? What is the y-intercept? What is the x-intercept? What does each mean?

3. Write the function as a transformation of the parent function $y = x$. What information do you note in this form?

Activity 2: Another Trend Line

Suppose you use distance as the independent variable in the United flight distance-time situation.

1. Set up the scatter plot so that distance is the independent variable.

2. Write the trend line with distance as the independent variable.

3. How does this trend line relate to the trend line in Activity 1?

4. What information do you note when the trend line is written with distance as the independent variable?

Reflect and Apply

Using the internet, find your own airline flight time and distance data. Find a trend line and interpret your findings.