3.1 Developing the Quadratic Model

Overview: Students can develop their understanding of quadratic functions with real-world examples. We develop this at first intuitively, moving from bouncing ball data and discovering that second differences are constant to functions and validating that idea.

Objective: **Mathematical Models with Applications TEKS:**
1B, 2A

Terms: Quadratic function, second differences, and function notation f(x)

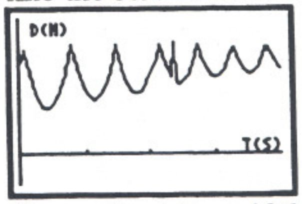
Materials: Graphing calculator, ball, motion detector, and data collection device

Procedures: Pair participants for these activities.

Activity 1: Ball Bounce

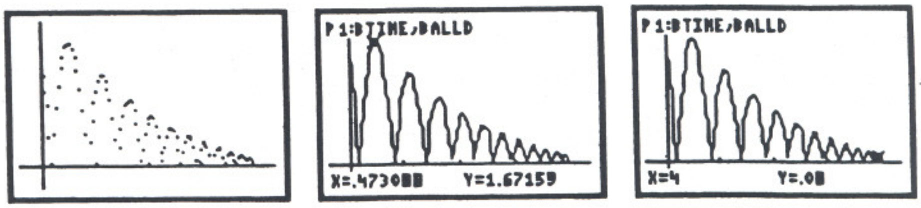
1. Bounce the ball a few times and ask participants to sketch a graph of time vs. ball height. Hold the motion detector about 0.5 meters above the ball. Run a program that will collect distance data for about 5-6 seconds. Drop the ball and collect the data. Follow the ball if it does not bounce straight up and down under the motion detector, but be sure to keep the motion detector at a constant distance from the floor. Repeat if necessary until you have at least five good bounces recorded. Display the data of time vs. ball height on the viewscreen calculator.

   Note that many programs, such as the one we used, are written so that the graph does not represent what really happened. Participants' guesses might more correctly look like the screen shot below:



   The program we used transforms the data as if the motion detector was on the floor with the ball bouncing on it. For this activity, a program like this makes the data easier to work with.

   Study your graph and ask the following questions. (The answers below are for the following sample.)



- What information can you get from the graph? [See the following questions.]
- How many times did the ball bounce? [12]
- In how many seconds did the ball bounce that many times? [4]
- About how high was the tallest bounce? The shortest? [1.67m, .1 m]
- How can we model one of the bounces? [with a quadratic]
- How can we tell if this is a reasonable model? [Let's explore differences. Do this using mental math for a few values on a blank transparency. Then set the mode on a graphing calculator to two decimal places and compute the rest of the differences. Look at the lists where your time and ball height data is stored. Take the data from one bounce, and use a list to find the first differences. Then use another list to find the second differences. An example follows:

TIME	BALLD	DIF1
.17	.03	.42
.22	.45	.35
.26	.80	.29
.30	1.09	.23
.34	1.33	.17
.39	1.50	.11
	1.62	.06

TIME(11) =.43008

BALLD	DIF1	DIF2
.03	.42	-.07
.45	.35	-.06
.80	.29	-.06
1.09	.23	-.06
1.33	.17	-.06
1.50	.11	-.06
1.62	.06	

DIF2(11)= -.0591

TIME	BALLD	DIF1
.47	1.67	-.00
.52	1.67	-.06
.56	1.60	-.12
.60	1.48	-.18
.65	1.30	-.24
.69	1.07	-.29
	.77	-.35

TIME(18) =.731136

BALLD	DIF1	DIF2
1.67	-.00	-.06
1.67	-.06	-.06
1.60	-.12	-.06
1.48	-.18	-.06
1.30	-.24	-.06
1.07	-.29	-.06
.77	-.35	

DIF2(18)= -.0652

- So if an equation is quadratic, what can we conclude about second differences? [Second differences are constant.]

In the second activity, we will verify this idea.

Activity 2: Second Differences

1. Ask the participants to use mental math to compute the table.
   - How can you tell it is not linear? [Take the first differences.]
   - How can you tell it is not exponential? [Take quotients.]
   - Take second differences.

0	$f(0) = 0^2 = 0$		
1	$f(1) = 1^2 = 1$	1	
2	$f(2) = 2^2 = 4$	3	2
3	$f(3) = 3^2 = 9$	5	2

2. Repeat the process from exercise 1.

x	$y = h(x) = 96x - 16x^2$	1st differences	2nd differences
0	$y = h(0) = 96(0) - 16(0)^2 = 0$		
1	$y = h(1) = 96(1) - 16(1)^2 = 80$	80	
2	$y = h(2) = 96(2) - 16(2)^2 = 128$	48	32
3	$y = h(3) = 96(3) - 16(3)^2 = 144$	16	32

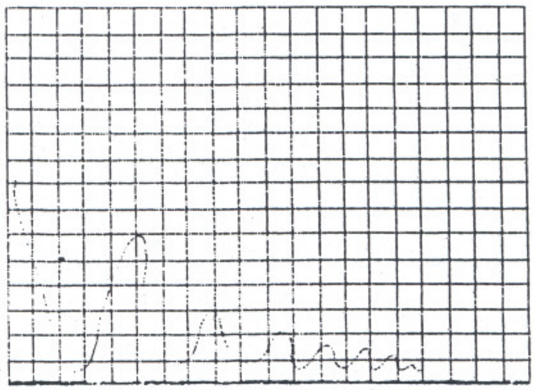
3. Ask participants to write their own quadratic function, build a table, and show that the second differences are constant. Have some participants share their work with the whole group.

Summary: Constant second differences indicate a quadratic relationship when the x-values are in an arithmetic sequence.

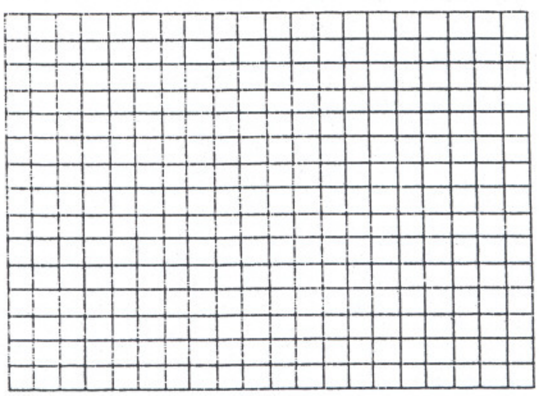
Activity 1: Ball Bounce

Use the graphs to sketch your predictions

1. Sketch your prediction for the graph of time vs. height. Label your axes including units.



2. Sketch a graph of the data collected from the ball bounce experiment. Label your axes including units.



CBL program reflects the graph

3. What information can you get from this graph?