Algebra I Clarifying Lessons: A Draining Swimming Pool

OLD Resources. These resources have NOT yet been updated to align with the revised secondary mathematics TEKS. These revised TEKS were adopted by the Texas State Board of Education in 2005, with full implementation scheduled for 2006–07. These resources align with the original TEKS that were adopted in 1998 and should be used as a starting point only.

What is a Clarifying Lesson?

A model lesson teachers can implement in their classroom. Clarifying Lessons combine multiple TEKS statements and may use several Clarifying Activities in one lesson. Clarifying Lessons help to answer the question "What does a complete lesson look like that addresses a set of related TEKS statements, and how can these TEKS statements be connected to other parts of the TEKS?"

TEKS Addressed in This Lesson

Foundations for functions: b.1.E; b.4A, B

Materials

Graphing calculator

Lesson Overview

Students analyze and develop symbolic representations to describe the amount of water left in a pool being pumped out after a given amount of time, with various pumping rates and initial quantities of water in the pool.

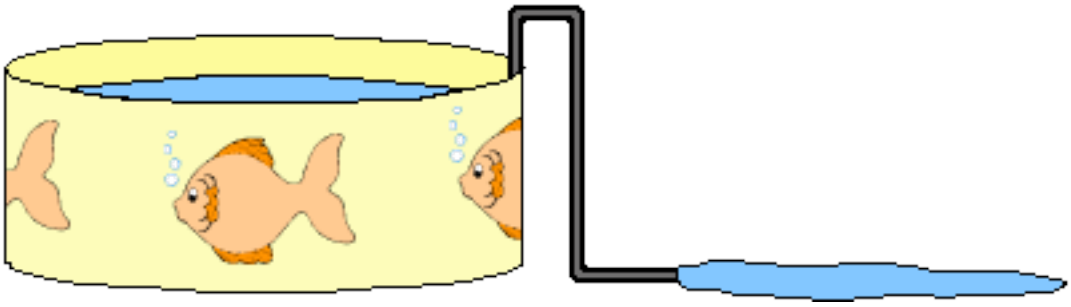
Mathematics Overview

Students interpret and make inferences from functional relationships while finding specific function values, simplifying polynomial expressions, transforming and solving equations, and factoring where necessary to solve given problems.

Set-up (to set the stage and motivate the students to participate)

1. Present the following situation to students:

   Suppose water is being pumped at a constant rate from a swimming pool.



2. Give students two representations of the amount of water (in liters) still in the pool in terms of the number, n, of hours that have passed since the pool was full.

   $f(n) = 4{,}300 - 215n$

   $f(n) = 215(20 - n)$

3. Have students analyze the given symbolic representations.

Teacher Notes (to personalize the lesson for your classroom)

Guiding Questions (to engage students in mathematical thinking during the lesson)

- How can you use the graphing calculator to show that these two formulas represent the same function? (b.1.E) (Draw the graphs of the two functions on the same grid and compare them.)
- How else can you show that they represent the same function? (b.4.B) (Using the distributive rule in (b) gives f(n) = 215(20) - 215n, the same as (a).)
- What are the units of the number 4,300? (b.1.E) (liters) The number 20? (hours) The number 215? (liters per hour)
- How much water was in the pool when it was full? (b.1.E) (The pool was full when n = 0.) Which formula shows this amount most clearly? (b.1.E) (Formula (a) shows directly that there were 4,300 liters in the pool when it was full.)
- How much water is in the pool after 1 hour of pumping (b.4.A)? (4,085 liters) after 2 hours of pumping? (b.4.A) (3,870 liters)
- What is the hourly rate of pumping? (b.1.E) (215 liters per hour) Which formula shows this rate most clearly? (b.1.E) (This rate is shown in both formulas (a) and (b).)

- How many hours will it take to empty the pool? (b.1.E) (The pool will be empty when f(n) = 0.) Which formula shows the answer to this question most clearly? (b.1.E, b.4.A) (Formula (b) shows clearly that this will happen when n = 20 hours.)

Teacher Notes (to personalize the lesson for your classroom)

Summary Questions (to direct students' attention to the key mathematics in the lesson)

- How would the formulas in (2) change if the amount of water in the pool when it was full was 3,225 liters and the pool was being emptied at the same pumping rate? (b.1.E) (In formula (a), 4,300 would change to 3,225; and in formula (b), 20 would change to 15.)
- How would the formulas in (2) change if the amount of water in the pool when it was full was 3,000 liters and it still takes 20 hours to pump the pool dry? (b.1.E) (In formula (a), 4,300 would change to 3,000. In formula (a) and formula (b), 215 would change to 3,000/20 = 150.)
- How would the formulas in (2) change if the rate of pumping water from the pool is 500 liters per hour? (b.1.E) (In formula (a), 215 would change to 500; and in formula (b), 215 would change to 500 and 20 would change to 26.)
- How would the formulas in (2) change if the pool started empty and was being filled at the same pumping rate? (b.1.E) (Formula (a) and formula (b) would both become f(n) = 215n. This formula is valid from n = 0, empty pool, to n = 20, full pool.)

Teacher Notes (to personalize the lesson for your classroom)

Assessment Task(s) (to identify the mathematics students have learned in the lesson)

Write corresponding formulas for a pool that has half the volume as the one in the original problem and is being pumped out at twice the rate. Use the formulas (and a graphing calculator, if appropriate) to determine how long it will take this pool to be pumped dry.

(Half the volume is 4,300/2 = 2,150 liters. Twice the rate is (2)(215) = 430 liters per hour. A formula for the water remaining in such a pool is g(n) = 2,150 - 430n. This formula corresponds to formula (a). Factoring out 430 leads to g(n) = 430(5 - n). This formula corresponds to formula (b). It shows that this pool will be pumped dry in 5 hours.)

Teacher Notes (to personalize the lesson for your classroom)