3.2 Gravity

Overview: The height of one bounce of a ball over time can be modeled by transforming a quadratic parent function.

Objective: **Mathematical Models with Applications TEKS:**
1B, 3A, 3C, 8B

Terms: Quadratic function, parent function, transformation, and gravity

Materials: Graphing calculator, data collection device, motion detector, and bouncy ball

Procedures: Show the graph of the ball bounce data from the last activity. We know that we can fit the scatter plot with a quadratic since the second differences are constant. How can we find the function? We need to know about transformations to do that.

Activity 1: Transformations of $y = x^2$

1a. Graph $y = x^2$ in a friendly window. Then graph $y = x^2 + 2$. Trace on $y=x^2$ to a point (not (1,1)) and then jump to $y = x^2 + 2$. Point out that the difference between the y-coordinates of the two points is 2. The parent function was shifted up 2. Now add the graph $y = x^2 - 1$ and show that the difference between the y-coordinates is now -1. The parent function was shifted down 1.

b. Repeat the above process with $y = 2x^2$ and $y = 0.5x^2$. The parent function is vertically stretched by a factor of 2 and vertically shrunk by a factor of 0.5, respectfully.

c-d. Have participants complete these parts in pairs and share their work with the whole group.

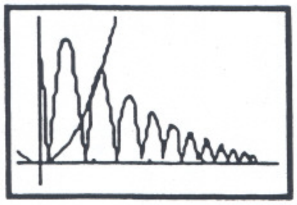
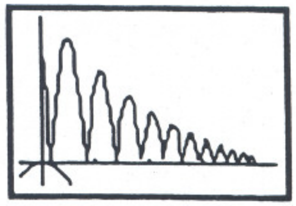
2. Do this activity as a group. Sketch $y = x^2$. Draw in the point (0,0). We will follow this point with each transformation.
   a. First shift the point (0,0) left 2, so it is now (-2,0). Draw and label this point. Sketch the new parabola. Now shift the point (-2,0) up 1, so it is now (-2,1). Draw and label this point. Sketch the resulting parabola.
   b. Have participants follow the procedure above to complete this part.
   c-f. Have participants complete these parts and present their work to the whole group.

Activity 2: Fit the Ball Bounce Data

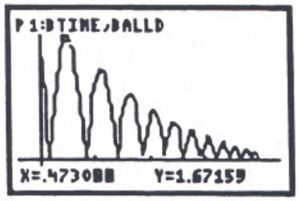
Give the data from the ball bounce you did earlier to the participants.

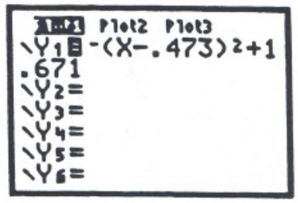
- How can we fit a quadratic to the first complete bounce? [A sample follows.]

Graph $y = x^2$ over the original graph. Reflect over the x-axis by graphing $y = -x^2$.

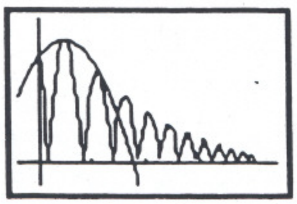
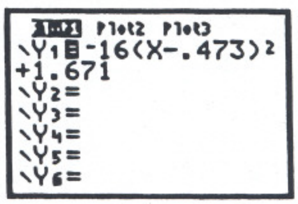



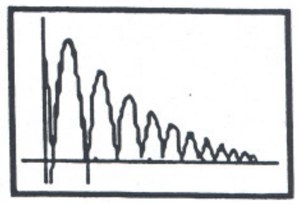
Trace to the vertex of the first complete bounce. Shift right 0.473 (the x-coordinate of the vertex). Shift up 1.671 (the y-coordinate of the vertex).





Now guess and check the stretch factor.



Does -16 have any particular significance? It is the force of gravity in the physics position equation $d = \frac{1}{2}at^2 + vt + d$, where a is the acceleration due to gravity, which is equal to -32 ft /sec^2 or -9.8 m/sec^2.

Now have participants find quadratics to model some of the other bounces. You may want to graph equations for all the bounces simultaneously if time permits.

Summary

Make it clear to participants that the graph is not a model of the path of the ball, only the vertical height over time. After you have a quadratic equation, you could graph the equation in parametric mode to show that it is just the vertical path of the ball.

Quadratic equations can be used to model the force of gravity on an object over time such as the ball in this activity. Transformations can be used to fit a parent function to data.

Activity 1: Transformations of $y = x^2$

1. Write a sentence describing each transformation on $y = x^2$.

   a. $y = x^2 + 2$ $\quad$ $y = x^2 - 1$

   b. $y = 2x^2$ $\quad$ $y = 0.5x^2$

   c. $y = -x^2$

   d. $y = (x + 2)^2$ $\quad$ $y = (x - 1)^2$

2. Using transformations (not memorized rules), sketch the following and then check with your graphing calculator.

   a. $y = (x + 2)^2 + 1$

   b. $y = 3(x - 2)^2$

   c. $y = -x^2 - 2$

   d. $y = \frac{1}{2}(x - 1)^2$

   e. $y = -(x - 1)^2 + 2$

   f. $y = -2(x + 3)^2 - 4$

3. Recall the position equation $h(x) = 96x - 16x^2$, where x is the time in seconds and h is the height of an object thrown straight up at 96 ft/sec. Find the vertex, and write it as a transformation of $y = x^2$. Expand to show the expression is equal to $96x - 16x^2$.