Chapter 2:

Linear Functions, Equations, and Inequalities

CDs for the Band

Bryan and his band want to record and sell CDs. There will be an initial set-up fee of $250, and each CD will cost $5.50 to burn. The recording studio requires bands to make a minimum purchase of $850, which includes the set-up fee and cost of burning CDs.

1. Write a function relating the total cost and the number of CDs burned.

2. Write and solve an inequality to determine the minimum number of CDs the band can burn to meet the minimum purchase of $850.

3. If the initial set-up fee is reduced by 50%, will the total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Scaffolding Questions:

- What will the cost be if they purchase only one CD? Two CDs? Ten CDs?
- What are the constants for this situation?
- What are the variables?
- Describe in words the dependency relationship between the variables.
- What does the $850 represent in this situation?

Sample Solution:

1. The total cost of recording CDs is a $250 set-up fee plus $5.50 times the number of CDs you want to purchase.

   $C = 250 + 5.50n$ where C represents the total cost and n represents the number of CDs

2. The total cost must be less than or equal to $850.
   Use the rule and put the values in a table.

Number of CDs	Total Cost
1	$255.50
10	$305.00
100	$800.00
110	$855.00

You know from the table that 110 CDs cost $855.00, which was just a little over the minimum fee of $850. Next calculate the cost of 109 CDs using the rule and find that the total cost equals

$250.00 + $5.50(109) = $849.50

109 CDs cost less than $850. They must purchase at least 110 CDs.

Another approach is to use the inequality to solve the problem.

$$250 + 5.50x \geq 850$$
$$5.50x \geq 850 - 250$$
$$5.50x \geq 600$$
$$x \geq 109.09$$

CDs must be purchased in whole number quantities. Therefore, the band can purchase 110 CDs.

3. If the set-up fee is reduced by 50%, it will be 0.50(250) or $125. The cost function becomes $C = 125 + 5.50n$. 50% of the original cost is

   $0.50(250 + 5.50n) = 0.50(250) + 0.50(5.50)n = 125 + 2.75n.$

   $125 + 2.75n \leq 125 + 5.50n$

   The new cost is more than 50% of the original cost.

Extension Questions:

- Suppose Bryan has found another company that charges a set-up fee of $200 and charges $6.00 per CD. Would this be a better company from which to purchase CDs if they have $850?

The cost function under these conditions is C = 200 + 6n

$$200 + 6x = 850$$
$$6x = 850 - 200$$
$$6x = 650$$
$$x = 108.33$$

They could purchase 108 CDs. This is not a better company to purchase from if they have $850.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

- Under what circumstances would the second company be a better choice for the band to use for producing their CDs?

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

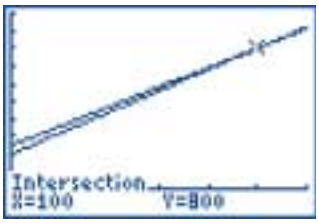
II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

The tables and graphs of the two functions may be compared to determine when they are equal in cost.



The functions have the same value when x is 100. The first company's cost is greater for values of x less than 100. The second company's cost is more for values of x more than 100.

If they are going to purchase less than 100 CDs, they should buy from the second company. If they are going to purchase more than 100 CDs, they should buy from the first company.

Explorer's Glide

When the space shuttle Explorer returns back to earth for landing, it travels in a long glide. The observer begins to record the height at a time when it is 100 km above the earth's surface.

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

1. Using symbols and words describe the relationship between the time in minutes and the height in kilometers.

2. Where was the shuttle two minutes before the observer began timing?

3. How long before the observer began timing was the shuttle at 102 km above the earth?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

Scaffolding Questions:

- What is changing in this situation?
- What would you expect the height to be in 5 minutes? Explain your reasoning.
- Is the relationship linear? How can you tell?
- What is the rate of change?
- What is the *y*-intercept?

Sample Solution:

1. Determine the rates of change from the table.

		10	5	5	2
time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84
		-2.8	-1.4	-1.4	-0.56

The rate of change is -0.28 km per 1 minute.

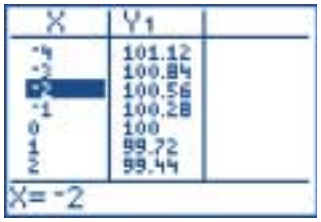
The height, *h*, is the starting height plus the rate of change times the number of minutes, *m*. The height of the Explorer in kilometers is 100 kilometers minus 0.28 kilometers per hour times the number of minutes.

$h = 100 - 0.28m$

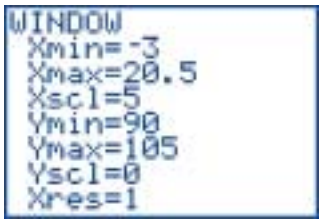
2. If the descent had begun at this constant rate at least two minutes before the timing began, the time would be represented by $m = -2$, it would have been at $100 - 0.28(-2)$ or 100.56 kilometers above the earth.

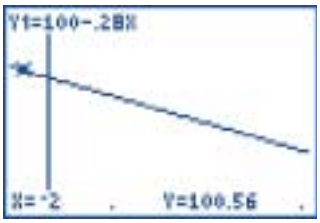
We could also use the table or the graph.

Enter the rule $y = 100 - 0.28x$.
Look at the table of values to find the value of y when $x = -2$.



The window may also be adjusted so that the graph may be traced to find the value when $x = -2$.

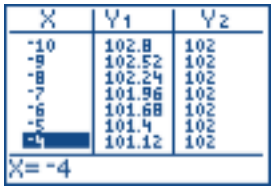


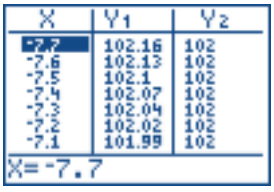


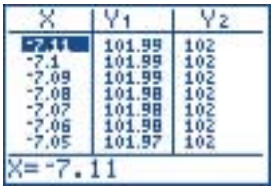
The height of the object when the time is two minutes before the observer began timing would be 100.56 km.

3. The question is asking when is $102 = 100 - 0.28x$. Solving for x gives a value of about -7.14. This means that if the glide had begun at this rate at least 7.14 minutes before timing began, then the shuttle would have been at 102 km above earth.

   To determine the time when $y = 102$, one could also examine a table of the function $y_1 = 100 - 0.28x$ and the function $y_2 = 102$. Continue to increment the x value to smaller increments until y is 102.







The value is between -7.16 and -7.13. The table rounds the values to the nearest hundredth.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors necessary in problem situations; and

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

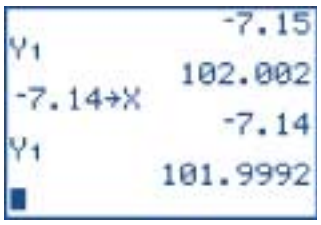
Connections to Algebra I: 2000 and Beyond Institute:

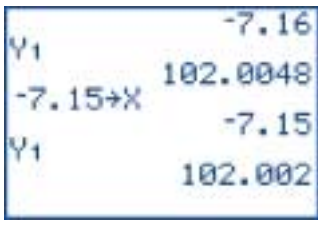
II. Linear Functions
1 The Linear Parent Function
1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

To determine more precisely one may compute the values.





The time is between 7.15 minutes and 7.14 minutes before the observer began timing.

Extension Questions:

- What does the *y*-intercept mean in this situation?

The y-intercept represents the height at time zero.

- What is the *x*-intercept, and what does it mean for this problem situation?

The x-intercept is approximately 367.14. It represents the number of minutes after the observer began recording time that the height is 0; that is the time when the shuttle would have hit the earth if it had continued to descend at the same rate.

- Is this a realistic model for the descent of a shuttle?

This is not a realistic situation. In reality the shuttle must decrease its speed as it gets closer to landing.

Student Work

1. The shuttle is descending at a rate of 0.28 km per minute.

$\frac{y_2 - y_1}{x_2 - x_1}$ Use the slope formula

$\frac{95.8 - 97.2}{15 - 10}$ Set up the equation inserting the values of any two points.

$\frac{-1.4}{5} = -0.28$ Simplify the equation.

The rate of increase is -0.28 km per minute, or it is descending at 0.28 km per minute.

2. 100.56 km

$y = mx + b$ Use the slope-intercept formula

$y = -0.28x + 100$ You got your slope from number 1 and since you have the point $(0, 100)$, so you know the y intercept.

$y = -0.28(-2) + 100$ Set x to -2 (two minutes ago) and solve

$y = 100.56$

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3. $7\frac{1}{7}$ minutes

$y = -0.28x + 100$ Use the equation from number 2:

$102 = -0.28x + 100$ Insert 102 for y

$2 = -0.28x$ Solve

$\frac{200}{-28} = x$

$-7\frac{1}{7} = x$

He was at 102 km $7\frac{1}{7}$ minutes ago.

Geothermal Energy

Whenever water comes into contact with heated underground rocks, geothermal energy is generated. The underground temperature of the rocks depends on their depth below the surface. The temperature t in degrees Celsius is estimated by the function $t(d) = 35d + 20$, where d is the depth of the rocks in kilometers.

1. Describe the graph of the function. Explain how the constants in the function are related to the graph and to the problem situation.

2. Identify the domains and ranges for this function and for the problem situation.

3. Find the temperature of the rocks at a depth of 3 kilometers.

4. Find the depth if the temperature of the rocks is 195 degrees Celsius.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations

Scaffolding Questions:

- If the depth of the rocks is 1 kilometer, what is the temperature?
- Find the temperature if the depth of the rocks is 2 kilometers; 3 kilometers.
- What is the relationship between the temperature at 1 kilometer, and at 2 kilometers? At 2 kilometers and 3 kilometers?
- Why can you NOT have negative values for the domain of the problem situation?
- What does the 35 mean in the equation?
- What does the 20 represent in the equation?
- What is the relationship between the depth and the temperature?

Sample Solution:

1. The temperature of the rocks depends on the depth of the rocks. For every kilometer of depth, the temperature increases 35 degrees Celsius. Because the rate of change is constant, the function is linear.

   The graph of the function will be a line with a positive slope and with a y-intercept value of 20. The intercept value of 20 means that at a depth of 0 kilometers the temperature is 20 degrees Celsius. The slope is the rate of change in the temperature per kilometer of depth. The rate of change is 35 degrees Celsius from every 1 kilometer of depth.

2. The domain represents the depth of the rocks. The problem states that geothermal energy is generated wherever water comes into contact with heated underground rocks. The positive values greater than zero would indicate depths of rocks in kilometers. A value of zero would represent the surface, not underground, and would not be included. The negative values would actually be representing kilometers above the surface and would not qualify either. The domain would be values greater than zero, or $x > 0$.

   The range represents the temperature in Celsius. At the surface (0 kilometers), the temperature of rocks is 20 degrees Celsius. Because the rocks must be underground, values greater than 20 are the only ones that satisfy the condition for the geothermal energy or $y > 20$.

3. Substitute the value $d = 3$ into the equation $t = 35d + 20$ to find the temperature of the rocks at 3 kilometers.

$t = 35(3) + 20$
$t = 105 + 20$
$t = 125$ degrees C.

At the surface (0 kilometers), the temperature of the rocks was 20 degrees Celsius because

$t = 35(0) + 20$
$t = 0 + 20$
$t = 20$ degrees C.

4. If the temperature of the rocks is 195 degrees Celsius, the depth can be found algebraically:

$$\begin{aligned} t &= 35d + 20 \\ 195 &= 35d + 20 \\ 175 &= 35d \\ 5 &= d \end{aligned}$$

The depth of the rocks at 195 degrees Celsius is 5 kilometers.

Extension Questions:

- Express the temperature of 195 degrees in Fahrenheit, given the formula $F = \frac{9}{5}C + 32$, where C = temperature in Celsius, and F = the temperature in Fahrenheit.

The temperature 195°C is converted to Fahrenheit using the formula

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(195) + 32$$
$$F = 351 + 32$$
$$F = 383$$

195 degrees C is equivalent to 383 degrees F.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

- How would the original equation, $t = 35d + 20$, change if the temperature t must be expressed in degrees Fahrenheit?

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.3 Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Convert from Celsius to Fahrenheit and substitute the Fahrenheit expression for Celsius in the function rule.

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$C = \frac{5}{9}(F - 32)$$

$t = 35d + 20$, t in Celsius

$$\frac{5}{9}(F - 32) = 35d + 20$$

$$F - 32 = \frac{9}{5}(35d + 20)$$

$$F = \frac{9}{5}(35d + 20) + 32$$

This rule defines the relationship between the distance, d, and the temperature in Fahrenheit, F.

Hot-Air Balloon

At the West Texas Balloon Festival, a hot-air balloon is sighted at an altitude of 800 feet and appears to be descending at a steady rate of 20 feet per minute. Spectators are wondering how the altitude of the balloon is changing as time passes.

1. What function relating the variables best describes this situation?

2. Make a table of values and/or graph to show the balloon's altitude every 5 minutes beginning at 5 minutes before the balloon was sighted until the balloon lands.

3. How high was the balloon 5 minutes before it was sighted?

4. How long does it take the balloon to reach an altitude of 20 feet? How long does it take the balloon to land?

5. A second balloon is first sighted at an altitude of 1200 feet and is descending at 20 feet per minute. How does the descent and landing time of the second balloon compare with that of the first balloon? What does this mean graphically?

6. A third balloon is first sighted at an altitude of 800 feet but is descending at 30 feet per minute. How does the descent and landing time of the third balloon compare with that of the first balloon? What does this mean graphically?

7. At the instant the first balloon is sighted, a fourth balloon is launched from the ground rising at a rate of 30 feet per minute. When will the first and fourth balloon be at the same altitude? What is that altitude? What does this mean graphically?

Teacher Notes

Materials:
One graphing calculator per student

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

Scaffolding Questions:

- What are the constants in the problem? What quantities vary?
- What quantity will be the dependent variable? The independent variable?
- What kind of function models the situation? How do you know?
- What decisions must you make to build a table for the function?
- What decision must you make to graph the function?
- How can you find the balloon's height at any given time?
- How can you find the time it takes the balloon to reach a given height?

Sample Solution:

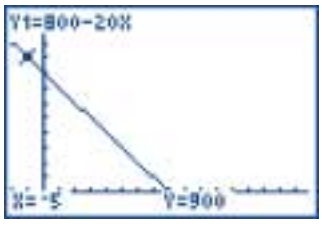
1. The starting height, 800 feet, is decreased by 20 feet per minute. The height, h, equals 800 minus 20 times the number of minutes, m.

   $h = 800 - 20m$

2. The time 5 minutes before it was sighted is represented by -5.

m	$800 - 20m$	h
-5	800 – 20(-5)	900
0	800 – 20(0)	800
5	800 – 20(5)	700
10	800 – 20(10)	600
15	800 – 20(15)	500
20	800 – 20(20)	400

The graph may also be used to examine the situation.



3. The value of y is 900 when x is -5. Therefore, the balloon was at 900 feet 5 minutes before it was first sighted.

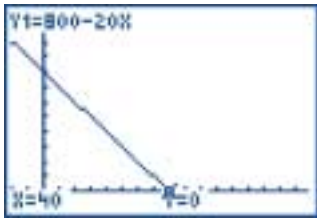
4. Solve for m:

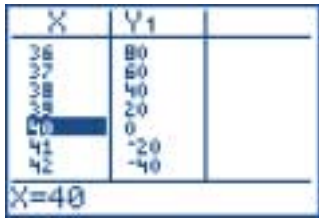
$$\begin{aligned} 800 - 20m &= 20 \\ -20m &= -780 \\ m &= 39 \end{aligned}$$

It takes the balloon 39 minutes to descend to 20 feet above the ground.

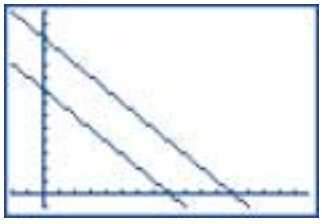
Solve $800 - 20m = 0$ for m to get $m = 40$. The balloon lands in 40 minutes.

The graph or table may also be examined to determine when the height is 0.





5. The balloon is at a higher altitude but descending at the same rate. It will take longer to land. The second function is $y = 1200 - 20x$. The graphs have different y-intercepts and x-intercepts. The graphs will be parallel lines since they have the same slope.



(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept
1.3 Exploring Rates of Change

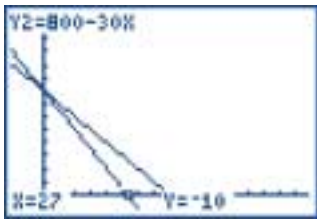
Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

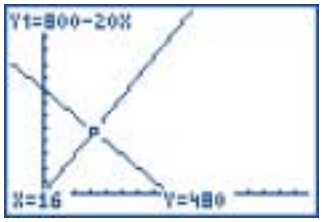
Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

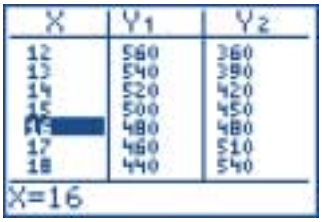
6. The third balloon starts at the same height as the first but is descending faster. Therefore, the third balloon will land sooner. The third function rule is $y = 800 - 30x$.

   The graphs have the same y-intercept but different x-intercepts. The x-intercept for the third balloon is less than that of the first balloon. The graph for the third balloon's descent will be steeper than that for the first balloon.



7. The function for the fourth balloon is $y = 30x$. To see if they are ever at the same altitude, explore with tables or graphs, or solve $800 - 20x = 30x$ to get $x = 16$. Sixteen minutes into descent/launch, both balloons will be at the same height, 480 feet.





Extension Questions:

- If the function of the motion of a fifth balloon had been $y = 700 - 20x$, how would the movement of the balloon have been different from the first?

The balloon would have been sighted at a height of 700 feet instead of 800 feet. The rate of descent would have been the same as the rate of descent of the first balloon.

- Would the fifth balloon have landed sooner or later than the first balloon? Explain how you know.

If it started at a lower altitude and descended at the same rate, it would land sooner. The x-intercept would be 700 divided by 20, or 35 seconds.

Student Work

1) $y = 800 - 20x$
Starting height · 20 feet per minute

2)

feet y	minutes x	
900	-5 ←	5 minutes before sited at 900
800	0 ←	when it was sited
700	5 ←	5 minutes after it was sited
600	10 ←	10 minutes after sited
500	15 ←	15 minutes after sited
400	20 ←	20 minutes after sited

3) 900 feet in the air from the table
I made a graph on the calculator and traced

4) 39 minutes at $x = 20$
40 minutes at $x = 0$

5) The second balloon is going later because its starting at 1200 and the first one was sited at 800 There is a (400 feet difference)

6) The Third balloon is descending at 30 ~~feet~~ per minute and the first one was going 20 feet per minute so the ~~T~~third balloon is going to get to the ground faster

7) The 4th and 1ST balloon are going to meet at 16 ~~feet~~ minutes at 480 feet. I graphed $y = 800 - 20x$ and $y = 30x$ and found the intersection

Making Pizzas, Making Money

The CTW Pizza Company is planning to produce small square pizzas. It will cost them $2.00 to make each pizza, and they will sell them for $5.00 a piece.

1. Express the profit earned as a function of the number of pizzas sold.

2. Graph the function rule and describe the relationship between the two variables.

3. What is the slope of the graph, and what does it mean in the context of the situation?

4. Discuss at least two methods for finding the number of pizzas that need to be sold to make a profit of $180.

5. CTW found a cheaper supplier, and now it costs $0.50 less to make each pizza. Describe how this will change the function rule, the graph, and the table, and explain how you know.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

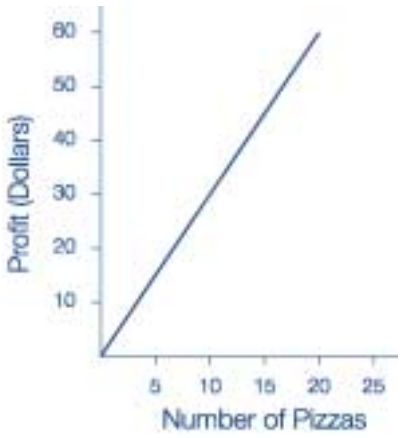
Scaffolding Questions:

- What is the profit for one pizza? For two pizzas? For three pizzas?
- What are the variables in this situation?
- What is the profit per pizza?
- What does the table look like?
- How much profit can be made selling 50 pizzas?
- If your goal is to make a profit of $300 a day, how many pizzas must you sell each day?

Sample Solution:

1. The profit from making pizzas can be determined by subtracting the cost to make each pizza from the selling price. Therefore, the function rule for the profit, p, will be $p = 5x - 2x$ or $p = 3x$ where x represents the number of pizzas.

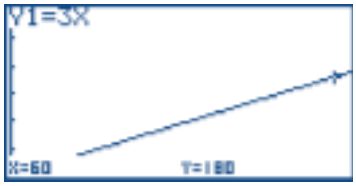
2.



The profit in dollars is the number of pizzas multiplied by $3. There will be a $3.00 profit per pizza. The more pizzas sold, the more profit made.

3. The slope is the profit in dollars per number of pizzas. It can be determined from the graph by looking for the rate of change; the profit goes up $3.00 for every pizza sold.

4. Use the function $p = 3x$. When p is 180, the rule becomes $180 = 3x$. Since 3 times 60 is 180, they must sell 60 pizzas to make a profit of \$180. The table or graph may also be used to answer the question.



5. The new cost is \$1.50 per pizza. The profit for each pizza would be $5x - 1.50x$ or $3.50x$. The profit has now increased by \$0.50 per pizza, therefore, the profit per pizza is now \$3.50. The b value in the function rule $y = mx + b$ is still 0, but the m will increase by \$0.50. The slope of the graph will now be greater, because for every pizza you sell you now make \$3.50 instead of \$3.00. The table will also show an increase of \$3.50 in the y value for every increase in 1 of the x value.

Extension Questions:

- Describe how to determine the slope from the table.

Calculate the rates of changes by finding the difference of two y values, divide by the corresponding differences in the x values, and look for a constant rate of change.

- Describe how to determine the slope from the graph.

You can determine the slope from the graph by finding the ratio of the vertical change to the horizontal change between any two points on the line.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

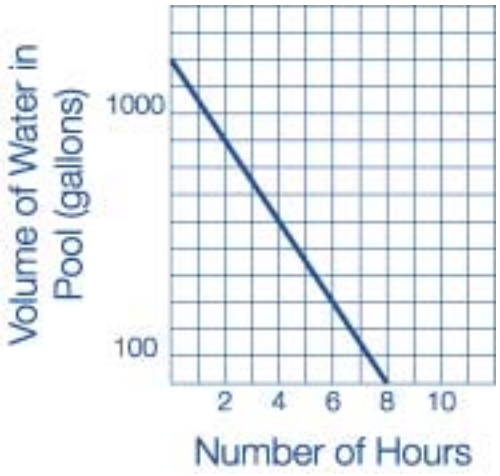
II. Linear Functions
1 Linear Functions

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Pool Problem



1. The graph shows the relationship between the amount of water in a pool and the number of hours that have elapsed since a pump began to drain the pool. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.

2. How much water would be in the pool after 4 hours and 20 minutes?

3. How many hours after they began draining the pool would it contain 720 gallons of water?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Define the independent variable and the dependent variable for this problem situation.
- What type of relationship is described by the graph?
- How much water was in the pool when the pumping started? What part will this number play in the function rule?
- How much water was in the pool after two hours? Four hours? Six hours? Organize your response in a table.
- At what rate is the amount of water decreasing per hour?
- Use the rate of change and the starting volume in the pool to write a function rule.

Sample Solution:

1. The amount of water in the pool at time zero is 1200 gallons.

   The water is being drained at a constant rate, because the graph is the graph of a straight line. It takes 8 hours to drain the pool. The rate per hour would be 1200 gallons divided by 8 hours or 150 gallons per hour. Because the water is draining, the rate of change is -150 gallons per hour.

   The amount of water in the pool is the starting value plus the rate times the number of hours.

   Let w be the amount of water in the pool at time t in hours,

   $w = 1200 + (-150)t$
   $w = 1200 - 150t$

   where t is any number between 0 and 8, inclusive.

2. The time is 4 hours and 20 minutes or $4\frac{1}{3}$ hours.

   $$w = 1200 - 150\left(4\frac{1}{3}\right) = 1200 - 650 = 550$$

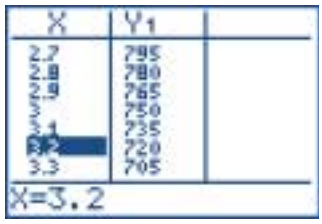
   The amount of water in the pool after 4 hours and 20 minutes is 550 gallons.

3. The amount of the water in the pool will be 720 gallons when $y = 720$.

$$720 = 1200 - 150t$$
$$t = 3.2$$

There will be 720 gallons of water in the pool after 3.2 hours.
0.2 (60) = 12
The time is 3 hours and 12 minutes.

A table or graph could also be used to determine when the amount of water is 720 gallons. Set the table minimum at 1 and increments at 0.1, and scroll down the table to find the value when $y = 720$ at $x = 3.2$. In 3.2 hours the amount in the pool would be 720 gallons.



Extension Questions:

- What is the domain for the function rule you have written?

The domain is the set of all real numbers.

- Describe the domain for this problem situation and explain why you selected this domain.

The domain is the set of all real numbers from 0 to 8 inclusive. The domain values must be a non-negative number and must give non-negative range values. The pool is empty after 8 hours.

- How much time would have elapsed if the pool is half-empty?

The original amount of water in the pool was 1200 gallons. The amount of water is 600 gallons at 4 hours. Note that this is one-half the time it takes to empty the pool.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Developing Mathematical Models
1.2 The Y-intercept
1.3 Exploring Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

- Will this relationship work if you are asked about how long it takes to empty one-third of the water? Explain your reasoning.

It would take one-third of the time it takes to drain the pool. There is a proportional relationship between the time and the portion of the water that has been drained.

The time it takes to drain the pool is $\frac{1200 \text{ gallons}}{150 \text{ gallons per hour}}$ *or 8 hours.*

If one-third of the pool is drained, two-thirds of the pool volume remains.

$$\frac{2}{3}(1200)1 = 1200 - 150x$$

$$150x = \frac{1}{3}(1200)$$

$$x = \frac{1}{3} \bullet \frac{1200}{150} = \frac{1}{3}(8)$$

Let f be the fractional part of the pool drained. The part remaining is 1-f.

$$(1-f)1200 = 1200 - 150x$$

$$150x = 1200 - (1-f)1200$$

$$150x = f\,1200$$

$$x = f\left(\frac{1200}{150}\right) \text{ or } 8f$$

Thus, if the amount drained is (f)(1200), the time it takes is (f)(8) or f times the amount of time it takes to drain the pool.

- If the pool started at 1500 gallons, but emptied at the same rate, how would that affect your graph?

The only value changed in the function is the y-intercept.

y = 1500 – 150 x

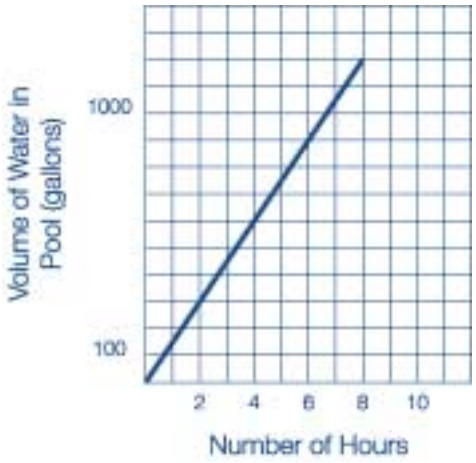
The graph would be a straight line parallel to the original line, but with a y-intercept of 1500.

- If the pool started at the same amount, but emptied at 100 gallons per hour, how would the graph be changed?

The rate of change or slope is -100.
The function would be y = 1200 – 100 x

- Suppose an empty pool was being filled at the same rate and with the same capacity of 1200 gallons. Sketch the graph to represent this situation and write the function to represent this new situation.

The function would be y = 150x where x varies from 0 to 8. Since the capacity of the pool is 1200 gallons, the graph terminates at the point (8,1200); the graph is a line segment.



Student Work

Pool Problem

1. The graph shows that for every two hours that pass, 300 gallons are drained out of the pool. This is an inverse relationship, meaning that as time (or the x-variable) goes on, the y-variable, or the gallons of water, decreases. An equation that could go with the graph is $y = -150x + 1200$. The 1200 is how much water the pool starts out with, and -150 is the rate at which the water is decreasing each hour. As you can also see, the graph is a linear relation, because it moves in a straight line and changes at a constant rate. X is the variable that stands for the number of hours passed and y is the amount of water remaining in the pool.

2. First, figure out what 4 hours and 20 minutes is in fraction or decimal form. 20 minutes is out of a whole of 60 minutes in one hour, therefore, $\frac{20}{60} = \frac{1}{3}$. This means 4 hours and 20 minutes = $4\frac{1}{3}$. Now, place the hours $4\frac{1}{3}$ into the equation $y = -150x + 1200$ as the x-variable. -150 is the rate at which the water is decreasing and 1200 is the water that was there to begin with. 1200 is also the y-intercept.

$y = -150(4\frac{1}{3}) + 1200$

$y = -650 + 1200$

$y = 550$ gallons of water remaining after 4 hours and 20 minutes.

3. Because 720 is how many gallons is remaining, you replace the y-variable in the equation with 720, because that is the number you're solving for. So, $720 = -150x + 1200$. You then solve the equation by subtracting 1200 from both sides.

$720 = -150x + 1200$
$-1200 \quad -1200$
$\frac{-480}{-150} = \frac{-150x}{-150}$
$3.2 = x$
hours draining

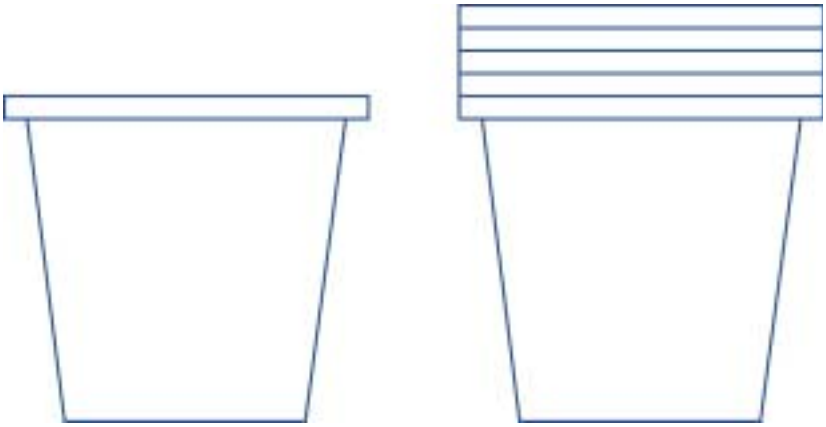
Then, you divide both sides by -150. To find how many minutes .2 equals, put it into decimal form: $\frac{2}{10}$ and set it equal to $\frac{x}{60}$. The solve by cross-multiplying: multiply 10 w/ x and 60 with 2. Divide each side by 10.

$\frac{2}{10} = \frac{x}{60}$
$\frac{120}{10} = \frac{10x}{10}$
$12 = x$
minutes

Now, you know that it takes 3 hours and 12 minutes for the pool to contain only 720 gallons.

Stacking Paper Cups

The figure shows drawings of one paper cup and five paper cups that have been stacked together. The cups are shown at one-half of the real size. Use a centimeter ruler to help answer the questions.



1. Create a function rule that gives the actual height of a stack of cups in terms of the number of cups in the stack. Define the variables. Explain in detail how you created your rule.

2. What would be the total height of 12 stacked cups? Justify your solution.

3. How many cups would fit stacked in a space 1 meter in height? Justify your solution.

4. Find a function that gives the number n of cups in a stack in terms of the height h of the stack.

5. If a new stack is created with the base of the cups remaining the same but the height of the lip of the cup doubled, will the new stack be more than, less than, or equal to twice the height of the original stack?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

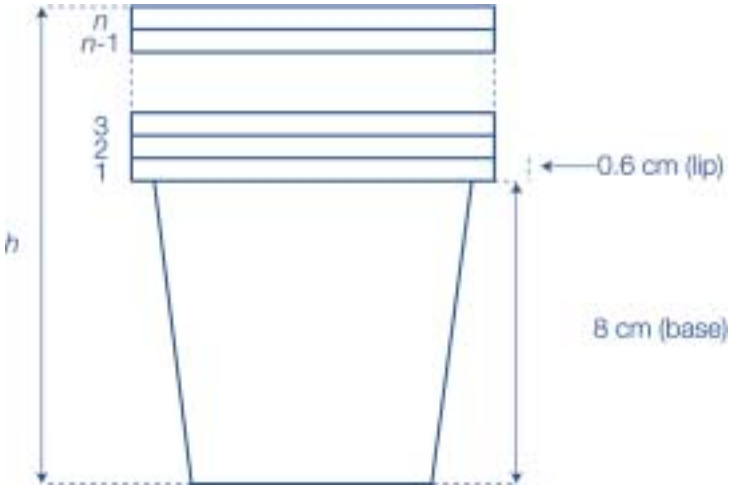
(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- How will you start?
- What is the measurement of one cup?
- What is the measurement of two cups?
- What is the measurement of one lip?
- What are the variables?
- What is the dependent variable?
- What is the independent variable?
- Describe how you might create a table to help you determine the function rule.

Sample Solution:

This solution is based on the following diagram of a stack of cups with all measurements doubled.



1. The height h of a stack of n cups can be broken down into the 8 cm "base" of the bottom cup plus n times the 0.6 cm "lip" of each cup. We can therefore represent the height h as a function of the number n of cups as

   $h(n) = 8 + 0.6n$

This is a "height function" that gives the height h in terms of the number n. The slope of the function is 0.6 cm per cup.

2. To find the height of a stack of 12 cups we can evaluate the height function at $n = 12$:

   $h(12) = 8 + 0.6(12) = 15.2$ cm

3. To find how many cups fit into a space 100 centimeters high we can set the height function equal to 100:

   $100 = 8 + 0.6n$

   This is an equation that we can solve for n:

$$n = \frac{100 - 8}{0.6} = 153\frac{1}{3}$$

Since we can't have $\frac{1}{3}$ of a cup, we see that 153 whole cups will fit into this 100 centimeters space. As a check one can see how much space 153 cups occupy by evaluating the function at $n = 153$:

$h(153) = 8 + 0.6(153) = 99.8$ cm

There is a space of 0.2 cm left over when 153 cups are put into a 100 cm space.

One may also enter the function rule $y = 8 + 0.6x$ into the graphing calculator. Examine the table and look for the y column value of 100. When x is 153, $y = 99.8$. When x is 154, the y value is 100.4. There would be 153 cups in the stack.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

4. To express the number of cups as a function of the height, solve the rule for *n.*

$$h = 8 + 0.6n$$
$$h - 8 = 0.6n$$
$$\frac{h-8}{0.6} = n \quad \text{or}$$
$$n = \frac{h-8}{0.6}$$

This is a "count function" that gives the number *n* in terms of the height *h*.

5. If the thickness of the lip of each cup is doubled, the rule for the height of a stack of cups becomes

$h(n) = 8 + 2(0.6)n$ or
$h(n) = 8 + 1.2n$

Twice the height of the original stack would be $2(8 + 0.2n)$ or $16 + 1.2n$. $8 + 1.2n$ is less than $16 + 1.2n$. The height is less than the height of the doubled stack.

Extension Questions:

- What is the rate of change (slope) of the function rule and what does it mean in this situation?

The rate of change is 0.6cm per one cup. The rate of change is the change in the height of the stack per one cup.

- What limits the domain in this situation?

The domain values must be counting numbers. There may not be a fractional number of cups or a negative number of cups.

- What are the *y*-intercept and *x*-intercept and what do they mean in this situation?

The y-intercept of the graph of the line y = 8 + 0.6x is 8, but this point (0,8) would not be a point plotted in this problem situation because it means a cup without a lip. The measurement of the cup without the lip is 8 centimeters.

The x-intercept is $\frac{-8}{0.6} = -\frac{40}{3} = -13\frac{1}{3}$, *but the point* $\left(-\frac{40}{3}, 0\right)$ *has no meaning in this problem situation, because there cannot be a negative number of cups.*

- Does the graph add any additional information?

The graph of the function is a line, but the graph of the problem situation is a set of points in the first quadrant. The domain of the problem situation is the set of counting numbers. Thus, the points must have first coordinates that are counting numbers. The graph on a graphing calculator may be used to answer questions about the situation, to evaluate the function at specific values, and to solve equations related to the function.

- If the height of the cup were 10 centimeters and the height of the lip stayed the same, how would it have changed the function for this situation?

The height of this new cup without the lip would be 10 – 0.6 or 9.4. The function rule would become y = 9.4 + 0.6x.

- If the problem had said the cup was drawn to the actual size of the cups, how would that have changed your function rule?

The values of the intercept and the slope would have been reduced by one-half. The function rule would have been y = 4 + 0.3x.

- If the problem had said the cup was one-third of the actual size, how would that have changed your function rule?

The measurements would have been multiplied by 3 instead of 2. The height of the cup would have been 12.9 centimeters. The lip would measure 0.9 cm. The function rule would become y = 12 + 0.9x.

- Another way of writing the count function is

$$n(h) = \frac{h}{0.6} - \frac{8}{0.6} = -13\frac{1}{3} + 1\frac{2}{3}h$$

$$n(h) = -13\frac{1}{3} + 1\frac{2}{3}h$$

- What does the slope of this rule mean in the context of the problem?

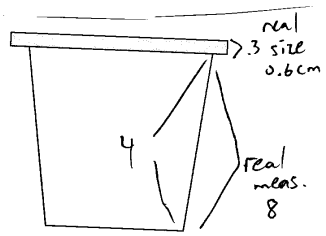
The slope of this function is $1\frac{2}{3}$ *cups per centimeter, which means that* $1\frac{2}{3}$ *cup lips fit in 1 centimeter.*

Student Work

① $y = 0.6x + 8$

Let y = the total height in centimeters

Let x = number of cups



The scale is 1/2 → Six millimeters are added each time a cup is added to the original base part of cup (8cm)

② $y = 0.6 \times 12 + 8$

$y = 15$ cm

③ $100 = 0.6x + 8$

$92 = 0.6x$ 153 cups because you can't have 1/3 of a cup.

$153\frac{1}{3} = x$

④ $\frac{y-8}{0.6} = \frac{0.6n}{0.6}$

$\frac{5}{3}h \rightarrow \frac{40}{3} = n$

⑤ Let $x = 10$

$0.6 \times 10 + 8 = 14$ cm The new stack is less than twice the height of the original

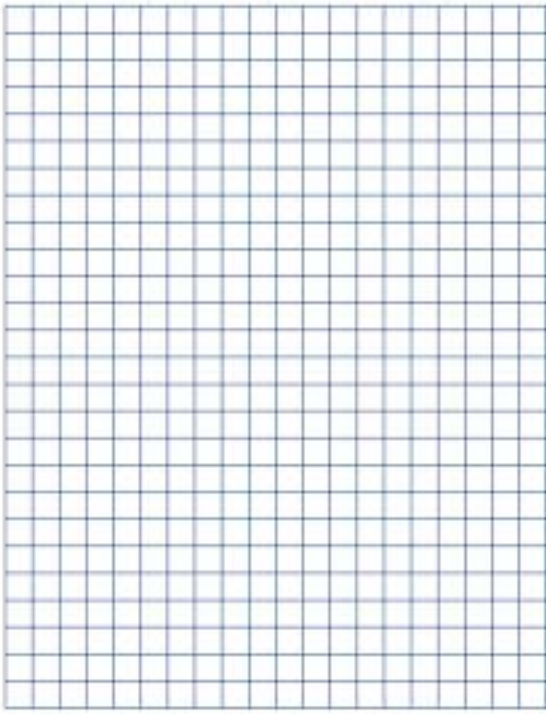
$1.2 \times 10 + 8 = 20$ cm

Stretched Spring

Data is collected in an experiment to determine the relationship between the length of a spring and the mass of an object hanging from it. The length of the spring depends on the mass of the object. The table below left gives a sample of the data.

Length versus Mass

Mass (kg)	Length (cm)
50	5.0
60	5.5
70	6.0
80	6.3
90	6.8
100	7.1
110	7.5
120	7.7
130	8.0
140	8.6
150	8.8
160	9.2
170	9.5
180	9.9
190	10.3



1. Construct a scatterplot of the data. Describe the functional relationship between the length of the spring and the mass suspended from it verbally and symbolically.

2. Predict the length of the spring for a suspended mass of 250 kilograms.

3. Predict the mass that stretches the spring to 15 centimeters.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

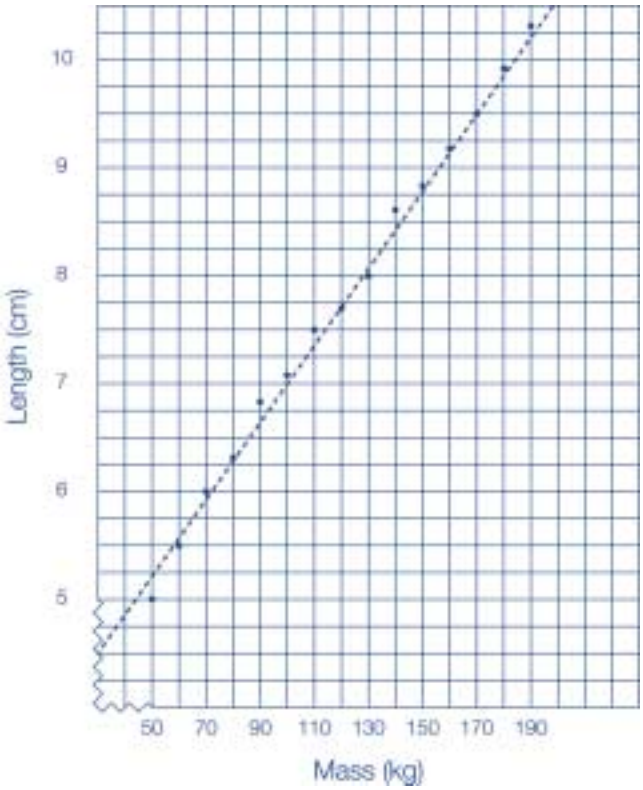
Scaffolding Questions:

- How will you organize the data that is collected?
- What will you need to consider to construct a scatterplot of the data?
- What will you need to consider to determine a reasonable interval of values and scale for each of the axes?
- What function type (linear, quadratic, exponential, inverse variation) appears to best represent your scatterplot?
- What do you need to know to determine a particular function model for your scatterplot?

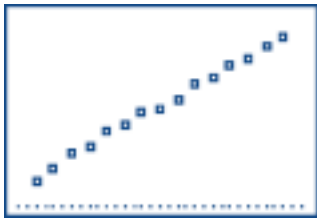
Sample Solution:

1. Create the scatterplot.

   The scatterplot is nearly linear.



The data points may also be entered into a graphing calculator to create the scatterplot.



The consecutive difference in the length values may be computed using the list feature.

The average of these consecutive differences is approximately 0.38. The difference in the consecutive mass values is 10. The rate of change may be approximated as 0.38 divided by 10 or .038. Using 0.038 centimeters per kilogram as the rate of change, a trend line is of the form $y = 0.038x + b$. Use any other data point to find a possible value for b. If the point (50, 5) is used, the value is 3.1.

$$5 = 0.038(50) + b$$
$$b = 5 - 0.038(50) = 3.1$$

$$y = 0.038x + 3.1$$

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

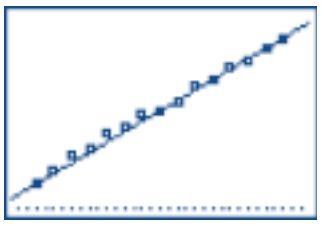
(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.

The graph of this line is an approximate trend line for the data.



(Note: Using the regression features of the calculator, the line of best fit is $y = 0.036x + 3.37$.)

2. To determine the length of mass, evaluate the function for $x = 250$.

$$\begin{aligned} y &= 0.038\,(250) + 3.1 \\ &= 12.6 \text{ cm} \end{aligned}$$

A mass of 250 kilograms will stretch the spring to a length of 12.6 centimeters.

3. To predict the mass that stretches the spring to 15 centimeters, use the function and solve the resulting equation:

$$\begin{aligned} 0.038x + 3.1 &= 15 \\ 0.038x &= 11.9 \\ x &= 313.16 \text{ kg} \end{aligned}$$

A mass of about 313.16 kilograms stretches the spring to a length of 15 centimeters.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities

Extension Questions:

- In your experiment, how did the mass suspended from the spring change and, in general, how did this affect the length to which the spring stretched?

The mass suspended from the spring started at 50 kilograms and increased by 10 kilograms each time until we reached a mass of 190 kilograms. The initial amount of stretch (at 50 kilograms mass) was 5 centimeters and increased by small amounts (0.2 centimeters to 0.5 centimeters) with each additional 10 kilograms of mass added to the spring.

- Since the mass increased in increments of 10 kilograms and the "stretch length" increased each time in the range from 0.2 centimeters to 0.5 centimeters, what did this suggest the functional relationship between spring length and mass would be?

It should be a linear relationship. As the mass increases in constant amounts, the amount by which the spring's length increases is nearly constant. This suggests a constant rate of change.

- How long is the spring when no mass is suspended from it?

Use the model, $y = 0.038x + 3.1$. When the spring has no mass attached to it, the value of x is 0 and y is 3.1 centimeters long.

- Suppose the initial length of the spring is changed to 6.8 centimeters, and we suspend mass from the spring in increments of 20 kilograms instead of 10 kilograms. How will this change the function that models this situation?

The impact of increasing the weight increments to 20 kilograms will not affect the amount of stretch in the spring. If the spring is of the same stretching ability, the rate of change would still be 0.038 centimeters per kilogram of mass. Changing the initial length of the spring to 6.8 centimeters will change the y-intercept to 6.8. The model of the situation would become $y = 0.038x + 6.8$.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

T-Shirts

A school organization has found four different places from which they may place an order for t-shirts. Each function below could represent the cost of placing a t-shirt order as a function of the number of t-shirts purchased.

A) $c = 5t$
B) $c = 3.25t + 55$
C) $c = 3t + 100$
D) $c = 6t - 55$

1. Write a scenario for each function.

2. Do all four functions fit a t-shirt situation? Explain your answer.

3. Make a table for each function.

4. Graph each function.

5. Describe the differences in the domain for the function and the domain for your problem situation.

6. Describe the differences in the range for the function and the range for the problem situation.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- What is the dependent variable?
- What is the independent variable for each situation?
- In situation A what must 5 represent?
- In situation B which constant represents the cost per t-shirt?
- In situation B what might the constant 55 represent?
- In situation C what does the 3 represent?
- In situation C what might the constant 100 represent?
- In situation D which constant represents the cost per t-shirt?
- In situation D what might the constant -55 represent?
- In situation D think about someone selling you the shirts at a constant rate but giving you a set discount.

Sample Solution:

1. A) Juan made a great deal with the manager of T-Shirts Plus. If the Math Club places an order of shirts, the cost will be $5.00 per shirt.

   B) The Spanish Club feels they have a better deal because they will get their t-shirts for only $3.25 each. They do have to pay a $55.00 set-up fee.

   C) The Math Club found another better deal. They will only pay $3.00 per shirt with a $100.00 set-up fee.

   D) The President of the Freshman class thinks he has the best deal: his father's friend will sell them shirts for $6.00 each and give him a $55.00 discount.

2. Yes, function A is the cost per shirt with no set-up fee, B and C both show the cost per shirt plus a set-up fee, and D shows cost per shirt with a discount.

3.

A) $c = 5t$

t	c
0	0
10	50
20	100
30	150
40	200
50	250
60	300

B) $c = 3.2t + 55$

t	c
0	55
10	87.50
20	120
30	152.50
40	185
50	217.50
60	250

C) $c = 3t + 100$

t	c
0	100
10	130
20	160
30	190
40	220
50	250
60	280

D) $c = 6t - 55$

t	c
0	-55
10	5
20	65
30	125
40	185
50	245
60	305

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

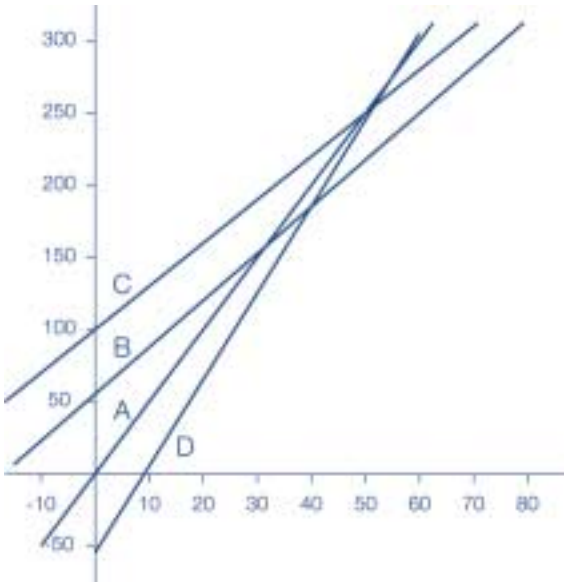
I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 Linear Functions
1.2 The Y- Intercept
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

4.



5. The domain of each function is all real numbers, because each function is a linear function. For the t-shirt situation the domain values must be whole numbers because shirts can not be purchased in fractions.

6. The range of each function is all real numbers. However, in the t-shirt scenario the amounts will be restricted to dollar values depending on the situation. For example, in situation B the amounts must be $55 plus a whole number multiple of $3.25.

Extension Questions:

- In situation A if the company had decided to give you a discount of $40, how would that change the equation?

The equation would become c = 5t – 40.

- From which t-shirt company should the group purchase the shirts if they are going to purchase 50 shirts?

By examining the table or the graph, one can see that the cost for 50 shirts is the least in situation B.

- Will situation B always give the least cost?

Situation C and B have the same cost at 180 shirts. After that number situation C has a smaller cost. This can be determined from examining the graph or table, or by solving symbolically.

Which Is Linear?

Four function rules were used to generate the following four tables:

I

x	y
-1	6
0	8
1	10
2	12
3	14

II

x	y
0	5
3	5
6	5
9	5
12	5

III

x	y
-2	-5
-1	-4.5
0	-4
3	-2.5
4	-2
5	-1.5

IV

x	y
-1	0.5
0	0
1	0.5
2	2
3	4.5
4	8
5	12.5

1. Which table(s) represent linear relationships? Explain how you decided.

2. Make a graph of the data in each table. Describe how the graphs are related.

3. Write a function for the linear relationships, and explain your thinking.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

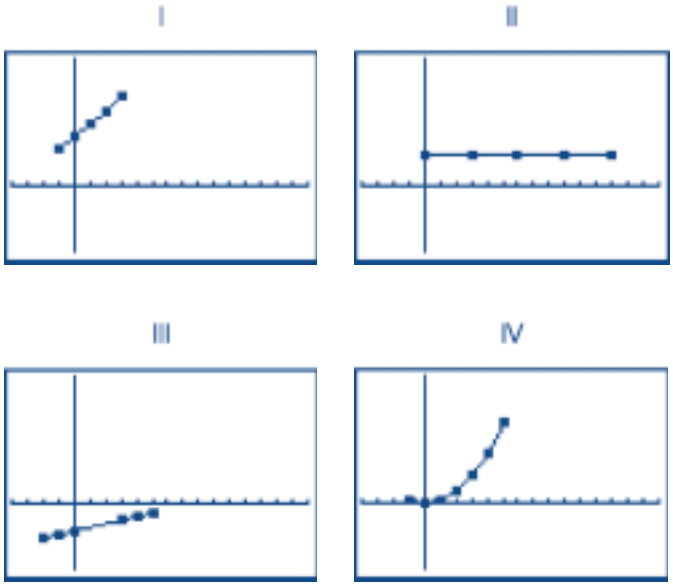
Scaffolding Questions:

- As the *x* values are increasing, what is happening to the *y* values?
- How are the patterns in the tables similar?
- How are the patterns in the tables different?
- What must be true about a function in order for it to be linear?
- How can you decide if a relationship is linear by looking at its table?
- What two numbers must you determine to write the linear function rule?

Sample Solution:

1. In Table I as *x* increases by 1, *y* increases by 2. In Table II *x* increases by 3, *y* stays constant. In Table III as *x* increases by 1, *y* increases by 0.5. In Table IV there is not a constant rate of change. Therefore, Tables I, II, and III represent linear relationships. The graphs of these sets of points form lines. As *x* increases by a constant number, *y* is also increasing by a constant number.

2. The scatter plots of the data are shown below in connected mode.



Three of the graphs show a linear relationship: I, II, and III. The graph of Table IV is not linear. The graph of Table I is the steepest linear graph. The graph of Table II has a slope of zero.

3. In Table I the rate of change is 2 because as x increases by 1, y increases by 2. The point (0,8) indicates that the line crosses the y-axis at 8, so 8 is the y-intercept.

   $y =$ starting value + the rate • x.
   $y = 8 + 2x$

   In Table II the rate of change is 0 because there is no change in y as x changes. The point (0,5) shows where the line crosses the y-axis. The function rule for Table II is $y = 5$.

   In Table III the rate of change is 0.5 because the ratio of the change in y to the change in x is 0.5. The point (0,-4) indicates that the line intersects the y-axis at -4.

   $y =$ starting value plus the rate times x
   $y = -4 + 0.5x$
   The function for Table III is $y = 0.5x - 4$.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Extension Questions:

- How can you decide if a relationship is linear by looking at its graph?

The graph of the data points forms a line whose slope can be determined.

- How can you decide if a relationship is linear by looking at its symbolic representation?

The rule shows the constant rate of change as the coefficient of the independent variable.

- When might you use a table to answer a question about a linear relationship?

Tables can be used when the values needed are part of the data given. Tables might also be used when they can be easily extended to find an answer; however, it is not always time-effective to extend a table. You may also set up a table using smaller increments for the domain values.

- When might you use a symbolic representation to answer a question about a linear relationship?

Symbolic representation can be used when a specific input or specific output value is required.