CORE

Algebra Assessments

Chapter 4:

Quadratic Functions

Fireworks Celebration

At a fireworks display celebration, a fireworks rocket is launched upward from the ground with an initial velocity of 160 feet per second. Spectators watch and wonder how high the rocket will go before it begins to descend back to the ground.

The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v is the initial velocity, and s is the initial height. Time t is measured in seconds, and height h is measured in feet.

1. What function describes the height, h in feet, of the rocket t seconds into launch?

2. Sketch a graph of the position of the rocket as a function of time into launch, and give a verbal description of the graph.

3. How high is the rocket after 3 seconds into launch? When does it reach this height again?

4. For the safety of the audience, the rocket, as it descends, should be set to explode at least 250 feet off the ground. The operator has a choice of fuses to use to explode the rocket. Fuse A will detonate the rocket between 3 and 5 seconds, Fuse B will detonate it between 4 and 6 seconds, and Fuse C will detonate it between 6 and 8 seconds. Which fuse should be used? Why?

5. Suppose the rocket is launched from the top of a 200-foot tall building. How will this change the position function for the rocket? How will the graph of the new position function compare with the graph of the first position function? What does the new graph tell you about the situation?

6. Suppose you are the operator and want to have the rocket launched from the ground to stay in the air 3 seconds longer (13 seconds instead of 10 seconds). How would you accomplish this? What effect will it have on the maximum height the rocket reaches?

Teacher Notes

Scaffolding Questions:

- What is the initial velocity and height in this problem?
- How can you rewrite the function so that it is easier to determine an appropriate window for graphing?
- How can you use your graph to answer questions about time and height?

Sample Solution:

1. The vertical motion formula is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second. The initial velocity, v, is 160 ft per second, and s, the initial height, is 0 because the rocket is thrown from the ground.

   $h(t) = -16t^2 + 160t$

2. To determine a reasonable window, find the x-intercepts by solving

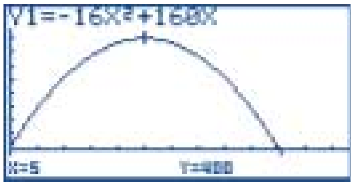
   $$h(t) = -16t^2 + 160t$$
   $$0 = -16t^2 + 160t$$
   $$0 = -16t(t + 10)$$

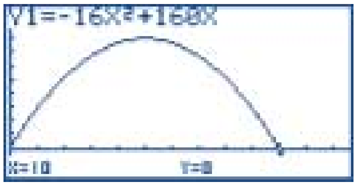
   $$-16t = 0 \quad \text{or} \quad t - 10 = 0$$
   $$t = 0 \quad \text{or} \quad t = 10$$

   The graph will be a parabola opening down, and the vertex is halfway between 0 and 10 at $t = 5$.

   $h(5) = -16(5)^2 + 160(5) = 400$

   The graph below shows the rocket rising to reach its maximum height of 400 feet in 5 seconds, then falling to hit the ground in 10 seconds.





Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

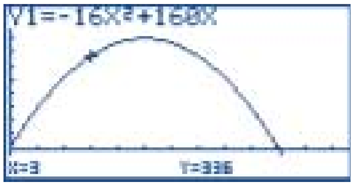
(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

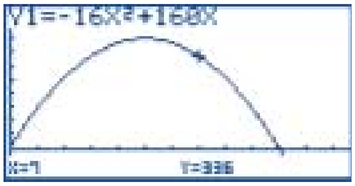
The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

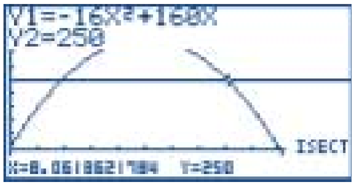
The student:
(A) uses symbols to represent unknowns and variables

3. By tracing the graph, we see that the rocket reaches a height of 336 feet when t = 3 seconds and again when t = 7 seconds.





4. By drawing the line y = 250 and finding its intersection points with the graph of the parabola, we see that the rocket is at least 250 feet off the ground between 2 and 8 seconds into launch.



This value could also be found by solving the equation

$$250 = -16t^2 + 160t$$
$$0 = -16t^2 + 160t - 250$$
$$0 = 16t^2 - 160t + 250$$
$$t = \frac{160 \pm \sqrt{(160)^2 - 4(16)(250)}}{2(16)}$$
$$t = \frac{160 \pm \sqrt{9600}}{2(16)}$$
$$t = 8.062 \quad \text{or} \quad t = 1.938$$

It does not make sense to detonate it when the rocket is rising. Therefore, it should be detonated at 8 seconds. The operator should use Fuse C since it has the time interval that includes 8 seconds.

5. Since the initial height of the rocket is now 200 feet instead of zero feet (on the ground) and the initial velocity is still the same, the new position function will be

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$; and

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

$h(t) = -16t^2 + 160t + 200$

The new graph will be the original graph translated up 200 units. Both graphs have the same shape and orientation. The new graph and the original graph are shown below.

The graphs have the same axis of symmetry $x = 5$, but the vertex of the new graph is (5,600). This makes sense because the original vertex has been translated up 200 units. By the same reasoning, the new y-intercept is (0,200). The x-intercepts for the new graph cannot be found as easily. By tracing or using the "zero" function of the calculator, one x-intercept is between 11 and 12, and the other is between -1 and -2.

The part of the graph that makes sense for the situation is the portion from the y-intercept to the x-intercept that is approximately (11,0). The graph shows that the rocket starts at a height of 200 feet, reaches a maximum height of 600 feet in 5 seconds, and then lands after a little more than 11 seconds.

6. We know that in the original function the gravitational constant, -16 feet per square second, does not change. Therefore, we should experiment with the initial velocity of the rocket. By "Guess and Check" changing of 160 feet per second and graphing, we see that the initial velocity needs to increase to about 200 feet per second and that the rocket reaches its maximum height of about 676 feet in about 6.5 seconds.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

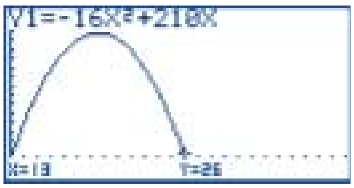
Connections to Algebra I: 2000 and Beyond Institute:

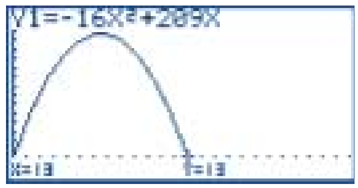
III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Functions
1.2 Transformations

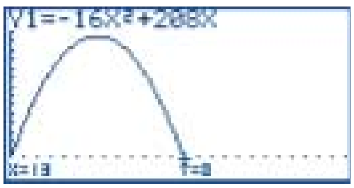
Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.







Another method uses the roots of the function. We see that the original time interval of 0 to 10 minutes shows up when we look at the original position function in factored form:

$h(t) = -16t(t - 10)$

Therefore, change the function to

$h(t) = -16t(t - 13)$
$h(t) = -16t^2 + 208t$

Now, the *x*-intercepts are (0,0) and (13,0), so that the rocket is in the air 13 seconds. The rocket reaches its maximum height in 6.5 seconds, and by evaluating the new function, the maximum height is 676 feet. By increasing the velocity, you have kept the rocket in the air longer and shot it to a greater height!

Extension Questions:

- What decisions must be made to determine an appropriate window for the graph of the function $ht = -16t^2 + 160t$?

The domain and range for the situation must be determined. This helps determine minimum and maximum values to use for x and y in the window, so that a complete graph of the function is viewed.

- What do you know about the shape of the graph of the function that helps you determine the domain and range? How will you use this information?

The graph is a parabola opening downward. One x-intercept is 0. Therefore the vertex of the parabola is above the x-axis. By finding the other x-intercept of the graph, we know to have x range in value from a little less than 0 to a little more than the second x-intercept. We get this second x-intercept by solving $0 = -16t^2 + 160t$ *by factoring and using the Zero Product Property.*

Once we know the x-intercepts, we know the x-coordinate of the vertex is halfway between them, and we can evaluate the function at that x-coordinate to get the maximum height. Then we know to have a range in value from a little less than 0 to a little more than the maximum height.

- How does changing the original height from which the rocket is launched change the function and its graph? What part of the graph of the mathematical function will make sense in the situation?

Launching the rocket from a point higher than ground level, e.g., s feet, changes the y-intercept of the graph to correspond to point (0,s). It also will translate the point that is the original maximum (vertex) up s units. The new graph will now have a negative x-intercept and a positive x-intercept, greater than the one before. Thus, the rocket reaches a greater maximum height and stays in the air longer. The part of the graph that makes sense in the situation is from the y-intercept to the positive x-intercept, since this is when the rocket leaves its launch site and when it lands.

- What parameter in the function cannot change? What does this tell you must change in order to launch the rocket from the ground and have it stay in the air longer?

Since the gravitational constant and initial height are constant, we must vary the initial velocity? By experimenting with different values, one can see that the initial velocity should be increased.

- Is it possible for a rocket to be launched from the ground and land on the ground in the same amount of time but go higher than the first rocket? Explain.

Since the rocket launches from the ground, its initial height is 0 and the function has the form

$$h(t) = -\frac{1}{2}gt^2 + vt + 0$$
$$h(t) = t\left(-\frac{1}{2}gt + v\right)$$

The t-intercepts are those values that give a height of zero.

$$h(t) = t\left(-\frac{1}{2}gt + v\right)$$
$$t = 0 \quad \text{or} \quad -\frac{1}{2}gt + v = 0$$
$$t = 0 \quad \text{or} \quad t = \frac{2v}{g}$$

Changing the initial velocity, v, changes the positive t-intercept of the graph so that the rocket is in the air a different amount of time. The constant g is the gravitational constant, -32 feet per square second, and cannot change if the rocket is being launched on earth! The rocket cannot go higher and land on the ground at the same time as the original rocket.

Golfing

The height *h* (in feet) above the ground of a golf ball depends on the time, *t* (in seconds) it has been in the air. Ed hits a shot off the tee that has a height modeled by the velocity function $f(t) = -16t^2 + 80t$. Sketch a graph and create a table of values to represent this function.

1. How long is the golf ball in the air?

2. What is the maximum height of the ball?

3. How long after it is hit does the golf ball reach the maximum height?

4. What is the height of the ball at 3.5 seconds? Is there another time at which the ball is at this same height?

5. At approximately what time is the ball 65 feet in the air? Explain.

6. Suppose the same golfer hit a second ball from a tee that was elevated 20 feet above the fairway. What effect would that have on the values in your table? Write a function that describes the new path of the ball. Sketch the new relationship between height and time on your original graph. Compare and contrast the graphs.

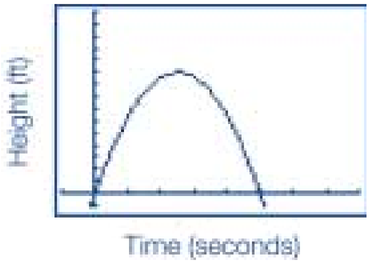
Teacher Notes

Scaffolding Questions:

- Describe your graph.
- Explain what values are reasonable for the domain and range in this situation.
- Describe the relationship between the height of the ball and the time.
- Describe the height of the ball over time.

Sample Solution:

1. The height *h* (in feet) above the ground of a golf ball depends on the time, *t* (in seconds) it has been in flight. Ed hits a shot off the tee that has a height modeled by the velocity function $f(h) = -16t^2 + 80t$. Sketch a graph and create a table of values to represent this function.



The graph of this function is a parabola that opens downward. It has a domain (representing the time in seconds) of $0 \le x \le 5$. The range (representing the height of the ball) is $0 \le y \le 100$.

The table of values is:

X	Y1
0	0
.5	36
1	64
1.5	84
2	96
2.5	100
3	96

X=0

X	Y1
3.5	84
4	64
4.5	36
5	0
5.5	-44
6	-96
6.5	-156

X=6.5

Using the table, it can be determined that the golf ball is in the air a total of 5 seconds.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

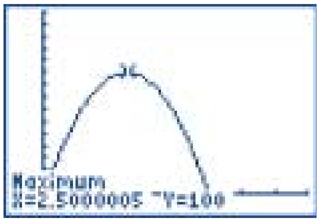
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

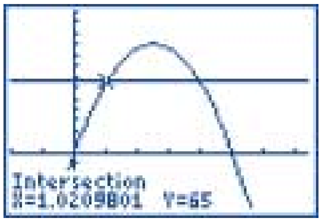
(B) relates the solutions of quadratic equations to the roots of their functions.

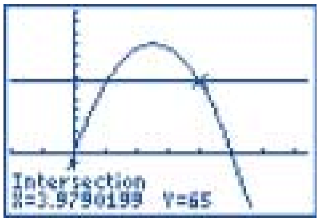
2. The table suggests that the golf ball reaches a maximum height of 100 feet (*y*) when the time (*x*) is 2.5 seconds. Another method of estimating the maximum height is to use the calculate function on the graphing calculator. Setting a lower and upper bound on the curve will yield a maximum height value of 100 feet and a corresponding time value that rounds to 2.5 seconds. (The trace feature may also be used to estimate the maximum height.)



3. The height of the ball at 3.5 seconds can be found in the table in the answer to question 1, when $x = 3.5$. At 3.5 seconds, the golf ball reaches a height of 84 feet.

4. The golf ball reaches a height of 65 feet at two different times. Because the graph is a parabola, the *y* value of the vertex is more than 65. The *y* value of 65 appears twice. The table above does not show an exact *y* value of 65 because the values are rounded. The value of 64 is found in the table at 1 second and 4 seconds. To find a more exact answer, the graphing calculator can be used. Results show very small differences in time from the table.





5. If the same golfer hit a second ball from a tee that was elevated 20 feet above the fairway, the values for the time remain the same in the table. The height values for the elevated golf shot are larger because the shot started 20 feet higher.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
  1.1 Quadratic Relationships
2 Quadratic Equations
  2.1 Connections
  2.2 Quadratic Formula

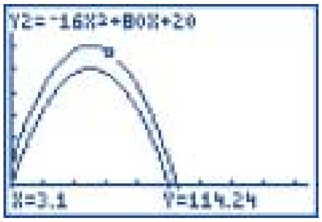
Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

The new function is $h = -16t^2 + 80t + 20$. The graph of the new function at time 0 seconds begins at 20 feet. This is 20 feet higher than in the original graph. The maximum for the new function is 20 feet higher than the original function. This can be found by tracing the function and looking in the table. For the first situation the golf ball hits the ground at 5 seconds. The second ball hits the ground at about 5.27 seconds.

The graph shows the original function and the new function.



Extension Questions:

- If the initial velocity had been 60 feet/second, how would the velocity function have been written?

Replacing the 80 in the original function with a 60 indicates the initial velocity is 60 feet per second. The function will be $f(t) = -16t^2 + 60t$.

- How would the graph of your function with an initial velocity of 60 compare to your original graph?

The new function has a maximum height of the ball at approximately 56.24 feet after about 1.9 seconds. The original function had a maximum at 100 feet after about 2.5 seconds.

- How does greater initial velocity appear to affect the flight of the ball?

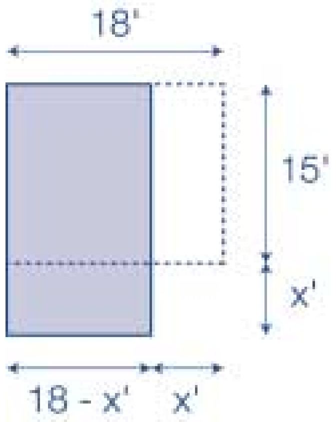
From the example it appears that the greater the initial velocity, the higher the maximum path of the ball over a longer time period.

- How do the x-intercepts of the two graphs compare?

If the velocity is increased, the x-intercept will increase, that is the ball will take longer to land on the ground.

Home Improvements

Ken's existing garden is 18 feet long and 15 feet wide. He wants to reduce the length and increase the width by the same amount, according to the diagram below.



1. Write a function that models the area of the new garden plot.

2. What value of *x* will produce a new area of 280 square feet? Justify your solution.

3. What value of *x* will produce a new area of 266 square feet? Justify your solution.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

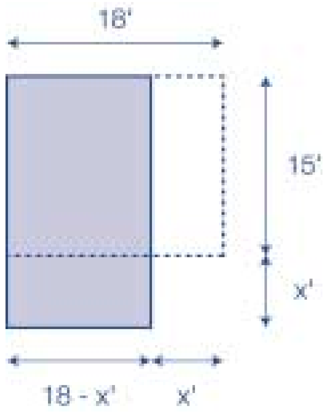
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- How can you find the area of any rectangle?
- What are the dimensions of the new garden plot?
- What methods might you use to solve your equation?
- How will the given values of the area relate to your algebraic area representation?

Sample Solution:

1. The new garden plot is a rectangle with dimensions of $(15 + x)$ by $(18 – x)$. The formula for finding the area of a rectangle is $A = l \bullet w$.



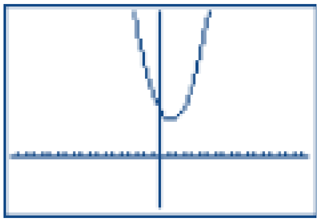
The function that models the area is:

$A = (15 + x)(18 – x)$
$A = 270 + 3x – x^2$

2. To find the value of x that will produce a new area of 280 square feet, substitute the new area for the variable A in the function above.

$$270 + 3x - x^2 = 280$$
$$0 = x^2 - 3x + 10$$

The graph of the function $y = x^2 – 3x + 10$ never crosses the x-axis, so there are no roots.



This means that there are no values for x that will produce a garden area of exactly 280 square feet.

Another way to solve this would be to check the discriminant. This shows that there are no solutions for this equation. The value of $b^2 - 4ac$ is less than zero.

$b^2 - 4ac = (-3)^2 - 4(1)(10) = 9 - 40 = -31.$

3. To determine values of x for which the garden has an area of 266 square feet, one can set the area function equal to 266 as follows.

$$\begin{aligned} 270 + 3x - x^2 &= 266 \\ 0 &= x^2 - 3x - 4 \\ 0 &= (x-4)(x+1) \\ x &= 4, -1 \end{aligned}$$

Both values will produce an area of 266 square feet when substituted back into the original equation.

$$\begin{aligned} (15+x)(18-x) &= 266 \\ (15+4)(18-4) &= 266 \quad \text{and} \quad (15+-1)(18--1) = 266 \\ (19)(14) &= 266 \qquad\qquad\qquad (14)(19) = 266 \end{aligned}$$

However, the value of x cannot be negative because the problem stated that he wanted to reduce the length and increase the width, so only the value of 4 makes sense for this situation. The dimension change that produces an area of 266 square feet is and increase of 4 feet.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

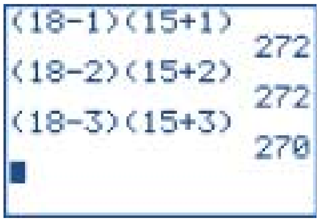
Extension Questions:

- What values of *x* will give Ken a garden with the dimensions of the original garden?

Ken's original garden was 18 feet by 15 feet. If the length and width are reduced and increased by the same amount (x), the value of x would have to be 3. If you reduce the 18 foot side by 3, it will result in a side of 15 feet. If you increase the 15 foot side by 3, it will produce a side of 18 feet. Therefore, the dimensions of the new garden will match that of the original garden.

- What values of *x* would produce a garden with maximum area?

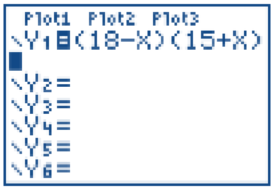
Using the area formula, the length and width can be decreased and increased by the same amount to find the largest area. Using the calculator, values of 1 foot and 2 feet produce the same and largest area.

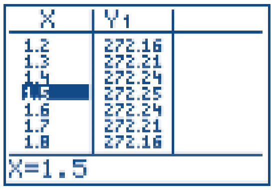


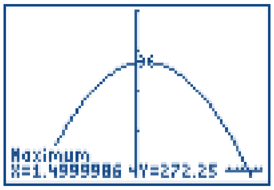
These values will produce dimensions of 16 feet by 17 feet. These whole number values are the closest to having the garden plot a square figure, which produces the maximum volume.

The area of the garden could be maximized more if decimal values are used. By increasing and decreasing the length by 1.5 feet, the area becomes 272.25 square feet.

The table and graph show this maximum value occurs when the increase is 1.5 feet.

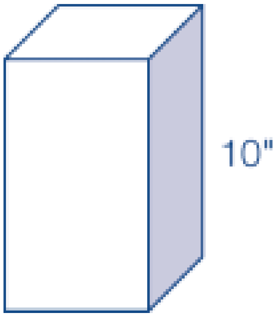






How Much Paint?

Emily has a can of paint that will cover 3800 square inches. She wants to build a small wooden box with a square base and a height of 10 inches. The paint will be used to finish the box.



1. Write a function to represent the total surface area of the box.

2. What equation will allow you to determine the dimensions of the box for which the surface area is 3800 square inches? Show how to solve the equation you wrote symbolically.

3. Describe how to solve the equation using a graph.

4. What is the measure of the base of the largest box Emily can build? Explain your answer.

Teacher Notes

Materials:
One graphing calculator per student.

Connection to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

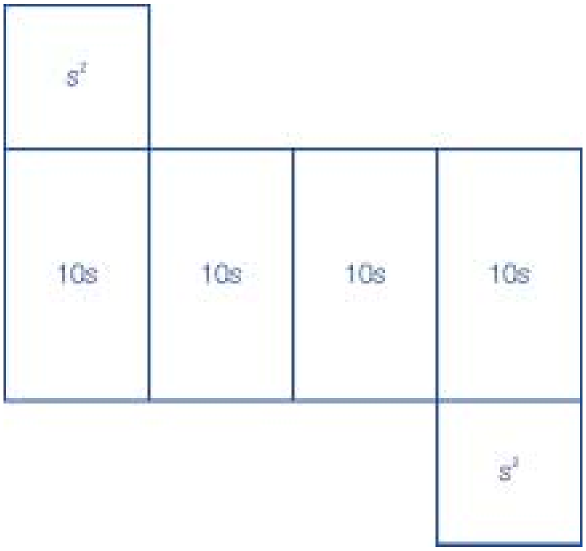
The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- How could you algebraically represent the area of each face of the box?
- How could you use the area of each face to help you represent the total area of the box?
- Describe how to use a graph to determine the number of roots.
- What are some different methods to solve for the side length?

Sample Solution:

1. Let s = the measure of a side of the square base. Then s^2 = the area of the square base. There are 2 square bases with area represented by $2s^2$.



The area of one rectangular side of the box is $10s$, using the formula of (base)(height). There are 4 sides on the box representing an area of $4(10s)$ or $40s$. The total area of the box equal $2s^2 + 40s$.
The function is $A(s) = 2s^2 + 40s$.

2. The total area of the box equal $2s^2 + 40s$. This is the area to be painted. The paint covers a total area of 3800 square feet. Solve the equation $2s^2 + 40s = 3800$.

Transform the equation into standard form:
$2s^2 + 40s - 3800 = 0$

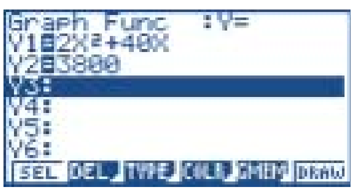
Use the quadratic formula with $a = 2$, $b = 40$, and $c = -3800$

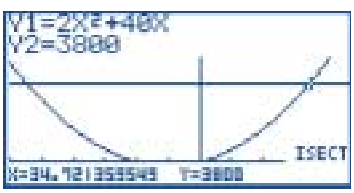
$$s = \frac{-40 \pm \sqrt{(40)^2 - 4(2)(-3800)}}{2(2)}$$

$s = 34.72$ inches or $s = -54.72$

There are two solutions, but only one makes sense for this situation. Since length cannot be negative, the measure of the side of the square base of the largest box should be about 34.72 inches.

3. Graph the function for the surface area, $y = 2x^2 + 40x$.
Graph the line $y = 3800$.





The graph shows the intersection points at approximately (–54.72,3800) and (34.72,3800). The x-value of the intersection points represents the possible lengths for the base edge on the box.

4. Since lengths cannot be negative, only x-values between 0 and 34.72 can be considered. The largest possible side length that will be within the paint coverage limit is 34.72 inches.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Extension Questions:

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

- Describe another graph that could be used to determine the measure of the base if 3800 square inches is to be covered.

The function $y = 2x^2 + 40x - 3800$ could be graphed. Determine for which values of x the function is 0.

- What is the meaning of this function $y = 2x^2 + 40x - 3800$ for the problem situation?

The rule represents the amount of the surface area less 3800 square feet. When the function value is zero, the surface area would be equal to 3800 square feet.

- How would the original function change if the height of the box had been 15 inches?

The function would be $A(s) = 2s^2 + 4(15)s$.

Insects in the Water

A biologist was interested in the number of insect larvae present in water samples as the temperature of the water varied. He collected the following data:

Temperature (C°)	0	10	20	30	40	50
Insect Population	20	620	950	920	670	75

1. Make a scatterplot of the data. Given that the value of b is 75, experiment with values for a and c in $y = ax^2 + bx + c$ to fit a quadratic function to your plot.

2. Write a verbal description of what the graph tells you about the insect population and the temperature of the water samples. What do the intercepts mean? When is the insect population greatest?

3. The water sample is considered to be mildly contaminated and does not need to be treated if the insect population is no more than 300. For what temperatures will this occur? Explain.

4. Suppose at 0° C, testing showed virtually no larvae present, and the model for this situation is the function $y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

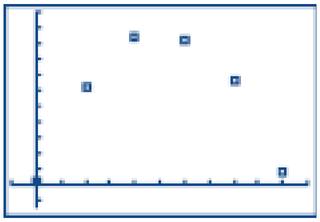
(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- How does the data in the table help you determine a reasonable window for your plot?
- What does the table tell you is a reasonable value for *c*? What does the shape of the graph tell you about the value of *a*?
- What is the function that most closely models this scatterplot?
- Experiment with values of *a* between -1 and -2.
- How can you use the graph of $y = 300$ to help you answer Question 3?

Sample Solution:

1. Scatterplot of population vs. water temperature:

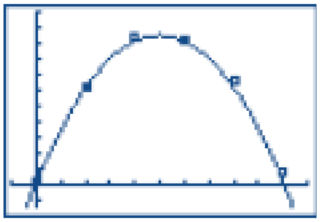


Since $y = 20$ when $x = 0$, the value for *c* in $y = ax^2 + 75x + c$ is 20. Since the scatterplot shows the larvae population increasing and decreasing, *a* must be negative.

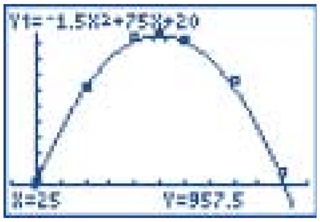
Graphing $y = -1x^2 + 75x + 20$ gives a parabola that opens wider than the plot appears and whose maximum value occurs at a greater temperature, 37.5º C, than the temperature that appears to give the maximum population for the scatterplot, between 10º C and 30º C.

Graphing $y = -2x^2 + 75x + 20$ gives a parabola that opens narrower than the plot appears and whose maximum value occurs at 18.75º C, which is too low.

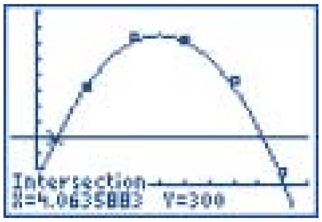
Trying a value close to $a = -1.5$, we find that a good fitting quadratic is $y = -1.5x^2 + 75x + 20$.

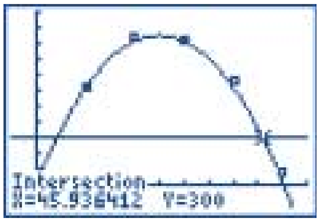


2. The *y*-intercept of the graph shows that at 0º C, there are 20 insect larvae in the water sample. As the temperature increases to 25º C, the population increases to 957 insect larvae in the water sample. This is shown by finding the coordinates of the vertex, (25,957.5), using the graph. Then the population decreases to 0 at about 50º C, the right *x*-intercept.



3. By graphing y = 300 along with the population graph and finding the points of intersection, we can determine the temperatures when the population is no more than 300.





(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$; and

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships
1.2 Transformations

III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

The graph shows that for temperatures up to 4º C, the population is no more than 300 insect larvae. Since this point is 21 units to the left of the axis of symmetry, $x = 25$, the other intersection point is 21 units to the right of the axis of symmetry, and, therefore, is (46,300). When the temperature is between 46º C and 50º C, the population will again be no more than 300 insect larvae.

4. Since the first model is in polynomial form and the second model is factored form, rewrite the second model in polynomial form:

$$y = -1.5x(x - 50)$$
$$= -1.5x^2 + 75x$$

Finding the x-intercepts and vertex of the graph either by calculator or analytically, we find that the second model has x-intercepts (0,0) and (50,0) and vertex (25,937.5). It is a translation of the first model, $y = -1.5x^2 + 75x + 20$, down 20 units. The first model accounted for the 20 larvae observed to be present at 0º C. Compared to the first model, the second model underestimates the number of insect larvae present at any temperature by 20 larvae.

Extension Questions:

- What trends in the table and the graph tell you what is happening with the insect larvae in the water samples?

The table shows that as the temperature increases to 20º C, the larvae population increases to 950 and then decreases to 75 at 50º C. The graph gives a more accurate picture. Since it is a parabola opening downward, the maximum number of larvae occurs at the vertex, which is when the temperature of the water is 25º C. The table shows the larvae population decreasing to 75 at 50º C. The graph shows the population decreasing to 20 at 50º C and no larvae present at a fraction of a degree hotter than 50º C.

- How can you graphically investigate when the insect larvae population is no more than 300?

"No more than 300" means "less than or equal to 300." By graphing the line $y = 300$, we can find the portion of the larvae population graph that lies below the line graph, including the intersection points. We can do this using calculator features such as trace or intersect.

- Suppose y = -1.5x(x – 50) + 20 is considered a usable model for predicting the amount of insect larvae present in water samples, and a second round of experiments shows that the population at each of the previous temperatures in the table doubles. How will this affect the scatterplot and the function to model this new scatterplot?

The new table will be

Temperature (C)	0	10	20	30	40	50
Insect Population	40	1240	1900	1840	1340	150

The original scatterplot will be stretched vertically by a factor of 2 since the larvae population doubles. All of the y values from the original function are multiplied by 2.

The new function will be $y = 2[-1.5x(x - 50) + 20]$ *or* $y = -3x(x - 50) + 40$.

- How does the new function in polynomial form compare with the original one?

The coefficients and the constant in the new function are twice those of the original function.

The original is $y = -1.5x(x - 50) + 20 = -1.5x^2 + 75x + 20$.
The new function is $y = -3x(x - 50) + 40 = -3x^2 + 150x + 40$.