

SUPPLEMENTAL

Algebra Assessments

Chapter 7:

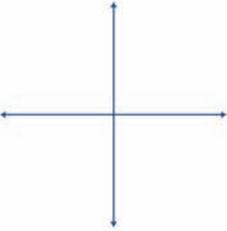
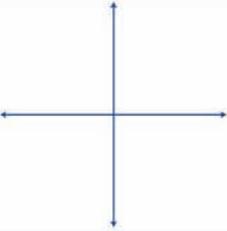
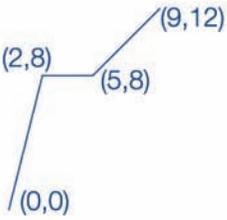
*Linear Functions, Equations,
and Inequalities*





Analysis of a Function

- I. If given a function, sketch a complete graph. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.
- II. Describe the domain and range for each mathematical situation. Explain your thinking.

Function	Graph or Table	Domain and Range										
1. $f(x) = 5 - 2x$		Domain: Range:										
2. $y = -2$		Domain: Range:										
3.	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>6</td> <td>6</td> </tr> <tr> <td>12</td> <td>9</td> </tr> </tbody> </table>	x	y	-2	2	2	4	6	6	12	9	Domain: Range:
x	y											
-2	2											
2	4											
6	6											
12	9											
4.		Domain: Range:										



- III. Write a summary of the functions, comparing their domains and ranges and their graphs.
- IV. Describe a practical situation that each of these functions might represent. What restrictions will the situation make on the mathematical domain and range of the function? How will the situation affect the graph of the mathematical function?



Teacher Notes

Scaffolding Questions:

- What type of function relates the variables?
- What is the dependent variable? What is the independent variable? How do you know?
- What are the constants in the function? What do they mean?
- What restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.



Sample Solution:

I & II:

(b.2) Foundations for functions.

The student uses the properties and attributes of functions.

The student:

(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

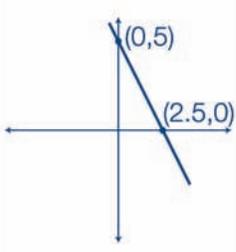
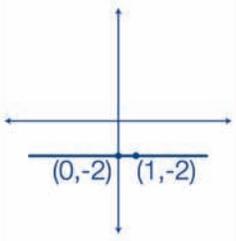
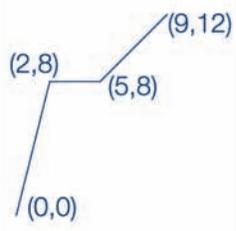
(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Function	Graph or Table	Domain and Range										
1. $f(x) = 5 - 2x$		<p>Domain: The domain is the set of all real numbers because $5 - 2x$ is defined for any value of x.</p> <p>Range: The range is the set of all real numbers since any number can be generated by $5 - 2x$.</p>										
2. $y = -2$		<p>Domain: The domain is the set of all real numbers because the function $y = -2$ means "y is equal to -2 no matter what x is." This is a constant function.</p> <p>Range: y is always -2, thus the range is just the number -2.</p>										
3. The table shows a constant rate of change of $\frac{1}{2}$ so this is a linear function. The y-intercept is (0,3). The function that models this set of points is $y = \frac{1}{2}x + 3$.	<table border="1" data-bbox="852 1270 982 1459"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>6</td> <td>6</td> </tr> <tr> <td>12</td> <td>9</td> </tr> </tbody> </table>	x	y	-2	2	2	4	6	6	12	9	<p>Domain: The domain is the set of given x values $\{-2, 2, 6, 12\}$.</p> <p>Range: The range is the set of y values $\{2, 4, 6, 9\}$.</p> <p>The domain and range of the function that models this data are both the set of all real numbers.</p>
x	y											
-2	2											
2	4											
6	6											
12	9											
4. <ul style="list-style-type: none"> If $0 \leq x \leq 2$, then the function is $y = 4x$. If $2 < x \leq 5$, then the function is $y = 8$. If $5 < x \leq 9$, then the function is $y = x + 3$. 		<p>Domain: The domain is the set of all real numbers x, $0 \leq x \leq 9$, since this is what the graph shows.</p> <p>Range: The range is the set of all real numbers, y, $0 \leq y \leq 12$, by the same reasoning.</p>										



III. Summary of the functions:

The functions in problems 1, 2, and 3 are linear functions, having the form $y = mx + b$. They all have as their domains the set of all real numbers because the expression for each function is never undefined. The functions in 1 and 3 have as their ranges the set of all real numbers because every real number can be generated by the expressions for those functions. The function in problem 2 has as its range the single number -2 because it is a constant function.

The graphs of the functions in problems 1, 2, and 3 are lines. The graph of the function in problem 1 has a y -intercept of (0,5) and an x -intercept of (2.5,0). The line “falls” from left to right because the slope is negative. This is a decreasing function. The graph of the function in problem 2 has a y -intercept of (0,-2) and no x -intercept. It is a horizontal line with slope zero. This is a constant function.

The graph of the function that models the data given in problem 3 has y -intercept (0,3) and an x -intercept of (-3,0). The line “rises” from left to right because the slope is positive. This is an increasing function.

The graph of the function in problem 4 consists of three linear segments, and so it requires three different functions to describe it.

For the first piece, $0 \leq x \leq 2$, and the graph is the corresponding part of the line with y -intercept (0,0) and slope $\frac{8-0}{2-0} = 4$. $y = 4x$. This graph “rises,” terminating at the point (2,8).

For the second piece, $2 < x \leq 5$, and the graph is a horizontal segment (slope = 0) terminating at (5,8). $y = 2$.

For the third piece, $5 < x \leq 9$, and the graph is the corresponding part of the line with slope $\frac{12-8}{9-5}$. The equation is of the form $y = 1x + b$.

Use the point (9,12). Substitute 9 for x and 12 for y .

$$12 = 1(9) + b. \quad b = 3$$
$$y = 1x + 3.$$

This graph starts at (5,8) and “rises.” The graph of the first piece is steeper than the graph of the third piece, because the slope of the first piece is greater than the slope of the third piece.

Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

2 Using Patterns to Identify Relationships

2.1 Identifying Patterns

3 Interpreting Graphs

3.1 Interpreting Distance versus Time Graphs

3.2 Interpreting Velocity versus Time Graphs

II. Linear Functions

1 Linear Functions

1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:

The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:

The student will graph problems involving real-world and mathematical situations.



IV. Description of a practical situation for each function:

The function $y = 5 - 2x$ could represent a small toy race car starting 5 feet away from the finish line and moving forward at 2 feet per second, where y = the distance in feet between the toy car and the finish line and x = the time in seconds the car has been moving. For this situation, the domain is the set of all numbers x , $0 \leq x \leq 2.5$ representing the time to start and complete the race. The range would be the set of all numbers y , $0 \leq y \leq 5$ representing the range of distance traveled by the car. The graph would simply be the segment from $(0,5)$ to $(2.5,0)$.

The function $y = -2$ could represent an ocean diver in the waters near a beach in Hawaii. The diver is swimming at a constant rate and is 2 meters below the water surface. For this situation, the domain is time, x in minutes, that the swimmer is at this depth. For example, the domain could be the set of all numbers x , $0 \leq x \leq 15$ and the range is $y = 2$. The graph would be a horizontal segment from $(0,-2)$ to $(15,-2)$.

The function $y = \frac{1}{2}x + 3$ could represent the allowance that a very young child gets each week. The parent puts \$3 in the child's piggy bank to start the child saving. Each week, the child gets a 50 cent allowance and adds it to the piggy bank. The child has been told that if he saves his allowance each week for 6 months, then he will get an increase. For this situation, the domain is the set of all values x , $x = 0, 1, 2, 3, \dots, 24$, because there will be roughly 24 weeks in the six-month period. The range will be the set of all values y , $y = 3, 3.5, 4, 4.5, \dots, 15$. The graph will be a discrete graph because it will simply be a plot of a set of 25 points.

The fourth function is the function consisting of three linear pieces. These could be defined as:

1. If $0 \leq x \leq 2$, $y = 4x$.
2. If $2 < x \leq 5$, $y = 8$.
3. If $5 < x \leq 9$, $y = x + 3$.

This function could represent a student's pace on a reading assignment. During the first two minutes, the student rapidly reads 8 short paragraphs of the 12 he has to read. He stops reading for 3 minutes so that he can reflect on what he just read. He realizes that he needs to read the remaining 4 paragraphs more carefully, so he finishes the remaining paragraphs at a slower rate. The domain for this situation is the time, x in minutes, that it takes the student to complete the reading assignment.



This will be the time interval from 0 minutes to 9 minutes. The range will be the amount of text the student has read during this time interval, 0 paragraphs to 12 paragraphs. The graph will be connected line segments: from (0,0) to (2,8), from (2,8) to (5,8), and from (5,8) to (9,12).

Extension Questions:

- For problems 1, 2, and 3, determine the equation of a line perpendicular to each of the given lines and having the same y-intercept.

If a function is not a horizontal line, find the slope of the line and determine the opposite reciprocal of this slope. If the line is a horizontal line, the perpendicular line will have undefined slope.

In problem one, the slope is -2; the slope of a perpendicular line would be $\frac{1}{2}$.

The equation of the line is $y = 5 + \frac{1}{2}x$.

In problem 2, the line is horizontal with y-intercept -2. The perpendicular line will be a vertical line. The slope of a vertical line is undefined. The line is of the form x equals a constant. Any vertical line would be perpendicular to $y = -2$. The equation would be $x = k$, where k is any number. If a line must contain the y-intercept point (0,-2), the equation would have to be $x = 0$.

For problem 3, the slope of the line is $\frac{1}{2}$. The perpendicular line would have slope -2. The equation of the line would be $y = -2x + 3$.

- Describe the domain of these three perpendicular lines.

The domain and range of the perpendicular lines in problems 1 and 3 would be all real numbers. The domain of the line $x = k$ is the number k. The range is all real numbers.

- Do these perpendicular lines represent functions?

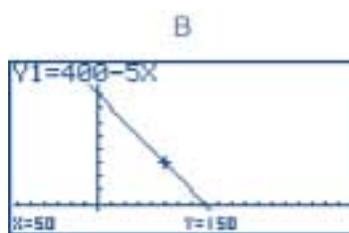
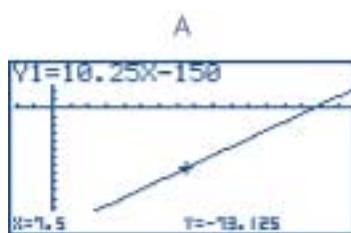
The perpendicular lines in problems 1 and 3 represent functions because for each x there is only one y value. However, $x = k$ does not represent a function because the x value 0 is paired with an infinite number of y values.





Create a Situation

Create and describe in detail a situation that each of the following graphs could represent:



Teacher Notes

Scaffolding Questions:

- What type of function are these graphs presenting?
- What are the constants in these functions?
- Are the functions increasing or decreasing?
- How can you use this information to help you describe a situation each function might represent?

Sample Solution:

- A. The function for the graph is given to be $y = 10.25x - 150$. The function rule implies that the rate of change is 10.25 in the y values for every unit change in the x value and the starting amount is -150. The following money situation could be modeled by this graph and function. You decide to start up a lawn mowing business. You borrow \$150 from your dad to buy a new mower. You charge \$10.25 for each lawn you mow.

The graph represents your cash assets when you have mowed x lawns. You will make a profit once the y values are positive. You are “in the red” until you mow the 15th lawn, since your “break-even point” (x -intercept) is between 14 and 15. Now you show a profit since your y -values are positive when you mow 15 or more lawns.

- B. The function for the graph is $y = 400 - 5x$. The starting value is 400, and the y value is decreased by 5 units for every increase of one in the x unit. This could represent the altitude of a skydiver whose parachute opens at 400 meters. The skydiver is gently drifting to a landing at a rate of 5 meters per second. The graph for this situation would just be the first quadrant region. The y -intercept, $(0,400)$ represents when the parachute opens. The x -intercept, $(80,0)$ represents the number of seconds it takes him to land. This could be extended to include the second quadrant region by assuming that $x = 0$ is when the skydiver is first sighted by someone on the ground and that he opened his parachute before that.

Extension Questions:

- What would happen to the graph of the function in Part A if the function were $y = 10.25x - 129.5$? How would this change the situation you described?

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.

The student uses the properties and attributes of functions.

The student:

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.



The graph will be a line with the same slope but with a different y-intercept, $(0, -129.50)$.

In the situation it could mean that you need to mow fewer yards because you found a mower that cost \$129.50.

- What would happen to the graph of the function in Part B if the function were $y = 400 - 4x$? How would this change the situation you described?

The graph will be a line with the same y-intercept. It will not be as steep since the slope is -4 instead of -5 . The x-intercept will change from $(80, 0)$ to $(100, 0)$.

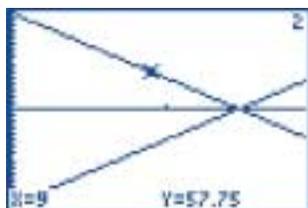
In the situation, it would mean that the skydiver is drifting to his landing at 4 meters per second, and lands in 100 seconds.

- For problems A and B, what would the graphs look like if you reflected the original graphs over the x-axis? How would this change the function describing the graph? How would it change the situation you chose to represent the graph?

The function for A would become $y = -10.25x + 150$ since reflecting over the x-axis is the same as multiplying the expression $10.25x - 150$ by -1 . This is a decreasing linear function. It could no longer represent a “money-earned” situation. It could represent a “money spent out of \$150” situation. For example, Jack has 150 dollars in his savings account. He withdraws \$10.25 each week. If he does not add any money to the account, y represents the amount of money in the savings account at x weeks.

$$y = 10.25x - 150$$

$$y = -10.25x + 150$$



Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

2 Using Patterns to Identify Relationships

2.1 Identifying Patterns

II. Linear Functions

1 Linear Functions

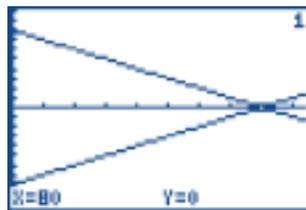
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:

The student will graph problems involving real-world and mathematical situations.

Similarly, the function for B would become $y = 5x - 400$. The graph would have a negative y-intercept, $(0, -400)$. This is an increasing function that starts at a negative value and could not represent the skydiver's altitude as he drifts to his landing. The altitude at time zero may not be negative. One must think of a situation that begins with a negative value. For example, Lance borrows \$400 from his sister, and pays her back at the rate of \$5 per week. If he continues to pay her at the constant rate, y represents the amount of money he owes her, and x represents the number of weeks. The x-intercept is 80; this means that after 80 weeks the amount he owes her is 0 dollars.

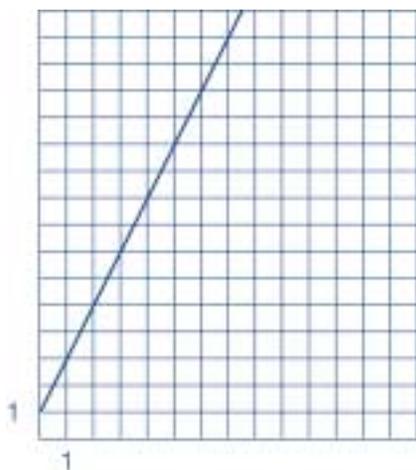


Finding Pairs

Functions are represented in different ways in letters A through L. Compare and contrast the function rules, tables, graphs, and the situations. Separate the letters into the six pairs of letters that then show representations of the same functional relationship.

A. $y = 2x - 1$

B.



E.

x	y
0	50
1	55
2	60
3	65
4	70
5	75

F. $y = 2x + 1$

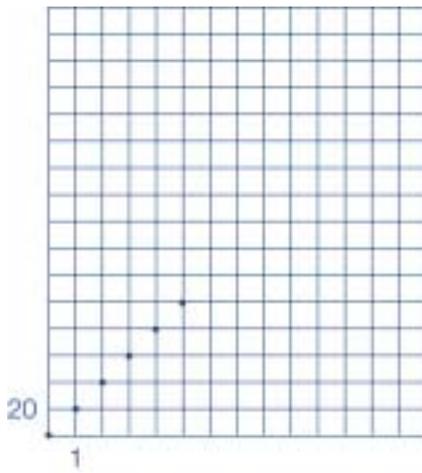
G. $y = 1.5x$

C. The huge beautiful plant was growing at a rate of 1 and a $\frac{1}{2}$ inches per week.

D. The Math Club found a place that will sell them t-shirts for \$5.00 each, but there is a set-up fee of \$50.



H.



J.

x	y
-2	1.5
-1	1.5
0	1.5
1	1.5
2	1.5
3	1.5

K.

x	y
0	0
1	20
2	40
3	60
4	80
5	100

I.

x	y
-2	-5
-1	-3
0	-1
1	1
2	3
3	5

L. $y = 1.5$



Teacher Notes

Scaffolding Questions:

- What type of relationship is described by the graphs, tables, and functions?
- What do the coefficients of x represent?
- What patterns do you notice in the tables? What is causing these patterns?
- What does it mean when you are subtracting a term?

Sample Solution:

Table D and situation E are a pair; the \$50 set-up fee is the entry $x = 0, y = 50$. The \$5.00 per shirt is evident in the table as increments of 5 under the y column.

Table I and function A are a pair. The point $(0, -1)$ from the table indicates a y -intercept of -1 . The increments of 2 under the y column, for every corresponding increment of 1 in the x column, mean that the slope is 2. The equation is $y = 2x - 1$.

Table K and graph H are pairs because the point $(0, 0)$ indicates that the graph passes through the origin. In the table y is increasing by a constant rate of 20, and the slope of the line is 20 because for every unit to the right there are 20 units up to get to a point on the line.

Table J and function L represent the same information. The table shows 1.5 under the entire y column, which means y is always equal to 1.5.

Function F and graph B match because the graph passes through point $(0, 1)$, which means the y -intercept is 1. The coefficient of x is 2, and 2 is also the slope of the line in graph B.

Function G and situation C are pairs. The coefficient of x is 1.5, and that is the same as the rate of change for the plant.

Extension Questions:

- How do the numbers in the functions affect the table?

The coefficient, m , of x in the function $y = mx + b$ is the slope or rate of change that can be determined from the table. The constant b in the function corresponds to the data point $(0, b)$.

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(C) describes functional relationships for given problem situations and writes functions or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve functions and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves functions, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.



(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

1 Linear Functions

- 1.1 The Linear Parent Function
- 1.2 The Y-Intercept
- 1.3 Exploring Rates of Change
- 1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:

The student will write linear functions (functions of lines) to model problems involving real-world and mathematical situations.

- Make a list of patterns you notice in the tables, and explain what causes the patterns.

In table E as x increases by 1, the values of y increase by 5. This is because the constant rate of change is 5. In a function rule it will show up as the coefficient of x .

The constant rate of change for table I is 2.

The constant rate of change for table K is 20.

The constant rate of change for table J is 0. The y values do not change as x changes. In the equation $y = mx + b$, the slope is zero. Therefore, $y = 0x + b$ or $y = b$ for every x .

The constant rate of change tells me the coefficient of x is 0. This indicates that the matching function does not have a term with an x .



First-Aid Supplies

Mark is the trainer for the Little Kids soccer team. He is at the store to pick up 50 bandages and 3 ice packs for the first-aid kit. Mark may spend at most \$12. Prices vary for the different ice packs, but every brand of bandage costs the same: \$4.50 for 50 bandages. The sales tax is 9%.

1. Write an inequality to identify the number of ice packs Mark can purchase. Identify your variable.
2. How much can Mark spend for each ice pack and keep within the \$12 budget?
3. Suppose the booster club gives Mark another \$10 to spend for ice packs. Describe your solution verbally and algebraically.



Teacher Notes

Scaffolding Questions:

- Can Mark spend less than the amount of money he has? Can he spend more?
- How does the tax rate affect your inequality?

Sample Solution:

1. The cost of the bandages is \$4.50. The cost of the ice packs depends on the price of the ice pack.

Let x = the price of one ice pack.

Since Mark needs 3 ice packs, the cost of the ice packs is 3 times the price of one ice pack or $3x$. The expression for the cost of the ice packs plus the cost of the bandages is $3x + 4.50$.

The sales tax is 9% of the cost or $0.09(3x + 4.50)$.

The total cost including the tax must be less than or equal to \$12.

$$(3x + 4.50) + 0.09(3x + 4.50) \leq 12.00$$

- 2.

$$3x + 4.50 + 0.27x + 0.41 \leq 12.00$$

$$3.27x + 4.91 \leq 12.00$$

$$3.27x \leq 7.09$$

$$x \leq 2.168195719$$

x represents a dollar amount and must be expressed to the nearest hundredth. If you round up to \$2.17, the cost would be

$$\begin{aligned} 3(2.17) + 4.50 + 0.09(3(2.17) + 4.50) &= \\ 11.01 + 0.09(11.01) &= 11.01 + 0.99 = 12.00 \end{aligned}$$

Mark can spend up to \$2.17 per ice pack.

3. If the booster club gives Mark an additional \$10 to spend for the ice packs, the only difference in the solution will be the total amount budgeted for purchase. Rather than being \$12, the new amount will be \$22. The additional money will allow Mark to purchase more ice packs at a cheaper price.



Let x = the price of an ice pack.

The total cost may now be less than or equal to \$12.00 plus \$10.00.

$$(3x + 4.50) + 0.09(3x + 4.50) < 22.00$$

Values are rounded to the nearest hundredth.

$$\begin{aligned} 3x + 4.50 + 0.27x + 0.41 &\leq 22.00 \\ 3.27x + 4.91 &\leq 22.00 \\ 3.27x &\leq 17.09 \\ x &\leq 5.226 \end{aligned}$$

If this answer is rounded to \$5.23 and Mark spent \$5.23 per ice pack, the cost would be \$22.01, so he can spend at most \$5.22.

Extension Questions:

- Suppose Mark found the bandages on sale for \$3, and he could spend no more than \$15. How much could he spend per ice pack?

If the bandages are on sale for \$3, and Mark can spend up to \$15, he could spend up to \$3.59 per ice pack. Values are rounded to the nearest hundredth.

Let x = the price of an ice pack.

$$\begin{aligned} (3x + 3.00) + 0.09(3x + 3.00) &\leq 15.00 \\ 3x + 3.00 + 0.27x + 0.27 &\leq 15.00 \\ 3.27x + 3.27 &\leq 15.00 \\ 3.27x &\leq 11.73 \\ x &\leq 3.59 \end{aligned}$$

- Would a 7% tax rate affect the number of ice packs Mark could purchase if he has a maximum of \$15 for the purchase?

Let x = the price of an ice pack.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.



$$\begin{aligned}
 (3x + 3.00) + 0.07(3x + 3.00) &\leq 15.00 \\
 3x + 3.00 + 0.21x + 0.21 &\leq 15.00 \\
 3.21x + 3.21 &\leq 15.00 \\
 3.21x &\leq 11.79 \\
 x &\leq 3.67
 \end{aligned}$$

Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Linear Functions

3 Linear Equations and Inequalities

3.3 Solving Linear Inequalities

Connections to Algebra End-of-Course Exam:

Objective 4:

The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 6:

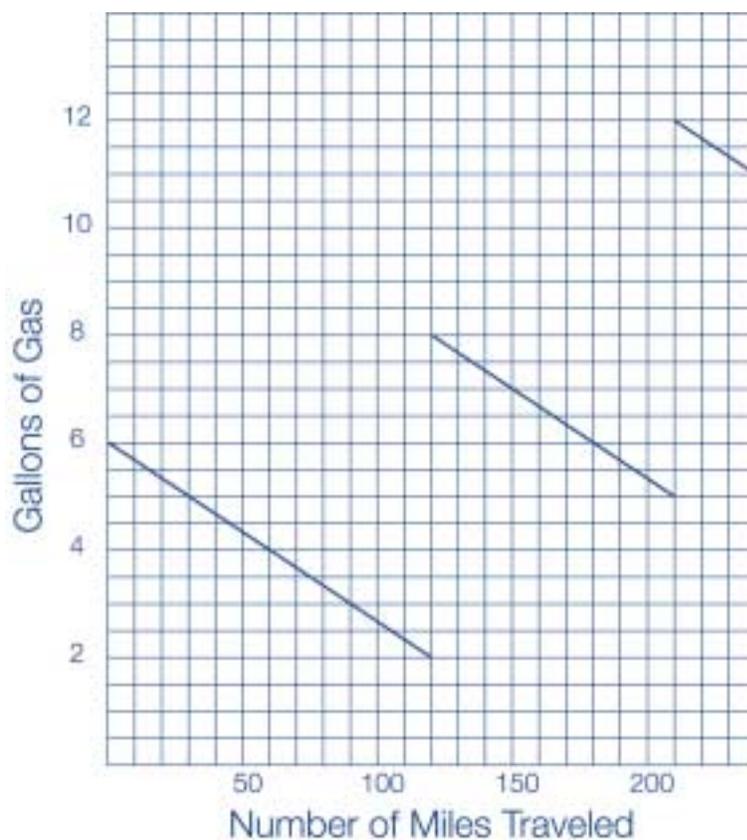
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

A 7% tax rate does not have a great impact on the price of the ice packs. Therefore, the number of ice packs Mark can purchase is not affected by the lower tax rate.



Gas Tank

The following graph shows how the amount of gasoline in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the shape of the graph for this situation. Include in your description an interpretation of the slopes of the segments.



Teacher Notes

Scaffolding Questions:

- How many phases do you see in the graph?
- How does the graph behave in each phase? What does this mean in the situation? How does the graph behave between phases? What does this mean in the situation?
- How does the amount of gas in the tank vary during the first 100 miles of the trip? During the next 120 miles? During the last 40 miles?

Sample Solution:

The gas tank starts out with 6 gallons of gas, and for the first 120 miles traveled drops at a steady rate to 2 gallons. At 120 miles, the number of gallons jumps to 8 gallons, which suggests stopping to get gas. Over the next 90 miles traveled (from mile 120 to mile 210), the gas amount drops steadily to 5 gallons. Again, at 210 miles the number of gallons jumps suddenly to 12 gallons and then drops steadily over the next 30 miles.

The capacity of the tank is at least 12 gallons since that is the maximum y-value we see. Thus, at the beginning of the trip the tank was not full, and on the first refill was not filled to capacity.

The rate of change in gas in all three phases is 1 gallon used per 30 miles

(slope of $-\frac{1}{30}$), so gas consumption (gallon/mile) is occurring at a steady rate.

Extension Questions:

- What does the graph tell you about the capacity of the tank?

The capacity of the tank has to be at least 12 gallons because that is the greatest amount of gas in the tank that the graph shows. However, the capacity could be more than that and the tank is not being filled to capacity each time.

- How would the graph be different if you know that the capacity of the gas tank is 15 gallons and the tank was filled to capacity at each of the stops?

Each of the segments would begin with a y-coordinate of 15.

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.

The student uses the properties and attributes of functions.

The student:

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.

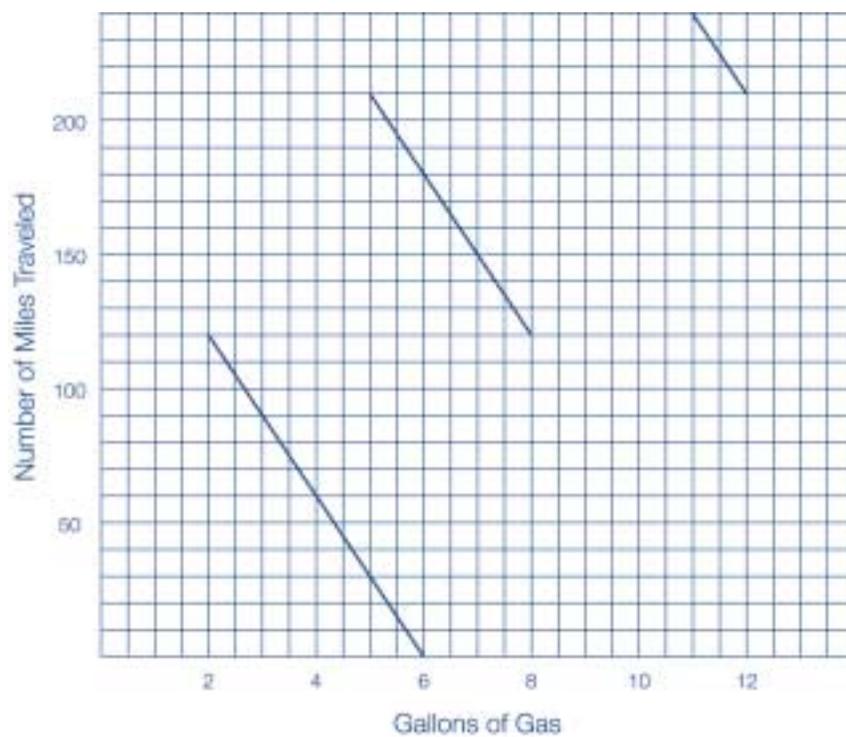
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.



- Create a new graph that would be the result of switching the independent and dependent variables of the original graph.



- What does the resulting rate of change (slope) in each phase now represent?

The rate of change would be miles traveled per gallon and is a decrease of 30 miles per gallon in each phase.

Texas Assessment of Knowledge and Skills:

Objective 1:

The student will describe functional relationships in a variety of ways.

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

3 Interpreting Graphs

3.1 Interpreting Distance versus Time Graphs

II. Linear Functions

1 Linear Functions

1.1 The Linear Parent Function

1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 2:

The student will graph problems involving real-world and mathematical situations.





Greetings

The school choir purchased customized cards from a company that charges \$100 for a set-up fee and \$2 per box of cards. The choir members will sell the cards at \$3 per box.

The function describing their profit, p dollars, for selling x boxes of cards is $p = 3x - (100 + 2x)$.

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?
2. How much money will the choir make if they sell 200 boxes? Show your strategy.
3. How many boxes must the choir sell to make a \$200 profit? Explain how you found your answer.
4. How many boxes must the choir sell to make a \$500 profit? Use a different strategy than the one you used in number 3.
5. How many boxes will the choir have to sell to break even?
6. The choir will not consider this project unless they can raise at least \$1,000. Write and solve an inequality that will help them determine if they should do this project.



Teacher Notes

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Scaffolding Questions:

- What does the word “profit” mean?
- What does the 3 in the expression $3x$ represent?
- What does x represent?
- What does p represent?
- Why are there parentheses used in the function rule?
- How can you use the distributive property to simplify the expression?
- Which of the variables are you given in question 2?
- Which of the variables represents \$200 in question 3?
- What does it mean to break even?
- Describe how you might use a table to answer question 3.
- Describe how you might use a graph to answer question 3.

Sample Solution:

1. The expression $3x$ represents the amount in dollars collected from the sale of x boxes. The $(100 + 2x)$ means that you have to pay \$100 plus \$2 per box that you sell.
2. If they are going to sell 200 boxes, you must evaluate the function for $x = 200$.

$$\begin{aligned}p &= 3x - (100 + 2x) \\p &= 3(200) - (100 + 2(200)) \\p &= 600 - (100 + 400) \\p &= 600 - 500 \\p &= 100\end{aligned}$$

They would make a profit of \$100.

3. Generate a table that shows the number of boxes and the amount of profit made. Use the table to determine the number of boxes that will make a \$200 profit.



Number of Boxes	Profit (dollars)
0	-100
100	0
200	100
300	200
400	300
500	400
600	500

You must sell 300 boxes to make a \$200 profit.

4. The symbolic method may be used to determine how many boxes the choir should sell to make a \$500 profit.

Simplify the rule.

$$p = 3x - (100 + 2x)$$

$$p = 3x - 100 - 2x$$

$$p = x - 100,$$

Substitute 500 for p .

$$500 = x - 100$$

$$x = 600$$

They must sell 600 boxes to make \$500.

5. To break even means that the cost equals the revenue or that the profit is 0.

$$p = x - 100$$

$$0 = x - 100$$

$$x = 100$$

The choir must sell 100 boxes to break even.

Texas Assessment of Knowledge and Skills:

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

3 Linear Equations and Inequalities

3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.



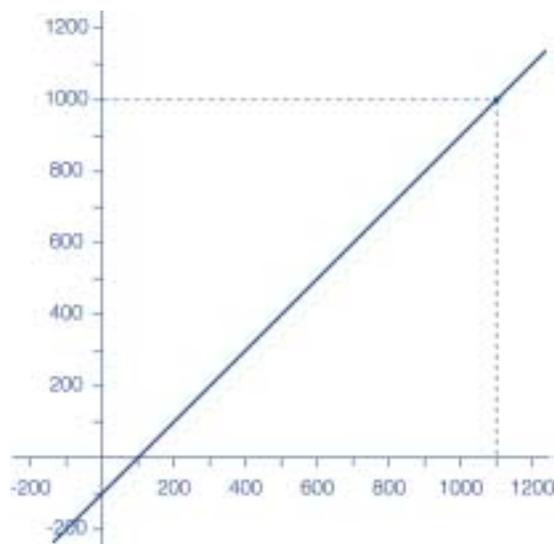
6. If they want to make at least \$1,000, it means that the profit must be greater than or equal to \$1,000.

$$\begin{aligned} p &\geq 1000 \\ x - 100 &\geq 1000 \\ x &\geq 1100 \end{aligned}$$

The choir must sell at least 1100 boxes of cards. If they feel they cannot sell at least 1100 boxes they should not do this project.

Another approach is to examine the graph.

The graphs of $y = 3x - (100 + 2x)$ and $y = 1000$ intersect in the point (1100, 1000). That means that when they sell 1100 boxes, the profit is \$1000. The graph of the profit is above the graph of $y = 1000$ for values of x greater than 1100. They must sell at least 1100 boxes to make a profit of at least \$1000.



Extension Questions:

- What will happen in this situation if the \$100 set-up fee is omitted?



You would have to pay less. The profit would be represented by $p = 3x - 2x$ or $p = x$. You will make \$1.00 per box. Now the y -intercept is zero. The rate of change is still one dollar per box.

- For another situation the profit is represented by $p = 3x - (30 + 2.50x)$. Describe the cost and selling process for this situation.

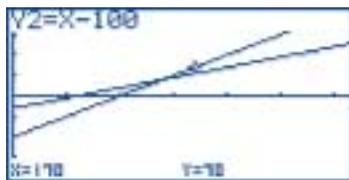
Profit is revenue minus cost. The expression $3x$ means that they are charging \$3 per box. The cost is represented by $30 + 2.50x$. They must be charged a set-up fee of \$30 plus \$2.50 per box.

- Under what conditions is the second situation better than the first?

Determine when the two are equal in value.

$$\begin{aligned}3x - (30 + 2.50x) &= 3x - (100 + 2x) \\0.5x - 30 &= x - 100 \\-0.5x &= -70 \\x &= 140\end{aligned}$$

Examine the graph to determine which function has the greater value after $x = 140$.



When x is greater than 140 the function $y = x - 100$ has the greater value. The profit is greater for the first situation for 140 or more boxes.

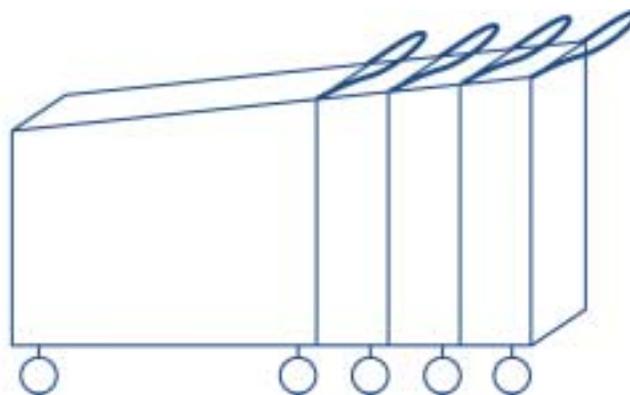




Grocery Carts

Randy must try to fit grocery shopping carts in an area that is 80 feet long and wide enough for the width of a grocery cart. He did some measurements to use in his computations. The table shows the length of a set of grocery carts as they are nested together.

Number of Grocery Carts	Length in Inches
1	37.5
8	116.25



Randy recently finished his algebra class and decided he could determine an expression for the length of the nested grocery carts.

1. What is a function for the length in inches in terms of the number of nested grocery carts?
2. What is the length of a nested set of 50 grocery carts?
3. How many carts would fit in a space 80 feet long if the space is wide enough for one grocery cart?



Teacher Notes

Scaffolding Questions:

- If one shopping cart is 37.5 feet long, how could you find the additional length for each nested grocery cart?
- Compute the finite differences in the table.
- What is the rate of change for the situation?
- Complete this new table with the missing values.

Number of Grocery Carts	Length in Inches Process	Length in Inches
1	37.5	
2	37.5 +	
3	37.5 +	
4	37.5 +	
5	37.5 +	
n		

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Sample Solution:

1. The length for one grocery cart is 37.5 inches. The rate of change is 11.25 inches for every additional grocery cart. The total length is 37.5 plus 11.25 for every additional grocery cart.

$$L = 37.5 + 11.25(n - 1) \quad \text{or} \\ L = 26.25 + 11.25n$$

where n is the number of grocery carts and L is the length of the set of grocery carts.

2. When the number of grocery carts is 50, the length is

$$L = 26.25 + 11.25(50) = 588.75 \text{ inches}$$

3. 80 feet is 960 inches. When is the length 960 inches?

A table or graph may be used to determine when y is 960. When a table is set with increments of 10, it shows that the value of x is between 80 and



90. When the table is set with increments of 1, the value of x that gives a y value of 960 is $x = 83$.



83 carts will fit in a length of 80 feet.

Extension Questions:

- What is a reasonable domain for the function you have created?

The function $L = 32 + 11.25n$ is a linear function. The domain of the function is all real numbers.

- What is a reasonable domain for the problem situation?

The domain for the problem situation represents the number of grocery carts and must be the set of positive integers. However, the domain is determined by the physical, logistical constraints of the situation, such as the available space for storage and customer capacity.

- How do the numbers in the equation relate to the physical grocery carts?

The 26.25 inches is the length of the cart that slides into the remaining carts each time; the 11.25 inches is the amount that hangs out for each new cart.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.



Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

2 Interpreting Relationships Between Data Sets

2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:

The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

- If the equation had been determined to be $L = 32 + 11.25n$ for a different set of grocery carts, what is the same about the two sets of grocery carts?

The portion that is added for each new grocery cart is the same, because the rate, 11.25, has not changed. However, the y-intercept has changed, so the part that is nested into the rest of the carts is not the same.



Hull Pressure

When a submarine descends into the ocean, the pressure on its hull increases in increments as given in the following table. (Pressure is measured in kilograms per square centimeter, and depth is measured in meters.)

Depth	0	300	600	900	1200	1500
Pressure	0	32	64	96	128	160

1. Describe verbally and symbolically a function that relates the depth of the submarine and the pressure on its hull.
2. How will the situation restrict the domain and range of the function?
3. What will be the pressure on the submarine's hull when it is at a depth of 1575 meters?
4. If the pressure on the submarine's hull is 240 kg/cm^2 , what is the depth of the submarine?



Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:**(b.1) Foundations for functions.**

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations; and

(G) relates direct variation to linear functions and solves problems involving proportional change.

Teacher Notes

Scaffolding Questions:

- How does the pressure change as the depth of the submarine increases?
- What is the initial pressure on the submarine's hull?
- What should the dependent variable represent?
- What should the independent variable represent?
- What is the rate of change in the pressure?
- How will you find the pressure for a given depth?
- How will you find the depth for a given pressure?

Sample Solution:

1. When the submarine is at the ocean's surface, the pressure on its hull is 0 kg/cm^2 .

For every 300 meters the submarine dives, the pressure on its hull increases by 32 kg/cm^2 .

A linear function with intercept $(0,0)$ and slope $m = \frac{32}{300} = \frac{8}{75}$

represents the situation, that is, $p = \frac{8}{75}d$, where p is the pressure in kg/cm^2 and d represents the depth in meters.

2. While the mathematical domain and range for this function are both the set of all real numbers, the situation restricts the domain to the real numbers from zero to the maximum depth the submarine can dive. The situation restricts the range to the real numbers from zero to the maximum pressure the submarine's hull can withstand. This would depend on the construction and size of the submarine.
3. If the depth of the submarine is 1575 meters, then $d = 1575$ and

$$p = \frac{8}{75}d = \frac{8}{75}(1575) = 168.$$

The pressure on the submarine's hull is 168 kg/cm^2 .



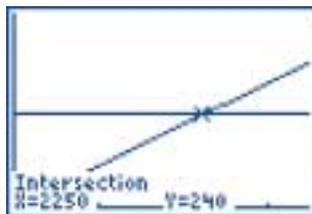
4. If the pressure on the submarine's hull is 240 kg/cm², then $p = 240$ and the following equation can be solved for d .

$$240 = \frac{8}{75}d$$

$$d = 240 \cdot \frac{75}{8} = 2250$$

The submarine's depth is 2250 meters.

The problem could also be solved by finding the intersection of the graphs of $y = 240$ and $y = \frac{8}{75}x$.



The value of x when $y = 240$ is 2250.

Extension Question:

- Is there a proportional relationship between the hull pressure and the depth? Explain how you know whether or not the relationship is proportional.

The graph of the function is a straight line that contains the point (0,0). Therefore, there is a proportional relationship between the hull pressure and the depth.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 1:

The student will describe functional relationships in a variety of ways.

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

- 1 Linear Functions
 - 1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:

The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:

The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 9:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.





Math-a-thon

Catrina is participating in the school math-a-thon to raise money for the end-of-year field trip. Her mother is donating \$25.00 to get her started. She will also receive 75 cents for every problem she answers correctly.

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.
2. Katrina's grandmother gives her an extra \$20.00 to add to her field-trip money. How would this change the previous situation's rule, graph, and table?
3. What part of the situation would you change in order to produce a lesser or greater slope? Explain how you know.



Teacher Notes

Scaffolding Questions:

- How much money will Catrina raise if she works 20 problems correctly?
- What are the constants in this situation?
- Describe the variables in this problem.
- What type of graph do you think this situation will produce?
- What role does the \$25.00 play in the graph of this situation?
- What does adding \$20.00 due to the graph of the situation?
- What is the rate of change for the original situation?
- What is the rate of change for the second situation?

Sample Solution:

1. The amount of money she will have is \$25.00 plus \$0.75 times the number of problems she gets correct. The function rule is

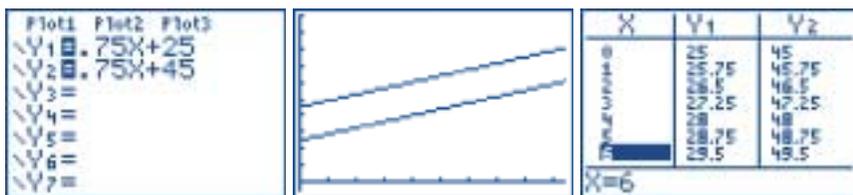
$$d = 0.75p + 25.00.$$

The \$0.75 is the amount of money Catrina will get for every problem she works correctly. The \$25.00 is the amount of money Catrina is going to get from her mother regardless of the number of problems she works. The variable p represents the number of problems worked correctly and the variable d is the total donation.

2. The \$25.00 in the first function rule will change to \$45.00 because now Catrina will start with \$25.00 plus \$20.00. The new rule is

$$d = 0.75p + 45.00.$$

The table now shows when x is 0, then y is 45 instead of 25. It still increases by 0.75 for every problem. The graphs will show parallel lines, one starting at $(0,25)$ and the other starting at $(0,45)$.



3. The amount of money Catrina receives per correct problem affects the rate of change. This rate of change determines the steepness of the line.

To get a greater slope in this function rule the amount of money Catrina receives per correct problem needs to increase. Anything more than \$0.75 will result in a greater slope, and anything less than \$0.75 will produce a line with a lesser slope.

Extension Questions:

- How do the domain of the function rule and the domain of the problem situation compare?

The domain for the function is all real numbers. However, in the problem situation the number of problems must be a whole number. The number of problems in the competition would be the maximum number she could get correct, so the domain of the first problem situation is a subset of the set of whole numbers.

- How do the graphs of the function rule and the situation compare?

The graph of the function would be a straight line, but the graph of the problem situation would be a set of points on a straight line in the first quadrant.

- Write another scenario that will produce a similar function rule, graph, and table.

Johnny has a basket with 20 apples and starts picking apples at a rate of 5 apples for every minute. How many apples will he have in 10 minutes?

- Jackie did not receive a starting donation; can she still collect as much money as Catrina? Explain your answer.

Yes, she can do more problems than Catrina, or she can collect more per problem she works or a combination of these two things.

(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Texas Assessment of Knowledge and Skills:

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 1:

The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:

The student will graph problems involving real-world and mathematical situations.

Objective 3:

The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:

The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.





Recycling

At Dobie Middle School, the number of cans, $n(p)$, collected for recycling after a baseball game depends on the number of people, p , who attend the game. At least 80 people attend each game. The approximate relationship is given by the function $n(p) = 2.5(p - 40) - 100$.

1. If 400 people attended the game for the semifinals of the district championship, how many cans would you expect to be collected? Show at least two different strategies to determine the number of cans.
2. If 300 cans were collected at a game, how many people would you expect to have attended the game? Solve this two different ways.
3. If 673 cans were collected at another game, how many people would you expect to have attended that game?



Teacher Notes

Scaffolding Questions:

- What does the 400 given in problem 1 represent?
- What does the 300 represent?
- How is problem 2 different from problem 1?
- Will it help you to construct a table to solve the problem?
- What will the graph look like? How can the graph help you answer the questions?

Sample Solution:

1. To determine the number of cans for 400 people, evaluate the function for $p = 400$.

$$\begin{aligned}n(p) &= 2.5(p - 40) - 100 \\n(400) &= 2.5(400 - 40) - 100 \\&= 2.5(360) - 100 \\&= 900 - 100 \\&= 800\end{aligned}$$

If 400 people attended the game, the band can expect to collect 800 cans.

A different approach to solve this would be to simplify the equation by using the distributive property. The equation will then be $n(p) = 2.5p - 100 - 100$ or $n(p) = 2.5p - 200$. Next make a table and find the value of c when $p = 400$.



X	Y1
400	800
401	802.5
402	805
403	807.5

FORM DEL ROW G-COL G-FLT 400

Looking at the table if 400 people attended the game, the band can expect to collect 800 cans.

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

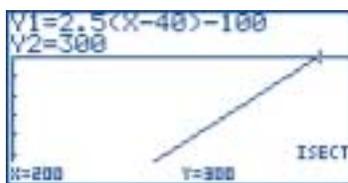
(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.



2. If 300 cans were collected using the table, it can be determined that 200 people attended the game that day. The equation can be used by substituting 300 for $n(p)$ and solve for p .

$$\begin{aligned} 300 &= 2.5p - 200 \\ 300 + 200 &= 2.5p \\ 500 &= 2.5p \\ 200 &= p \end{aligned}$$

Two hundred people attended the game the day the band collected 300 cans. Another approach is to use a graphing calculator: enter the functions $y = 2.5(x - 40) - 100$ and $y = 300$. The point of intersection on the graph may be found or the x -value may be located when the y -value is 300.



X	Y1	Y2
199	297.5	300
200	300	300
201	302.5	300
202	305	300

FORM DEL ROW X-CON G-FLT 200

3. To determine the number of people if 673 cans were collected, look for 673 in the table for the x -value.

X	Y1	Y2
348	670	300
349	672.5	300
350	675	300
351	677.5	300

FORM DEL ROW X-CON G-FLT 349

There are 349 people for 672.5 cans. This number does not have meaning because the number of cans and the number of people must be whole numbers.

The best estimate is for 350 people collecting 675 cans.

Texas Assessment of Knowledge and Skills:

Objective 4:

The student will formulate and use linear equations and inequalities

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

3 Linear Equations and Inequalities

3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.



Another approach would be the symbolic solution.

$$n(p) = 2.5p - 200$$

$$673 = 2.5p - 200$$

$$873 = 2.5p$$

$$p = 349.2$$

Since p represents the number of people it may not be a fraction. There would be more than 349.2 people to collect at least 673 cans.

Extension Questions:

- Explain the meaning of the slope of the function in this problem situation.

The slope is 2.5 cans per person; it represents the estimated number of cans each person would throw away during the game.

- Explain the meaning of the y-intercept of the function.

The y-intercept of the function is -200; that is $y = -200$ when $x = 0$. However, this value of y does not have a meaning in the context of the problem because the number of people is greater than or equal to 80.

- What is the x-intercept, and what is its meaning in the context of the problem?

The x-intercept is 80, the minimum number of people that attend the game.



Shopping

Celeste is going shopping to buy 2 pairs of shoes and some earrings. She can spend \$100 at the most. The shoes Celeste wants to buy cost \$24.99 per pair. Earrings cost \$12.99 a pair. What is the greatest number of earrings she can buy? The sales tax on the total sale is 8% of the amount.

1. Write an inequality to identify the number of earrings she can purchase.
2. Would it be possible for Celeste to purchase 4 pairs of earrings? Explain your answer.
3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for \$19.99?



Teacher Notes

Scaffolding Questions:

- Identify the variable and describe the situation verbally and symbolically.
- Can Celeste spend less than the amount of money she has? Can she spend more?

Sample Solution:

1. Let x = the number of pairs of earrings Celeste can buy
\$24.99 = the cost of one pair of shoes
\$12.99 = the cost of one pair of earrings

The cost of two pairs of shoes at \$24.99 each + x pair of earrings at \$12.99 may be represented by $2(24.99) + 12.99x$.

The tax of 8% on the sale is represented by $0.08 [2(24.99) + 12.99x]$.

The cost plus the tax may total to no more than \$100. This inequality describes the restriction:

$$2(24.99) + 12.99x + 0.08[2(24.99) + 12.99x] \leq 100.00$$

Use the distributive property to simplify before solving and round to the nearest hundredth:

$$\begin{array}{r} 49.98 + 12.99x + 4.00 + 1.04x \leq 100.00 \\ 53.98 + 14.03x \leq 100.00 \\ \underline{- 53.98} \qquad \qquad \underline{- 53.98} \\ 14.03x \leq 46.02 \\ x \leq 3.28 \end{array}$$

Celeste can buy no more than 3 pairs of earrings. She doesn't have enough money for 4 pairs of earrings, but she will have some money left.

If Celeste finds the shoes on sale for \$19.99 a pair, the inequality would change as follows:

$$2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] \leq 100.00$$

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.



Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{array}{r}
 39.98 + 12.99x + 3.20 + 1.04x \leq 100.00 \\
 43.18 + 14.03x \leq 100.00 \\
 \underline{- 43.18} \qquad \qquad \underline{- 43.18} \\
 14.03x \leq 56.82 \\
 x \leq 4.05
 \end{array}$$

Celeste would be able to buy 4 pairs of earrings and still have money left over.

Extension Questions:

- Suppose Celeste wants to have \$20 left. Describe and write your solution algebraically.

In order for Celeste to have \$20 left, the total amount she can spend has to be reduced by \$20. Rather than having \$100 to spend, she only has \$80. The inequality is:

$$\begin{array}{r}
 2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] \leq 80.00 \\
 49.98 + 12.99x + 4.00 + 1.04x \leq 80.00 \\
 53.98 + 14.03x \leq 80.00 \\
 \underline{- 53.98} \qquad \qquad \underline{- 53.98} \\
 14.03x \leq 26.02 \\
 x \leq 1.85
 \end{array}$$

Celeste would only be able to purchase 1 pair of earrings if she sets aside \$20.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

- 1 Developing Mathematical Models
 - 1.2 Valentine's Day Idea

II. Linear Functions

- 1 Linear Functions
 - 1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.



- How would your solution change if Celeste finds the shoes on sale for 25% off of the original price. Show your solution algebraically.

If Celeste finds the shoes on sale for 25% off the original price of \$24.99, the new shoe price could be found by realizing the shoes will cost 75% of the original price. Multiplying \$24.99 by 0.75 will give you the new shoe price. The solution is similar to the sample solution:

$$\$24.99 * 0.75 = \$18.74$$

The new inequality will be:

$$\begin{aligned} 2(18.74) + 12.99x + 0.08[2(18.74) + 12.99x] &\leq 100.00 \\ 37.48 + 12.99x + 3.00 + 1.04x &\leq 100.00 \\ 40.48 + 14.03x &\leq 100.00 \\ -40.48 &\quad -40.48 \\ \hline 14.03x &\leq 59.52 \\ x &\leq 4.24 \end{aligned}$$

Celeste would be able to purchase 4 pairs of earrings and still have some money left over.

- How would a 9% tax rate have an effect on the number of earrings Celeste could purchase at the original show price?

$$2(24.99) + 12.99x + 0.09[2(24.99) + 12.99x] \leq 100.00$$

Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{aligned} 49.98 + 12.99x + 4.50 + 1.17x &\leq 100.00 \\ 54.48 + 14.16x &\leq 100.00 \\ -54.48 &\quad -54.48 \\ \hline 14.16x &\leq 45.52 \\ x &\leq 3.21 \end{aligned}$$

A 9% sales tax rate will cost a little more, but Celeste will still be able to purchase 3 pairs of earrings and have a little money left.



Sound Travel

Many fishing boats and salvage ships are equipped with sonar to help them find shipwrecks and large schools of fish. Sound travels through water at about 1463 meters per second. By measuring the time taken for the sound waves to travel through the water from the boat to the fish it is possible to calculate the distance from the boat to the fish.

1. Write a function that is a model for the relationship between the number of seconds it takes the sound signal to return to the boat and the distance from the boat to the school of fish. Identify your variables.
2. Describe the graph of this function including the domain and the range. Explain how you know whether or not there is a direct variation between the number of seconds and the distance in meters.
3. Suppose the sound signal returned to the boat in 0.05 seconds. Estimate the distance to the school of fish.
4. If the distance from the boat to the school of fish is 24,000 meters, how long will it take the signal to return to the boat?



Teacher Notes

Scaffolding Questions:

- If the sound returns to the boat in 1 second, what is the distance to the school of fish? 2 seconds?
- What is the relationship between the distance to the school of fish at 1 second and at 2 seconds? At 2 seconds and at 3 seconds?
- What does the 1463 mean in the function?

Sample Solution:

1. Let d = the distance to the school of fish in meters.
Let t = the time in seconds for the sound to return to the boat.

The time for the signal to return to the boat depends on the distance in meters. The time is the dependent variable, and the distance is the independent variable.

The table shows the number of seconds and the distance in meters.

Time (seconds)	Distance (m)
0	0
1	1463
2	2926
3	4389

The function rule is linear because the rate of change is constant. The difference in the time in the table is 1 second. The difference in the distance is an increase of 1463 kilometers for every increase of one second. The function rule is $d = 1463t$.

2. There is a direct variation (proportional relationship) between the distance and the time in seconds because the graph of the function is a straight line that passes through the origin.

The domain of the function represents the time it takes for the sound to return to the boat. The time in seconds will have to be greater than

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

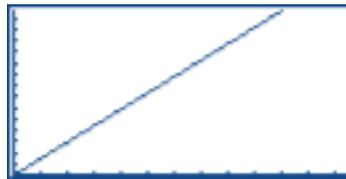
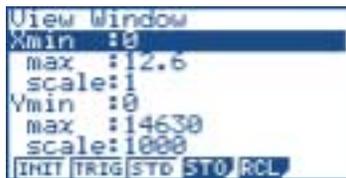
(G) relates direct variation to linear functions and solves problems involving proportional change.



0 or $x > 0$. The range represents the distance to the school of fish. The distance must also be greater than 0, or $y > 0$.

The rate of change in the distance is an increase of 1463 meters for every second. The slope of the equation is 1463, representing an increase of 1463 meters per second. The y -intercept is 0 because at 0 seconds, there is no distance to be recorded.

The graph of the function is:



If the sound signal returned to the boat in 0.05 seconds, the distance could be found by substitution into the function as follows:

$$d = 1463t$$

$$d = 1463(0.05)$$

$$d = 73.15 \text{ meters}$$

If the distance to the fish is 24,000 meters, it will take the sonar signal approximately 16.4 seconds to return to the boat. Because

$$d = 1463t$$

$$24000 = 1463t$$

$$16.4 = t$$

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:

The student will demonstrate an understanding of linear functions.

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions

3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 6:

The student will perform operations on and factor polynomials that describe real-world and mathematical situations.



Extension Questions:

- If the time is doubled, will the distance be doubled? Justify your answer.

Because there is a proportional relationship, the distance will be doubled if the time is doubled.

$$d = 1463t$$
$$1463(2t) = 2(1463t) = 2d$$

- Describe the difference in the two questions asked in problems 3 and 4.

In problem 3 you are given the domain value and asked to evaluate the function. In problem 4 you are given the function value (range value) and asked to find the domain value.

- Determine a function rule that expresses the time as a function of the distance. What type of relationship is this?

Solve the rule for t .

$$d = 1463t$$
$$t = \frac{d}{1463}$$

This function is also linear.



Taxi Ride

The cab fees in Chicago are \$1.40 for the first one-fifth mile and 20¢ for each additional one-tenth of a mile.

1. What is the longest distance you can travel for \$10.00?
2. How much will you have to pay if you need to get to a restaurant that is 20 miles away from your hotel? Solve using two different techniques.
3. If you want to include a 15% tip, what is the longest distance you can travel for \$10.00?



Teacher Notes

Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the variables in this situation?
- Which variable is independent?
- What kind of relationship is there between the two variables?
- How much money would you have to pay for the first mile?
- How much money would you have to pay for the second mile?
- What is the rate of increase per mile?
- What is the y -intercept?
- What is multiplied by 20¢ ? Could you use 40¢ instead? How?
- How many tenths are there in $1/5$?

Sample Solution:

1. Make a table to look for patterns using the information given, and from the pattern generate the rule for the situation. The pattern shows that the cost will increase by 40¢ for every additional fifth of a mile.

Miles	Cost (dollars)
$\frac{1}{5}$	1.40
$\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$	1.60
$\frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

Determine how many tenths are left after paying $\$1.40$ for the first fifth of a mile. Let the number of miles be represented by m .



The number of miles left after the first one-fifth of a mile would be $m - \frac{1}{5}$.
In each mile there would be 10 tenths of a mile.

The number of tenths of a mile left after the first one-fifth of a mile would be represented by $10(m - \frac{1}{5})$ or $10m - 2$.

The cost is 20 cents for every tenth of a mile or $10(m - \frac{1}{5})(0.20)$ or $2m - 0.40$.

The total charge would be

$$\$1.40 + 10(m - \frac{1}{5})(0.20) = \$1.40 + 2m - 0.40 = 2m + 1.$$

To determine how many miles can be traveled with \$10.00, substitute the \$10.00 for the cost and solve the inequality.

$$\begin{aligned} 10 &\geq 1.40 + 10(m - \frac{1}{5})(0.20) \\ 10 &\geq 1.40 + (10m - 2)(0.20) \\ 10 &\geq 1.40 + 2m - 0.40 \\ 10 &\geq 2m + 1 \\ 9 &\geq 2m \\ m &\leq 4.5 \end{aligned}$$

The greatest number of miles that could be traveled with \$10.00 is 4.5 miles.

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.



2. Generate a table for the amount for traveling whole miles distances.

Texas Assessment of Knowledge and Skills:

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

3 Linear Equations and Inequalities

3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Miles	Cost (dollars)
1	3.00
2	5.00
3	7.00
4	9.00
5	11.00
6	13.00

The first mile will cost \$3.00, but after the first mile the other miles will cost \$2.00 each. Continue the table until 20 miles, or use the patterns to figure the rule. The function for this situation is the cost per mile is \$2 for every mile but not the first mile that costs \$3.

In symbols: $c = 2(m - 1) + 3$

Substitute the 20 miles for the m and determine the cost.

$$c = 2(20 - 1) + 3$$

$$c = 2(19) + 3$$

$$c = 38 + 3$$

$$c = 41$$

It would cost \$41.00 to travel 20 miles.

Another way to solve this problem would be to use the rule for the tenth of a mile. Substitute the 20 for the m .

$$c = 1.40 + 10\left(m - \frac{1}{5}\right)(0.20)$$

$$c = 1.40 + 10\left(20 - \frac{1}{5}\right)(0.20)$$

$$c = 1.40 + 10(19.5)(0.20)$$

$$c = 41.00$$



3. If the cost is to include a tip of 15%, the computed cost must be multiplied by 1.15.

$$10 \geq [1.40 + 10(m - \frac{1}{5})(0.20)]1.15$$

$$10 \geq 1.61 + (10m - 2)(0.23)$$

$$10 \geq 1.61 + 2.3m - 0.46$$

$$10 \geq 2.3m + 1.15$$

$$8.85 \geq 2.3m$$

$$m \leq 3.847826$$

If you want to leave a 15% tip and travel under \$10.00, the longest distance you can travel is about 3.8 miles.

Extension Questions:

- What are reasonable domain and range values for this situation?

The domain values are every tenth of a mile after the first mile. The range values are \$1.40 and every increment of 20¢ after \$1.40.

- If a shuttle service charges a fee of \$50.00 to any location from the airport, under what circumstances would it be more cost-effective to take a taxi?

The question is when is the taxi going to cost less than \$50.00.

Examine the table for the function.

X	Y ₁	
24.3	49.6	
24.4	49.8	
24.5	50	
24.6	50.2	
24.7	50.4	
24.8	50.6	
24.9	50.8	
X=24.5		

It would be more cost-effective to take the taxi for anywhere less than 24.5 miles.



- If the rates increase, how would it affect the representation of your data?

If the rate per tenth of a mile increases, the table entries would be greater and the slope of the graph would be steeper. If the increase happened in the first fifth mile, the y-intercept would also change.



The Contractor

Lupe is a flooring contractor. He sets floor tile for a living. He submits a bid for each new job. Every time he bids for a job, he measures the area of the floor that he will tile and then figures out how much material he will need. He charges the following prices:

Subflooring:	\$1.27 per square foot
Tile:	\$6.59 per square foot
Adhesive:	\$31.95 per job
Grout:	\$55.95 per job
Labor:	\$125 base price plus \$0.79 per square foot

1. Write a rule to determine the total cost of the materials and labor for a typical job. Explain what the numbers and symbols in the rule mean.
2. Make a table and a graph that will help Lupe see the amount of money he should charge for jobs with various amounts of square footage.
3. If Lupe was awarded a job with an area of 550 square feet, what was the amount of the bid, based on the materials listed above?



Teacher Notes

Scaffolding Questions:

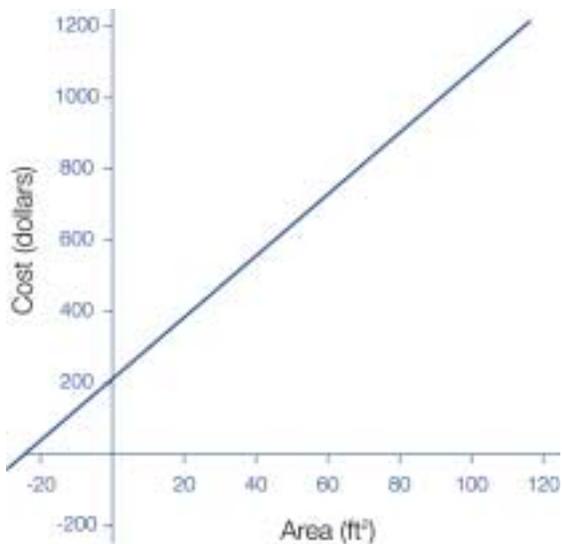
- What are the constants in this problem situation?
- What are the variables?
- Make a list of all the charges he must consider.
- How would he compute the cost for the tile?
- How would he compute the cost of the subflooring?
- How would he compute the labor charges?

Sample Solution:

1. The rule would be cost is equal to the cost of the subflooring plus the cost of the tile plus the cost of the adhesive plus the cost of the grout plus the cost of the labor. If the area is the variable x and the total cost of the job is c , then, $c = 1.27x + 6.59x + 31.95 + 55.95 + 125 + 0.79x$. This can be rewritten as $c = 8.65x + 212.90$; c is the total cost of the job, x is the area of the space to be tiled, and \$8.65 is the combined cost of the items per square foot of area. \$212.90 is the total of the fixed costs that do not depend on area.
2. Use the rule and put the values in a table:

Area (ft ²)	Cost (dollars)
0	212.90
50	645.40
100	1077.90
150	1510.40
200	1942.90
250	2375.40
300	2807.90
350	3240.40
400	3672.90
500	4537.90
600	5402.90
700	6267.90
800	7132.90
900	7997.90
1000	8862.90





3. Use the table and look for the x value of 550.

X	Y ₁
250	2375.4
300	2807.9
350	3240.4
400	3672.9
450	4105.4
500	4537.9
550	4970.4

X=550

If the area was 550 square feet, the cost will be \$4970.40.

(b.4) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:

(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.



Texas Assessment of Knowledge and Skills:

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions

3 Linear Equations and Inequalities

3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Extension Questions:

- Suppose Lupe wants to make a 20% profit on each job. Write a new rule that he can use to compute how much he should charge his customers including his profit.

The new price can be determined adding an additional 20% of the original cost to the original cost. Using symbols it could be $.20(8.65x + 212.90) + 8.65x + 212.90$ or $1.20(8.65x + 212.90)$.

- The price of the adhesive has increased to \$45.95 per job. How is this going to affect the cost of the jobs? How is it going to show up in the rule, the graph, and the table?

The adhesive cost is part of the fixed amount per job, \$212.90. The fixed costs will increase by the difference in the new cost and the original cost, $\$45.95 - 31.95$ or \$14. The new fixed costs will be $212.90 + 14$ or \$226.90. The table values will all increase by this same amount. In the graph, the y -intercept will be 226.90.

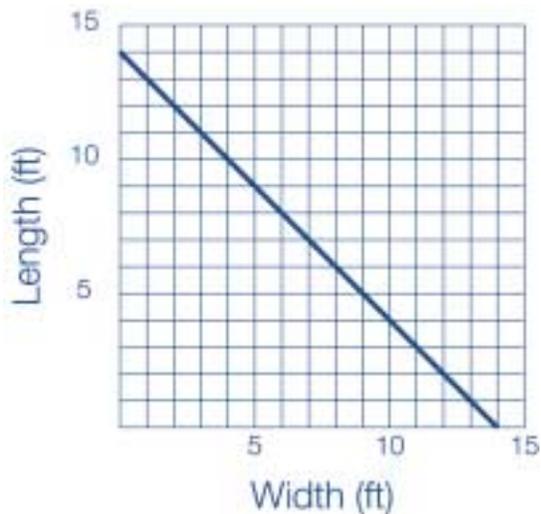
- If the charge for labor has changed to \$130 plus \$0.85 per square foot, how is this going to affect the function that represents the cost of the jobs?

The price for labor has a fixed amount, and it is also part of the charge per square foot. An increase in the fixed amount of the labor will affect the y -intercept. If the fixed amount was increased from \$125 to \$130, the y -intercept will be increased by 5. An increase in the charge per square foot charge will increase the slope of the graph. The increase from \$0.79 to \$0.85 will increase the slope of the graph by 0.06.

The function rule will be changed from $c = 8.65x + 212.90$ to $c = 8.71x + 217.90$.



The Garden



1. Lance had a certain amount of fencing to enclose his garden. He created a graph to represent the relationship between the length of the garden and the width of the garden. Describe verbally and symbolically the relationship between the length and the width.
2. What are reasonable domain and range values for this function?
3. Explain what the graph tells you about the perimeter of the garden.



Teacher Notes

Scaffolding Questions

- What type of relationship is described by the graph?
- Name some points on the graph. What does that tell you about the possible dimensions of the garden?
- Define the independent variable and the dependent variable for this problem situation.
- What would the length be if the width were 12 feet?
- What are the restrictions on the length and the width?

Sample Solution:

1. The graph is a line.

The starting value (y -intercept) is 14. The x -intercept is 14.

Other points are (1,13) and (2,12). The rate of change is a decrease of 1 in the length for every increase of 1 in the width. Thus, the slope of the line is -1.

$$l = -1w + 14$$
$$0 \leq w \leq 14$$

However, there would be no garden if the width were 0 or 14 feet.

2. From the graph you can see that both the length and the width must be positive numbers less than 14.

Width (ft)	Length (ft)
0	14
2	12
4	10
8	6
10	4
14	0

The points (0,14) and (14,0) do not have meaning in this problem situation. There would be no garden if either the width or the length was 0.



Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

3. Notice from the table that the length plus the width must be 14.

$$l + w = 14$$

or

$$l = -1w + 14$$

The perimeter is twice the sum of the width and the length.

$$2(l + w) = 28$$

The perimeter or amount of fencing is 28 feet.

Extension Questions:

- How is the domain for the function you have written different from the domain for the problem situation?

The domain for the function rule is the set of all real numbers. The domain for the problem situation is the set of all real numbers from 0 to 14.

- How would the graph have been different if the total amount of fencing had been 24 feet?

Twice the sum of the length and width would have been 24 feet. The sum of the length and the width would have been 12 feet. The x and y intercepts would have both been 12.

- How would the graphs have been the same if the total amount of fencing had been 24 feet?

The slope for both graphs would have been -1 .

- Describe the relationship between the area of the garden and the width of the garden.

*The area is the length times the width. The length is represented by $14 - w$.
 $A = (14 - w)w$*

(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 1:

The student will describe functional relationships in a variety of ways.

Objective 2:

The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:

The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

3 Interpreting Graphs

3.1 Interpreting Distance Versus Time Graphs

Connections to Algebra End-of-Course Exam:

Objective 2:

The student will graph problems involving real-world and mathematical situations.

Objective 3:

The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.





The Submarine

A submarine is cruising at 195 meters beneath the ocean's surface and begins rising toward the surface at 12 meters per minute.

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.
2. How long does it take the submarine to reach the ocean's surface? Explain.
3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this will change the function and the graph of the original situation. What is the new function, and how did you determine it?



Teacher Notes

Scaffolding Questions:

- What are the constants in the problem? What quantities vary?
- What quantity will be the dependent variable? The independent variable?
- Create a table and/or a graph to help verify your function rule for Problem # 1.
- What kind of function models the situation?
- What is the submarine's depth when it is at surface level?
- What equation will you write and solve?
- If the submarine must surface 5 minutes sooner, how long will it take to surface?
- What quantity in the original function rule must change?
- Will the submarine rise at the same rate? Slower? Faster?
- Try different rates in your original function rule. Use tables and/or graphs to estimate the rate at which the submarine needs to rise to get to the surface 5 minutes sooner.
- What equation can you write and solve to determine this new rate?

Sample Solution:

1. The submarine's position, D meters below the surface, depends on the time, t minutes, it has been rising. The submarine starts rising from 195 meters below the surface, so its initial position is -195. It is rising toward the surface at 12 meters per minute, so it is rising at a constant rate of 12 m/min. The function will be a linear function because the rate at which it is rising is a constant.

The distance will be the starting value plus the rate of change times the number of minutes.

This gives the function: $D = -195 + 12t$

2. When the submarine surfaces, its depth below the surface is 0 meters, so

$$\begin{aligned}D &= -195 + 12t \\0 &= -195 + 12t \\-12t &= -195 \\t &= 16.25\end{aligned}$$

So, it takes the submarine 16.25 minutes to surface if it starts at 195 meters below the surface and rises at a rate of 12 meters per minute.



Materials:

One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:

(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:

(A) uses symbols to represent unknowns and variables.

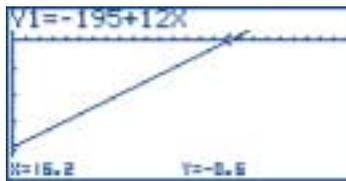
(c.1) Linear functions.

The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Another approach to answering the questions is to use a graphing calculator:

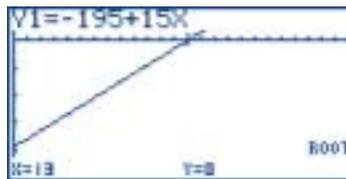


3. If the submarine needs to surface 5 minutes sooner, it needs to rise at a faster rate than 12 meters per minute. Before, it took the submarine 16.25 minutes to surface, and now it is to surface in 11.25 minutes.

Experiment with different rates (more than 12 meters per minute) by using tables and graphs.

If the submarine rises at 15 meters per minute, the equation is $D = -195 + 15t$.

From the graph and table it can be seen that it takes 13 minutes to reach the surface.



(c.2) Linear functions.

The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:

(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$;

(F) interprets and predicts the effects of changing slope and y -intercept in applied situations.

(c.3) Linear functions.

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:

(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.



Texas Assessment of Knowledge and Skills:

Objective 1:

The student will describe functional relationships in a variety of ways.

Objective 3:

The student will demonstrate an understanding of linear functions.

Objective 4:

The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions

1 Developing Mathematical Models

1.1 Variables and Functions

1.2 Valentine's Day Idea

2 Using Patterns to Identify Relationships

2.1 Identifying Patterns

II. Linear Functions

1 Linear Functions

1.2 Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 1:

The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

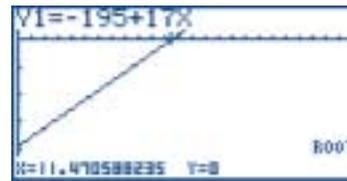
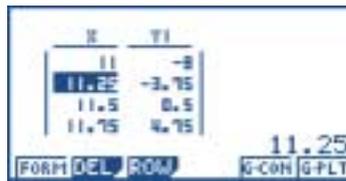
Objective 2:

The student will graph problems involving real-world and mathematical situations.

Objective 8:

The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

If the submarine rises at 17 meters per minute, the equation is $D = -195 + 17t$. It takes about 11.5 minutes to reach the surface.



If the submarine rises at 18 meters per minute, the equation is $D = -195 + 18t$. It takes about 10.8 minutes to reach the surface.



The submarine needs to surface at a rate between 17 and 18 meters per minute.

Another approach would be to let the rate be a variable.

$$D = -195 + rt$$

D must be equal to 0 when $t = 11.25$.

$$\begin{aligned} 0 &= -195 + r(11.25) \\ 195 &= 11.25r \\ r &= 17.3 \end{aligned}$$

The rate should be 17.3 meters per minute.

Extension Questions:

- How do the domain and range of the situation compare with the mathematical domain and range of the function representing this situation? What effect does this have on how you graph the situation?



The mathematical domain and range of the function are both all real numbers because it is a (nonconstant) linear function. The situation restricts the domain to 0 to 16.25 minutes and the range to -195 to 0 meters. Knowing the domain and range of the situation helps determine an appropriate window.

- What are the intercepts of the graph of the function? What information do they give about the situation?

The y -intercept is $(0, -195)$. The initial depth of the submarine is 195 meters below surface. The x -intercept is $(16.25, 0)$. It takes the submarine 16.25 minutes to reach the ocean surface.

- Suppose the submarine must rise from 195 meters below the surface to the ocean's surface within 10 to 20 minutes. How will this affect the rate at which the submarine rises toward the ocean surface?

In this case, the constant is a range in time to rise to the surface instead of the rate at which the submarine rises. So, let r be the rate, in meters per minute, that the submarine rises and D the depth, in meters, of the submarine.

$$D = rt - 195$$

$$\begin{aligned} \text{If } d = 0 \text{ at 10 minutes, } & 0 = r(10) - 195 \\ & 10r = 195 \\ & r = 19.5 \end{aligned}$$

$$\begin{aligned} \text{If } d = 0 \text{ at 20 minutes, } & 0 = r(20) - 195 \\ & 20r = 195 \\ & r = 9.75 \end{aligned}$$

The rate, r meters per minute, must range from 9.75 to 19.5.

- Suppose a student modeled this situation with the function rule $y = 195 - 12t$. What do the variables represent for this rule?

y would represent distance from the surface to the submarine. The rate of travel would be -12 feet per minute, because the distance from the surface to the submarine would be decreasing at a rate of 12 feet per minute.



Student Work

1. The equation $y = 12x - 195$ relates to the submarine's position and the time it's been rising. -195 meters is where it starts, which is the "y" intercept and it rises at 12 feet per minute which is "mx". you could either graph the function ^{or} use the table to solve for the function to find when the submarine reaches the surface.
2. It takes 16.25 minutes for the submarine to reach the surface because "x" is 16.25 when "y" is 0, which is the measure of the surface. $0 = 12x - 195$, $195 = 12x$, $16.25 = x$
3. It will change it to 17.3 per minute. the graph will be closer to the y-axis. my new function is:

$$y = 195 - (5\frac{2}{3}x)$$

I subtracted 5 from 16.25 and got 11.25 and then I divided 195 by 11.25 and got $5\frac{2}{3}$.

