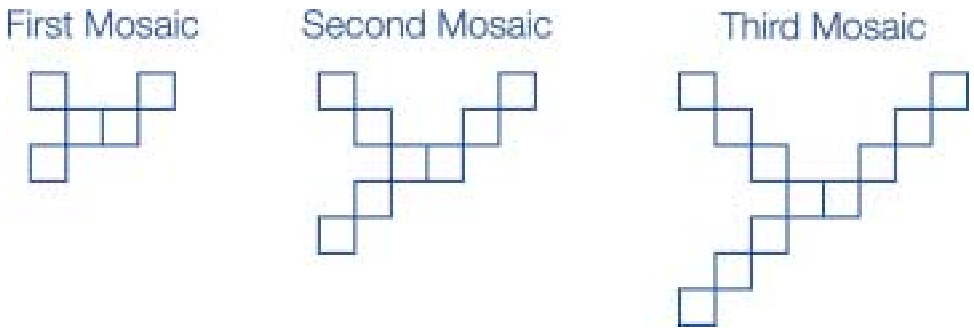
Chapter 1:

Function Fundamentals

Mosaics



Reuben created the first mosaic. He decided on a pattern and created the second and third mosaics.

1. Develop a rule to determine the number of tiles in the *n*th mosaic. Write a description of how your rule is related to the tile picture including a description of what is constant and what is changing as tiles are added.

2. How many tiles would be in the tenth mosaic?

3. Would there be a mosaic in his set that uses 57 tiles? Explain your reasoning.

Teacher Notes

Materials:
30 tiles per student (optional).
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- How many tiles would be in the next mosaic?
- What is remaining constant in the set of mosaics?
- What is the rate of change for tiles with respect to the mosaic number?

Sample Solution:

1. There are two tiles in the middle of the mosaic. For each mosaic there are three outside tiles added. The number of tiles, T, is two plus three times the mosaic number, n, or

   $T = 2 + 3n$

2. Evaluate the function for $n = 10$.

   $T = 2 + 3(10)$
   or 32 tiles

3. To ask when there are 57 tiles is to ask when is T equal to 57.

   $57 = 2 + 3n$
   $55 = 3n$
   $n = \frac{55}{3} = 18\frac{1}{3}$

   Since n must be a whole number, there would not be a mosaic with 57 tiles.

Extension Questions:

- How would the function rule have been changed if the middle of the mosaic contained four tiles?

The constant would be 4. The rule would be $T = 4 + 3n$.

- If the function rule had been $T = 2 + 4n$, describe the first two mosaics and the general rule.

There would be two tiles in the middle and one tile on each corner for the first mosaic. The second mosaic would have two tiles in the middle and two tiles at each of four corners. The general rule means that there are two tiles in the middle and four tiles added for each new mosaic.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns
  2.2 Identifying More Patterns

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Student Work

(1.) $2 + 3n = m$

2 in the equation represents the two blocks that are always there. The 3 shows the number of blocks connected to the other 2 blocks. The "n" tells you what mosaic number to put in. Then you would multiply 3 by the mosaic number. The "m" represents the total number of blocks in the mosaic. The constant is the two blocks that are always there. The "n" is changing. The total is also changing while the "n" changes.

mosaic number	2+3n	Total # of blocks
1	2+3(1)	5
2	2+3(2)	8
3	2+3(3)	11
4	2+3(4)	14
5	2+3(5)	17
6	2+3(6)	20
7	2+3(7)	23
8	2+3(8)	26
9	2+3(9)	29
10	2+3(10)	32

(2.)

$2 + 3(n) = m$

$2 + 3(10) = m$

$2 + 30 = m$

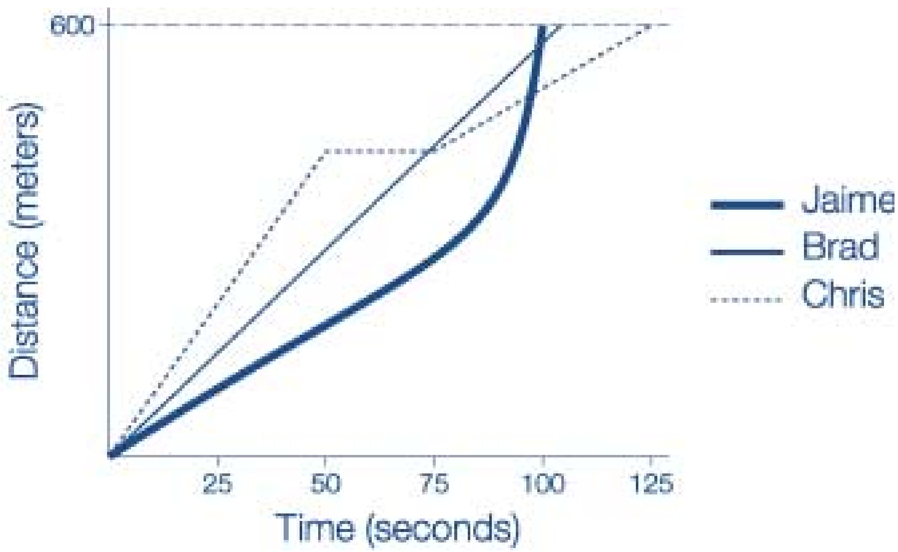
$32 = m$

There are 32 tiles in the tenth mosaic.

(3.) No, because there isn't a mosaic that has exactly 57 tiles. The 18th mosaic has 56 tiles and the 19th mosaic has 59 tiles.

The 600-Meter Race

The graph below describes what happens when three athletes, Jaime, Brad and Chris, enter a 600-meter race.



Give a detailed interpretation of each athlete's experience like sprints, slowdowns, and speed throughout the race, including estimates of time and distance.

Teacher Notes

Materials:
None required.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Can the race be broken up into phases? What would these phases be?
- What does the graph show you about each of the runners? Who leads when?
- How do you know? How do their speeds compare? How do you know?
- Where are the runners at the end of each phase?
- Who finishes first? Last? How do you know? What can you say about the runners' times?

Sample Solution:

Let t = the time ran in seconds and d = the distance ran in meters. The graph indicates the race has four phases; the estimated time intervals of these phases are [0,50], [50,75], [50,100], and [100,125].

From 0 to 50 seconds:
During the first phase, Chris is in the lead, followed by Brad and then Jaime, since Chris has the greatest distance values for this interval, Brad has the second greatest distance values, and Jaime has the least distance values for this interval. Chris and Brad are running at constant rates since they traveled farther in the 50-second time interval. Chris is running faster than Brad since his graph has the greater slope.

From 50 to 75 seconds:
During the second phase, Chris stops suddenly (sharp turn in graph) and remains still for about 25 seconds. His graph is horizontal, showing no change in distance. Brad gains the lead and travels faster than Jaime and surpasses Chris at about 75 seconds. Jaime remains behind the other two runners because his distance values are less than Brad and Chris for any given time in the interval.

From 75 seconds to about 100 seconds:
Chris starts to run again and he runs at a constant but slower rate because the linear graph shows less slope. Now Brad is in the lead, followed by Chris and then Jaime. Jaime is increasing his speed because the curve is increasing faster. At about 100 seconds, the graphs of Brad and Jaime intersect; Jaime catches up with Brad and wins the race.

From 100 seconds to about 125 seconds:
Brad finishes the race just after 100 seconds. Chris completes the race at just about 125 seconds.

Extension Questions:

- What type of function could represent the distance ran by each athlete as a function of time?

Jaime's graph might be modeled with a quadratic function. Brad's graph appears to be linear and increasing, except at the very end of the race. There it appears to curve upwards. Chris's graph would have to be defined in three pieces: first, linear and increasing, then constant, and, finally, linear and increasing with less slope than the first piece.

- For the functions described above, how do the mathematical and situation domains compare?

The mathematical domain for each function described would be the set of all real numbers. The situation domains are limited to a finite time interval starting with a time of zero seconds and ending with the time it takes the last athlete to complete the race.

- Describe the range for the problem situation.

The range for the situation would be between 0 and the total distance in the race, 600 meters.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations of Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns
3 Interpreting Graphs
3.1 Interpreting Distance versus Time Graphs

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

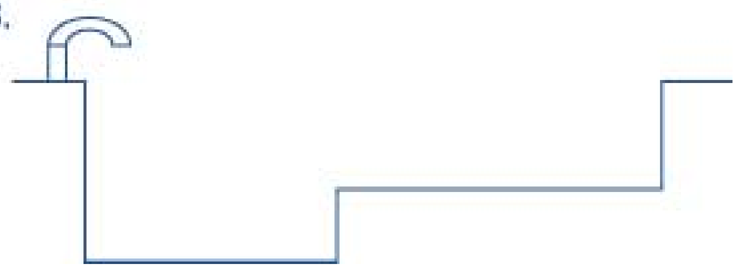
Objective 2:
The student will graph problems involving real-world and mathematical situations.

Swimming Pools

The cross sections of two swimming pools that are being filled at a constant rate are shown below.

A.

B.



1. For each pool, write a description of how the depth in meters, d, of the water in the pool varies with time in minutes, t, from the moment the empty pool begins to fill.

2. Sketch a graph to show how the depth of the water in each pool varies with time from the moment the empty pool begins to fill.

Teacher Notes

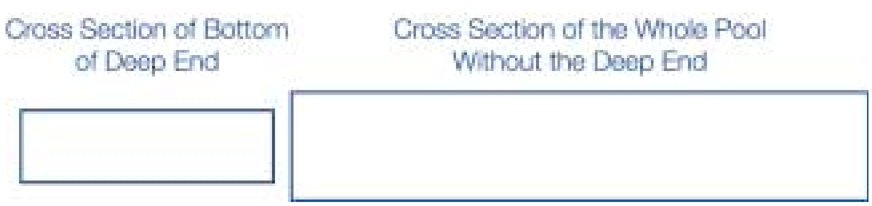
Scaffolding Questions:

- What section of the pool will be filled first?
- How are these sections different in each pool?
- How are the pools different from each other?
- How are the pools the same?
- What should the graph look like?
- How will the graphs be different?
- How will the graphs be the same?

Sample Solution:

1. For Pool A, the cross section of the pool is a rectangle. If water is flowing into the pool at a constant rate, the height of the pool will rise at a constant rate.

   For Pool B, the two sections to consider are the bottom of the deep end of the pool up to where the shallow end starts and then the rest of the pool. Both of these portions are rectangular prisms.



   The water depth will increase at a certain constant rate until the water level reaches the deep end edge of the shallow end of the pool. Then, as the water begins to fill the shallow end, the water depth increases at a slower but still constant rate until the pool is full. This is because the deep end of the pool is a prism with a smaller base than the base of the prism that is the whole pool.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

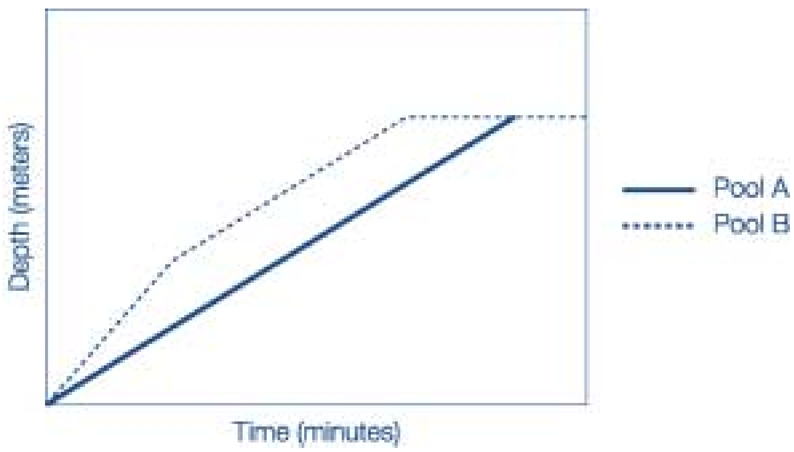
The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

2. Possible graphs:



Extension Questions:

- How are the graphs of the pools related?

If the length, depth, and width of the pools are the same and water is being poured into the pools at the same rate, when the deep section of the Pool B is full, the last portion of the graph will be parallel to the graph representing Pool A.

- Describe the portion of the graphs after the time when the pools are filled.

Since Pool B will hold less water, it will fill up sooner than Pool A. When a pool is full, the depth becomes constant and the graph is horizontal after the point in time when the pool is full. If the depths of the pools are the same, the graph representing Pool B will become horizontal before the graph of Pool A.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

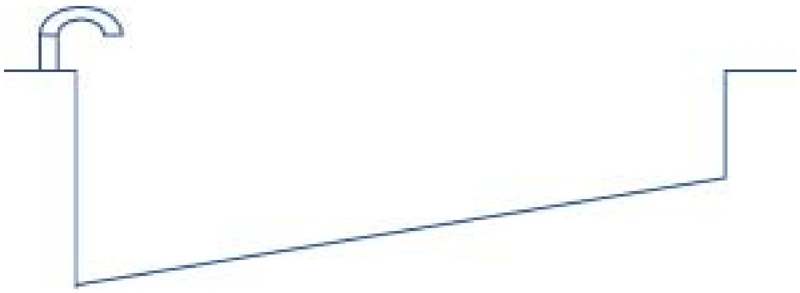
Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.1 Variables and Functions

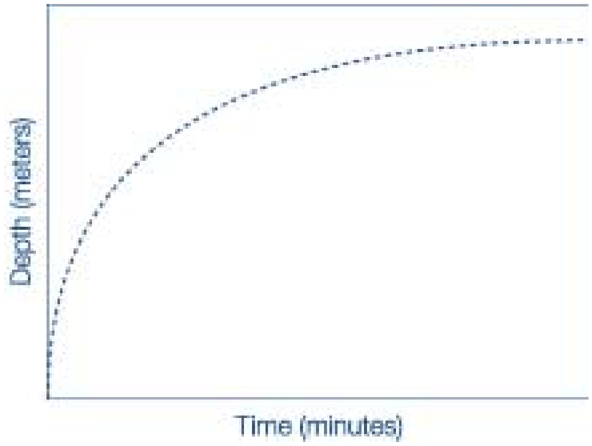
Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

- Describe the graph if the cross section of the swimming pool had been a trapezoid.



The graph would be a curve. The height on the deep end would rise quickly. As the deep end of the pool fills, the volume of water "widens." The height rises more slowly as the surface area of the water in the pool increases.



Student Work