Figurative Fun

Concept(s): Pattern building. Finding rules. Problem solving using patterns.

TEKS:

Geometric Patterns: The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures.

The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

Congruence and the geometry of size: The student analyzes properties and describes relationships in geometric figures.

Materials:

Transparencies of the three pattern charts (optional)
Optional: Geo-All (dir:sketches.file:figfun)

Notes:

Estimated time: 30 – 60 minutes

Encourage students to find several different ways to find the formulae.

Possible Answers:

Square Numbers

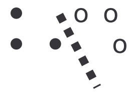
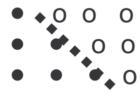
Term	1	2	3	4	58	100	n
Numerical Value of term	1	4	9	16	3364	10000	n^2
Process: Square area formula	● 1	● ● ● ● 2^2 or 1 + 3	● ● ● ● ● ●. ● ● ● 4^2 or 1 + 3 + 5				

Oblong Numbers

Term	1	2	3	4	58	100	n
Numerical Value of term	2	6	12	20	3422	10100	n(n+1) or $n^2 + n$
Process: Rectangle area formula	● ●	● ● ● ● ● ● 2 * 3 or $2^2 + 2$	● ● ● ● ● ● ● ● ● ● ● ● 3 * 4 or $3^2 + 3$				

Oblong number can be obtained by adding a square part to an extra column of size n: $n^2 + n$, or, by multiplying length x width: n(n + 1). Here is an opportunity to show the equivalence using the distributive property.

Triangle Numbers

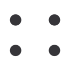
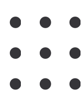
Term	1	2	3	4	58	100 -	n
Numerical Value of term	1	3	6	10	1711	5050	$\frac{n(n+1)}{2}$
Process: (Oblong formula) / 2							

Triangle numbers can be obtained by taking the equivalent oblong number (a rectangle) and dividing by 2. This fits the triangle area formula (1 x w)/2, or (b x h)/2.

The triangle pattern can be looked at another way:
1,[1+2], [1+2+3], [1+2+3+4],…,[1+2+3+…+(n-2)+(n-1)+n]
This brings to mind the Gauss method for finding the sum of a sequence of consecutive numbers:

Add the first term and last term: n + 1
Add the second term and second last term: 2 + (n-1) = n + 1
Add the third term and third last term: 3 + (n-2) = n + 1. Continue to pair the numbers up in this manner. You will have (n/2) pairs of terms, each sum being (n + 1). The total will be n(n + 1)/2. Again, we see the triangle formula, obtained a different way.

Square Numbers

Term Number	Visual Form	Written Description	Process Column	Numerical Value of Term
1		A square made of 1 dot.		
2		A square made of 4 dots.		
3				
4				
58				
100				
n				

Oblong Numbers

Term Number	Visual Form	Written Description	Process Column	Numerical Value of Term
1	• •	A rectangle made of 2 dots.		
2	• • • • • •	A rectangle made of 6 dots.		
3	• • • • • • • • • • • •			
4				
58				
100				
n				

Triangle Numbers

Term Number	Visual Form	Written Description	Process Column	Numerical Value of Term
1	•	A triangle made of 1 dot.		
2	• • •	A triangle made of 3 dots.		
3	• • • • • •			
4				
58				
100				
n				