Chapter 6:

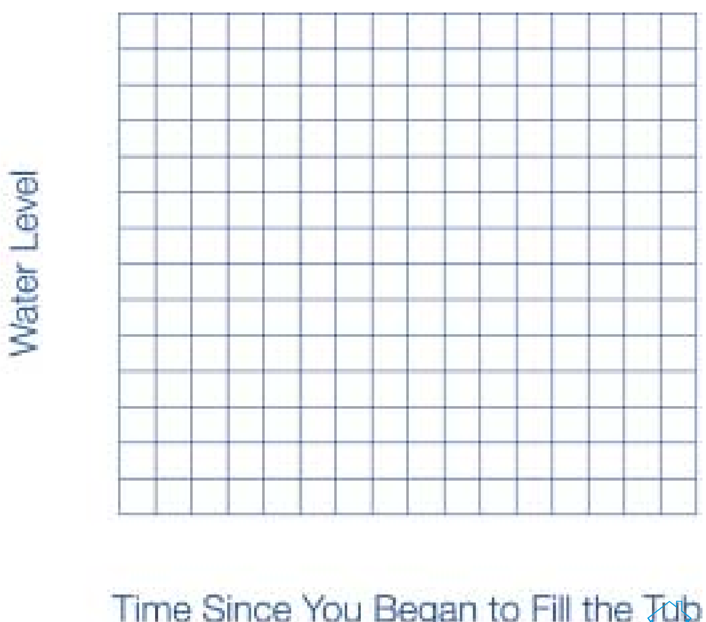
Function Fundamentals

Bathing the Dog

It's time for Shadow, your German Shepherd, to get a winter bath. Shadow does not enjoy getting a bath! You fill the bathtub halfway full, put Shadow in the tub, and begin to bathe her. Shadow tries to escape and gets halfway out of the tub. You pull her back into the tub and finish the bath. You get her out and then drain the tub. Assume a lengthwise cross section of the tub is trapezoidal with the tub sides nearly vertical.

1. Describe how the water level in the tub will vary before, during, and after Shadow's bath.

2. Sketch a graph of your description. Clearly label significant points on the graph.



Teacher Notes

Materials:
None required.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships; and

(E) interprets and makes inferences from functional relationships.

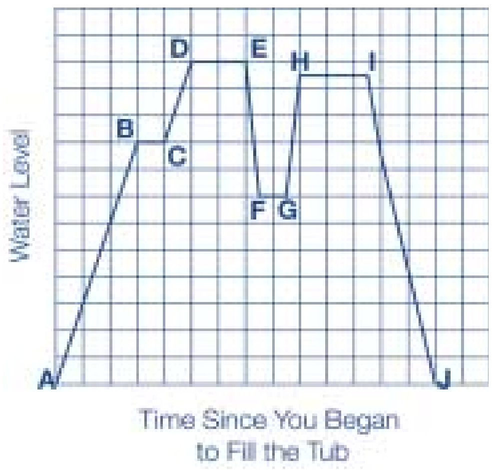
(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

Scaffolding Questions:

- What is the water level when the tub starts to fill?
- How will the shape of the tub affect how the water level changes as the tub is filled?
- What happens to the water level when you put Shadow into the tub?
- What happens to the water level if Shadow climbs halfway out of the tub?
- What happens to the water level when you get Shadow completely back into the tub?
- What happens to the water level when you finish bathing Shadow and she gets out?
- How will the shape of the tub affect how the water level changes as the tub is drained?

Sample Solution:



First, the water level starts at (0,0) because the tub initially is empty. It rises at a nearly constant rate because the tub walls are nearly vertical (A to B on graph). In other words, the tub is basically a cylinder with an oval base. The water level stays constant for a moment while you go get Shadow (B to C on graph).

Second, the water level jumps up suddenly when you put Shadow into the tub (C to D on graph) because her body mass displaces water. Also, since she does not like her bath, she thrashes around, (D to E on graph).

Third, Shadow climbs halfway out of the tub for a moment, trying to escape her bath. This makes the water level drop (E to F on graph). Although she is halfway out of the tub, you continue to bathe her (F to G on graph).

Fourth, you get Shadow back into the tub so the water level jumps back up, but not as high, because by now some of the water has splashed out of the tub (G to H on graph). You finish bathing her (H to I on graph).

Finally, Shadow's bath is done. She gets out, and you pull the plug. As the tub drains, the water level drops at a nearly constant rate (I to J on graph).

Extension Questions:

- Is it possible to write a function rule to describe this situation? Why or why not?

It would be extremely difficult to write a function rule for this situation. The graph would need to be described by a different function for each phase of the graph. Also, the graph would change the next time Shadow gets a bath. The graph depends on a number of factors. How fast is the tub filling? Is the tub filling at a constant rate? How long will Shadow stay in the tub before she tries to climb out?

Data for one bath could be collected and a function fit to each of the various stages. However, the function for each stage would change with the next bath that Shadow gets!

- How would the graph change if the sides of the bathtub were graduated, that is if the sides of the bath tub widen more from tub bottom to top?

As the tub fills, the water level would rise more slowly. The graph would represent an increasing function whose graph is concave down. As the tub empties, the water level would drop more rapidly. The graph would represent a decreasing function that is concave down.

- When a dog enjoys getting a bath, the dog does not try to get out of the tub or thrash around. How would the graph change if Shadow enjoyed getting a bath?

The portion of the graph when Shadow is in the tub would be nearly horizontal because the water level would not change very much.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations of Functions
1 Developing Mathematical Models
1.1 Variables and Functions

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

- Explain the functional representations you used in describing this situation.

A verbal description was used to describe how the water level in the tub was related to the time (and events) for the bath of Shadow. A graph was used to describe how the water level in the tub varied as time during the bath elapsed. The water level is a function of the time needed for the bath. Thus, the water level is the dependent variable and time is the independent variable.

Distance and Time

The graph below represents distance as a function of time.

Create a situation this graph might represent. Choose appropriate units for time and distance. Describe your situation in detail.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Describe some situations in which the variables could be distance and time.
- For the situation you choose, what are reasonable units for time and distance?
- Describe how to break the graph up into phases.
- What do the graphs in the phases show you in terms of the function increasing, decreasing, or being constant? What does this mean in your situation?
- Can you determine the slope of the graph in each phase? What will this mean in your situation?
- What are the *x*- and *y*-intercepts for the graph? What do they mean in the situation you chose?

Sample Solution:

A wind blows a leaf off a tree branch about 8 feet above the ground. The wind swirls the leaf upwards at a constant rate of 2 feet per second for one second. Now the leaf is 12 feet above the ground. The wind slows down. The leaf swirls upwards at a constant rate of 1 foot per second, reaching a height of 15 feet. From 5 seconds to 10 seconds the wind subsides. The leaf falls at a steady rate of 2 feet per second to 5 feet above the ground and lands on another tree branch. It stays on the branch for two seconds until a slight breeze catches the leaf and it falls to the ground at a steady rate of 5 feet per 2 seconds. The leaf's journey from tree branch to ground lasted 14 seconds.

Extension Questions:

- Consider the phases: ① 0 to 2, ② 2 to 5, ③ 5 to 10, ④ 10 to 12, and ⑤ 12 to 14. In which phases is the function increasing? Decreasing? Constant? What does this tell you about the slope of each phase?

The function is increasing in phases ① and ②, so the line slopes upward, and the rate of travel is positive.

The function is decreasing in phases ③ and ⑤, so the line slopes downward, and the rate is negative.

In phase ④ the distance remains constant; the line is horizontal; the rate of travel is 0 feet per second.

- What does the slope of each phase mean in the situation you created?

Since it equals change in distance between two points divided by change in time for the two points, it represents velocity or speed and direction at which the object travels.

- How would you interpret this graph if the dependent quantity was velocity instead of distance?

In Phase ①, an object is moving with constant positive speed. In phase ②, the velocity is still steady but slower than before. In phase ③, the distance is now decreasing at a steady rate; the velocity is a negative number. In phase ④, the velocity is zero because the object does not move. Finally, in phase ⑤, the distance decreases; the velocity is negative.

- If the first phase on the graph had been from the point (0,12) to the point (2,12), how would that change your description of the graph?

The object was still for the first two seconds before it started to move because the distance remained constant.

- Take the information from the graph and create a graph of the velocity of the object as a function of time. The velocity is the speed at which the object traveled.

The velocity can be found for each phase.

Time Interval	Velocity
0 to 2	2
2 to 5	1
5 to 10	-2
10 to 12	0
12 to 14	-2.5

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns
3 Interpreting Graphs
  3.1 Interpreting Distance versus Time Graphs

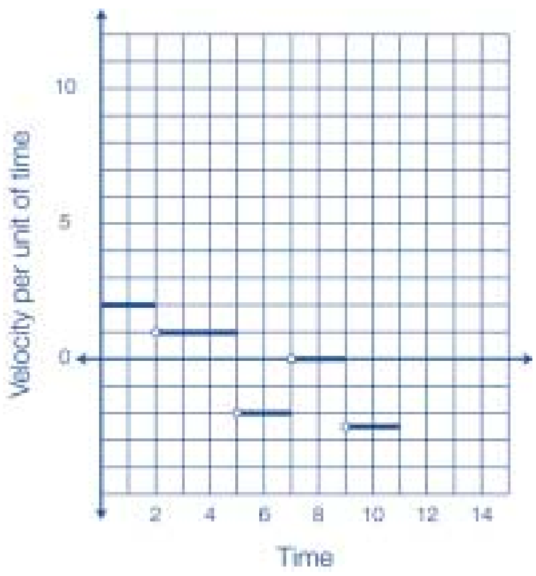
II. Linear Functions
1 Linear Functions
  1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

The graph is a series of horizontal line segments. There is an open circle at one end to indicate that a value of x may not have two function values.



Extracurricular Activities

Determine a function that represents each of the following situations. Describe the mathematical domain and range of the function and a reasonable domain and range for the situation. Use a table, graph, picture or other representation to explain your choice of domain.

A. You join a fitness club that charges an annual fee of $150 plus a facilities-use fee of $3.50 per visit. Your total annual cost is a function of the number of times you visit the club.

B. Over the past summer, you earned $500 by providing in-home pet care service to your neighbors when they went on vacation. You deposited this money in your savings account. Because you are an aspiring musician, you decide to join a CD music club. Their introductory offer allows you to order up to $325 on your first order with a minimum monthly payment of $25.78. No interest will be charged if you make your monthly payments on time and order no other CDs before paying off your first order. Your initial order totals $322.25. Your music club balance is a function of the number of months you pay on the balance.

C. A hiker walking across a desert mesa arrives at the edge just in time to see a hot-air balloon launched from the desert floor, 100 meters below the mesa. The balloon rises at a steady rate of 10 meters per second and can cruise at a maximum height of 300 meters above the mesa. The balloon's vertical distance from the mesa, as it rises to cruising altitude, is a function of the number of seconds that have passed since it was launched.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

For each situation,

- What are the constants?
- What is the dependent variable?
- What is the independent variable?
- In situation A, if you made 5 trips to the fitness club, how would you compute the cost?
- In situation B, as the number of payments increases, what happened to the balance?
- What type of function (linear, quadratic, exponential, inverse variation) relates the variables? How would you determine this?
- What restrictions does the function place on the independent variable?
- Should you use all real numbers for the domain? Why or why not?
- What representation would best help you see the domain and range?

Sample Solution:

A. The cost is equal to $150 plus $3.50 times the number of visits. The function rule is $f(n) = 150 + 3.5n$, where n = the number of visits during the year and $f(n)$ = the total annual cost.

The mathematical domain and range for this function are both the set of all real numbers because the function is a nonconstant, linear function.

A table of values helps show the domain and range that make sense for the situation.

Number of visits, n	Annual cost (dollars)
0	150
1	153.50
2	157
3	160.50
4	164
...	...
n	$150 + 3.50n$

The number of visits to the fitness center must be a whole number. Since the function value, *f*(*n*), gives cost with initial value of $150 and $3.50 per visit, *f*(*n*) must be 150 plus whole number multiples of $3.50. There must be a maximum number of visits, *n*, depending on how many times you can go in a year.

To summarize, the domain for the situation is {0, 1, 2, 3, 4,..., *n*} where *n* is the maximum number of visits per year, and the range is {150, 153.5, 157, 160.5,..., 150+3.5*n*}. Both the domain and the range are finite sets, limited by the number of visits you make in a year. The graph for the situation would show this as an increasing, dotted plot.

B. The music club balance is $322.25 minus 25.78 times the number of payments that you have made. The function rule is $c(n) = 322.25 - 25.78n$, where n = the number of months you pay on your music club balance and $c(n)$ = your music club balance. Since this is a nonconstant, linear function, the mathematical domain and range for this function are both all real numbers.

The domain values for the problem situation must be whole numbers. The number of payments must be a whole number. Suppose you make no additional charges on your music club account, and you make a payment of $25.78 each month.

The table below helps describe the domain and range that make sense for the situation.

Number of Payments	Music Club Balance
0	322.25
1	296.47
2	270.69
3	244.91
...	...
11	38.67
12	12.89

Your music club balance "zeroes out" at the thirteenth month, when you pay the final balance of $12.89.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

To summarize, the domain for the situation is the set {0, 1, 2, 3,..., 13}, and the range is the set {322.25, 296.47, 270.69,..., 12.89, 0}. Both domain and range are finite sets. The graph of the situation consists of only 14 points plotted in the coordinate plane. The graph of the mathematical function is a line with an infinite number of points.

C. The initial vertical distance from the balloon to the mesa at the time of the launch can be represented by -100. The vertical distance is increasing at a rate of 10 meters per second. The height of the balloon is the starting vertical distance plus 10 times the number of seconds that have elapsed since the launch.

$h(t) = -100 + 10t$, where t = time in seconds into launch and
$h(t)$ = height in meters of the balloon with respect to the mesa

Both the mathematical domain and range for the function are all real numbers since this is a linear function.

The domain for the situation, time t in seconds into launch, is the set of all t values where $0 \leq t \leq 40$. The range for the situation, height in meters into launch, is the set of all values $h(t)$ where $-100 \leq h(t) \leq 300$.

The domain is time and changes continuously from 0 seconds to 40 seconds because the balloon starts at time 0, and it takes 40 seconds to reach the altitude of 300 meters. The range is distance and changes continuously from -100 meters to 300 meters. The graph of the situation will be a line segment instead of a set of discrete points.

Extension Questions:

- What kind of function is needed to model each of the situations?

Each situation is modeled by a linear function since each situation involves a constant rate of change.

- What is the parent function for these functions? How does knowing the parent function for these functions help you determine the mathematical domain and range of the function for each of these situations?

The parent function is $y = x$. The domain and range of the parent function are the set of all real numbers. The only special case is a linear function that is *constant, which restricts the range to a single value. Multiplying x by m and adding b to get y = mx + b does not change the domain (set of x - values) or the range (set of y - values). It just changes the graph of the function in terms of where it crosses the y - axis and its slope.*

- In Situation B, describe how the domain and range would change if you changed the rate in the problem but not the initial value.

If you increase the rate at which you pay the balance, the domain and range will shorten, since you will pay off the credit card balance faster. If you decrease the rate, that is if you pay less each month, the domain and range will lengthen; it will take longer to pay off the credit card debt.

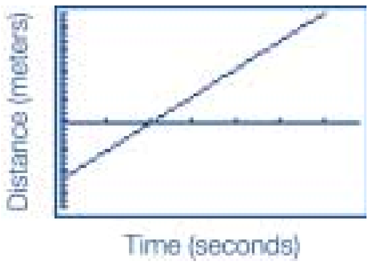
- In Situation C, describe how the domain and range will change if the launch altitude and cruising altitude of the balloon changed. Call this the "launch to cruise distance."

If the "launch to cruise distance" increases, the domain will be a longer time interval and the range will be a longer distance interval. If the "launch to cruse distance" decreases, the domain will be a shorter time interval and the range will be a shorter interval.

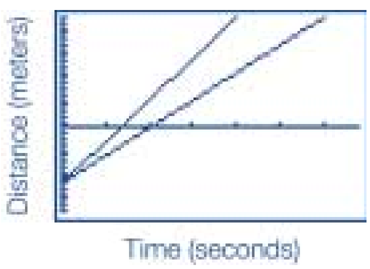
- In Situation C, describe how the domain and range will change if the launch to cruise distance remains the same but the balloon rises at a different rate.

If the balloon rises faster, it will take less time to cover the same distance. This means the domain will be a shorter time interval. The range interval will be the same. If the balloon rises more slowly, it will take more time to cover the same distance. This means the domain will be a longer interval of time, but the range interval will be the same. The graphs of these situations would look like this:

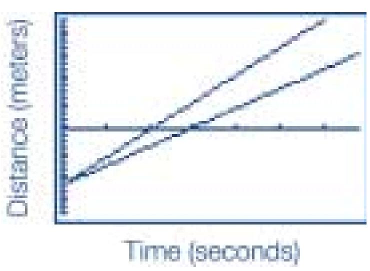
The original situation:



Faster rate:



Slower rate:



Making Stuffed Animals

Pocket sized stuffed animals cost $20 per box (100 animals) plus $2,300 in fixed costs to make them.

1. What does it cost to make 200 boxes of stuffed animals? 250 boxes? 320 boxes?
2. How many boxes of stuffed animals can be made with $5,000? $50,000? Explain how you found your solution.
3. How can you use a graph to predict the cost of 425 boxes?

Teacher Notes

Materials:
One graphing calculator per student.
One sheet of graph paper per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the variables?
- Identify the dependent variable.
- Identify the independent variable.
- What does the $2,300 fixed cost mean?
- What is the cost of one box of stuffed animals?
- What is the cost of three boxes of stuffed animals?

Sample Solution:

1. Compute the cost of the boxes and put the values in a table.

Number of Boxes	Cost of Boxes (dollars)
0	2,300
100	4,300
200	6,300
300	8,300
400	10,300
500	12,300

Every 100 boxes has an additional cost of $2,000 so 50 boxes would have an additional cost of $1,000. Use the table to find that 200 boxes cost $6,300 then add $1,000 to find the cost of 250 boxes. The cost of 250 boxes is $7,300. To find the cost of 320 boxes use the fact that 50 boxes cost an additional $1,000; so 10 boxes would have cost $200. The cost of 320 boxes would be 8300 + 200 + 200 = 8700.

2 The cost is 2300 plus 20 times the number of boxes. The function is

$c = 2300 + 20b$, where c is the cost in dollars and b is the number of boxes.

$$\begin{aligned} 5000 &= 2300 + 20b \\ 2700 &= 20b \\ 135 &= b \end{aligned} \qquad \begin{aligned} 50000 &= 2300 + 20b \\ 47700 &= 20b \\ 2385 &= b \end{aligned}$$

3. Graph the line *y* = 2300 + 20*x*. Set an appropriate viewing window. Trace along the line to find the value of *y* when *x* is 425.

The cost of 425 boxes is $10,800.

Extension Questions:

- How are the graphs of the problem situation and the graph of the function rule different?

The graph of the problem situation is a set of points with x values that are counting numbers. The number of boxes may not be negative or a fraction. The graph of the function rule is a line.

- How does the $2,300 fixed cost affect the graph?

The $ 2,300 is the cost of zero boxes. It is the point (0,2300) on the graph line. However, it would not be a point on the graph of the situation. You would not purchase zero boxes.

- What limits the domain in this situation?

The domain values must be counting numbers, because the stuffed animals are sold by the whole box.

- What makes a relationship linear?

There is a constant increase or decrease in how one variable is related to the other variable. That is there is a constant rate of change.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 The Linear Parent Function
1.2 Y-Intercept

- Create an example of another linear relationship.

Linear situation:
If Jack makes $7.50 per hour, the amount of money Jack makes depends on the number of hours he works.

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

- Create an example of a nonlinear relationship.

Nonlinear situation:
If Jack is driving on a highway at varying speeds, the number of miles traveled depends on the time.

Student Work

Making Stuffed Animals

B = number of Boxes, T = Total Cost

The base cost is $2,300 so you have to add that to other work. However many boxes you make they cost $20 each for however many boxes you make.

Formula: 20B + 2300 = T

① 20B + 2300 = T Start with the formula

20(200) + 2300 = T Substitute B with 200 (the number of boxes)

4000 + 2300 = T Do the Multiplication and the Addition

$6300 of cost = T You will get the Total cost

②a. 20B + 2300 = T Start with the formula

20B + 2300 = 5000 Substitute

20B = 2700 Subtract 2300 from both sides then divide by 20

B = 135 boxes Find the number of boxes

b. 20B + 2300 = T Start with the Formula

20B + 2300 = 50000 Substitute

20B = 47700 Subtract 20 from both sides

B = 2385 boxes Divide by 20

③ This problem involves a linear relationship

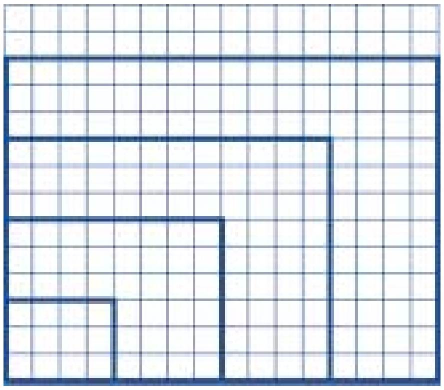
You can tell that it does from a table because the relationship between the numbers is going up at a constant rate.

You can tell from a graph because the line goes up at a constant rate and is using a formula to graph it.

From a function rule you can tell it is a linear relationship due to the corresponding rule and it forms a line relationship

④ I could use a graph to find the cost of 425 boxes by graphing the function. Once you have the graph, find 425 on the x-axis and go up to the line and once you reach the line go across to the y-axis and you will find how much it costs.

Nested Rectangles



A set of similar rectangles has been placed on a grid.

1. Write a function rule that shows how the width of a rectangle depends on its length. Consider the length to be the measure of the horizontal side and the width to be the measure of the vertical side.

2. Is this a proportional relationship? Explain how you know.

3. Could a rectangle with dimensions 10 units by 8 units belong to this set? Justify your answer using two different methods.

4. Name four other rectangles that would belong to this set.

5. Describe verbally, symbolically, and graphically the relationship between the length of the rectangle and the perimeter of the rectangle.

6. Describe verbally, symbolically, and graphically the relationship between the length of the rectangle and the area of the rectangle. Compare this relationship to the relationship between length and perimeter. Is either of these relationships a direct variation?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- What are the length and width of the smallest rectangle?
- What is the relationship between these two numbers?
- What are the length and width of the second rectangle?
- What is the relationship between these two numbers?
- How could you organize the information you are collecting?
- What conditions must occur if there is a proportional relationship?
- How do you find the perimeter of the rectangle?
- In question 5 what is the dependency relationship?
- What are the variables to be considered in question 5?

Sample Solution:

1. Count the length and the width of each rectangle and record the information in a table.

Length	Width
4	3
8	6
12	9
16	12

The ratio of the width to the length is 3:4.

$$\frac{w}{l} = \frac{3}{4}$$

$$w = \frac{3}{4}l$$

The other rectangles also satisfy this relationship.

Length	Process	Width
4	$\frac{3}{4}(4)$	3
8	$\frac{3}{4}(8)$	6
12	$\frac{3}{4}(12)$	9
16	$\frac{3}{4}(16)$	12

The relationship between the length and the width is that $w = \frac{3}{4}l$.

2. The relationship is a proportional relationship (a direct variation) because the equation is of the form $y = kx$ where k is a constant. The ratio of the length to the width in any given rectangle is a constant.

3. Check the values of 10 for the length and 7.5 for the width in the equation.

$$\frac{3}{4}(10) = 7.5$$

The rectangle with measurements 10 units and 8 units would not belong to this set. If the length is 10 units, the width must be 7.5 units.

Another approach is to ask if the two ratios are equal.

$$\frac{8}{10} \neq \frac{3}{4}$$

Therefore, this rectangle would not be similar to the given rectangles.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(G) relates direct variation to linear functions and solves problems involving proportional change.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

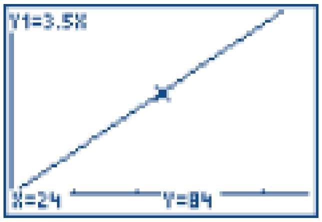
4. Possible answers are shown in the table.

Length	Process	Width
5	$\frac{3}{4}(5)$	3.75
9	$\frac{3}{4}(9)$	6.75
13	$\frac{3}{4}(13)$	9.75
14	$\frac{3}{4}(14)$	10.5

5. The perimeter is twice the length plus twice the width.

$$p = 2l + 2w$$
$$w = \frac{3}{4}l$$
$$p = 2l + 2\left(\frac{3}{4}l\right) = 2l + \frac{3}{2}l = \frac{7}{2}l$$
$$p = \frac{7}{2}l$$

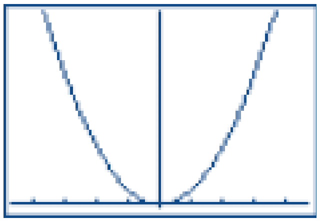
The perimeter is three and one-half times the length.
The graph of the function is a straight line.



6. The area of the rectangle is the product of the length and the width.

$$A = lw$$
$$w = \frac{3}{4}l$$
$$A = l\left(\frac{3}{4}l\right) = \frac{3}{4}l^2$$
$$A = \frac{3}{4}l^2$$

The area is three-fourths of the length squared.
The graph is a parabola with a vertex at the origin.



The portion of the parabola that makes sense in this situation is the portion in quadrant I.

The relationship for length and perimeter is linear, but this relationship between length and area is not linear. The length and perimeter relationship is a direct variation because it is of the form $y = kx$. Its graph is a line that passes through the origin.

Extension Questions:

- What restrictions must be placed on the domain of the function $w = \frac{3}{4}l$ for this problem situation?

The length may be any positive real number. The measurement of a side of a rectangle may not be negative or zero.

- If a rectangle, similar to the original rectangles, is created by doubling the length of one of the rectangles, how is the perimeter of the new rectangle related to the perimeter of the original rectangle.

The perimeter of the original rectangle is given by the formula

$$p = \frac{7}{2} l$$

The perimeter of the new rectangle would be

$$P = \frac{7}{2}(2l)$$

$$P = 2\left(\frac{7}{2} l\right) = 2p$$

The perimeter of the new rectangle would be twice the perimeter of the original rectangle.

- If a new similar rectangle is created by doubling the length of one of the rectangles, how is the area of the new rectangle related to the area of the original rectangle.

The area of the rectangle in this set is given by the formula

$$A = \frac{3}{4} l^2$$

If the length is doubled, the formula becomes

$$\text{New Area} = \frac{3}{4}(2l)^2$$

$$\text{New Area} = \frac{3}{4}(2^2 l^2)$$

$$\text{New Area} = \frac{3}{4} \cdot 4l^2$$

$$\text{New Area} = 4\left(\frac{3}{4} l^2\right)$$

The new area is 4 times the area of the original rectangle.