SUPPLEMENTAL

Algebra Assessments

Chapter 8:

Interacting Linear Functions, Linear Systems

Bonnie's Dilemma

Bonnie and Carmen are lab partners in a chemistry class. Their chemistry experiment calls for a 5-ounce mixture that is 65% acid and 35% distilled water. There is no pure acid in the chemistry lab, but they did find two mixtures that are labeled as containing some acid. Mixture A contains 70% acid and 30% distilled water. Mixture B contains 20% acid and 80% distilled water. How many ounces of each mixture should they use to make a 5-ounce mixture that is 65% acid and 35% distilled water? Justify your solution using symbols, tables, and graphs.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Scaffolding Questions:

- If you take 1 ounce Mixture A, how much of this is distilled water? Describe how you determine that amount.
- How many ounces of acid are there in 4 ounces of Mixture A?
- How many ounces of distilled water are there in 2 ounces of the Mixture B?
- What are the variables in this situation?

Sample Solution:

Create a table to compute possible combinations of the two mixtures and the percent of acid and of water in the new mixture.

Amount of Mix A	Amount of Mix B	Amount of acid in New Mix	Amount of distilled water in New Mix	% of New Mix that is acid	% of New Mix that is distilled water	Is it 65% acid and 35% distilled water?
1	5 – 1 = 4	0.7(1) + 0.2(4) = 1.5	0.3(1) + 0.8(4) = 3.5	1.5 out of 5 = 30%	3.5 out of 5 = 70%	Too much water
2	5 – 2 = 3	0.7(2) + 0.2(3) = 2	0.3(2) + 0.8(3) = 3	2 out of 5 = 40%	3 out of 5 = 60%	Too much water
3	5 – 3 = 2	0.7(3) + 0.2(2) = 2.5	0.3(3) + 0.8(2) = 2.5	2.5 out of 5 = 50%	2.5 out of 5 = 50%	Too much water
4	5 – 4 = 1	0.7(4) + 0.2(1) = 3	0.3(4) + 0.8(1) = 2	3 out of 5 = 60%	2 out of 5 = 40%	Too much water
5	0	0.7(5) + 0 = 3.5	0.3(5) + 0 = 1.5	3.5 out of 5 = 70%	1.5 out of 5 = 30%	Not enough water

The value must be between 4 and 5.

4.5	0.5	0.7(4.5) + 0.2(0.5) = 3.25	0.3(4.5) + 0.8(0.5) = 1.75	3.25 out of 5 = 65%	1.5 out of 5 = 35%	The correct amts are 4.5 Mix A and 0.5 Mix B

Another approach using symbols requires that the variables be defined. The variables are the amount of coffee to be used from each mix.

x = the number of ounces of Mixture A
y = the number of ounces of Mixture B

The total amount must be 5 ounces.

Equation 1: $x + y = 5$

The amount of acid in Mixture A plus the amount of acid in the Mixture B must be 65 percent of 5 ounces. The amount of acid in the Mixture A may be expressed as $0.7x$. The amount of acid in the Mixture B may be expressed as $0.2x$.

Equation 2: $0.7x + 0.2y = 0.65(5)$

Similarly, the amount of Distilled water in the Mixture A plus the amount of Distilled water in the second blend must be 35 percent of 5 ounces.

Equation 3: $0.3x + 0.8x = 0.35(5)$

To solve symbolically, use two of the equations and the substitution method:

$x + y = 5$
$0.3x + 0.8y = 0.35(5)$

$$\begin{aligned} y &= 5 - x \\ 0.3x + 0.8(5 - x) &= 0.35(5) \\ 0.3x + 4 - 0.8x &= 1.75 \\ -0.5x &= -2.25 \\ x &= 4.5 \\ y &= 5 - 4.5 = 0.5 \end{aligned}$$

She should take 4.5 ounces of Mixture A and 0.5 ounces of the Mixture B.

Graphs may also be used to solve the problem. To use the graphing calculator to graph or make a table, solve the equations for y:

$y = 5 - x$

$$y = \frac{0.65(5) - 0.7x}{0.2}$$

$$y = \frac{0.35(5) - 0.3x}{0.8}$$

Graph the equations and make a table of values to find the common point. The common point is (4.5,0.5). This means that she should take 4.5 ounces of Mixture A and 0.5 ounces of the Mixture B.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities

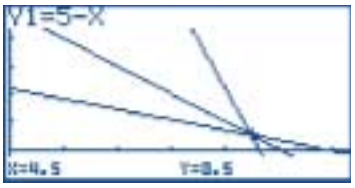
Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.



Extension Questions:

- Was it necessary to have three equations to solve the problem?

The second and third equations were complements of each other because the result formed the amounts that remained when the given portions were removed. For instance, 70% of the water in one mixture meant 30% of the acid was in the mixture. Three equations are not necessary.

- Does it matter which pair of equations is used?

From the graph it is evident that it does not matter which pair of equations are used. If you use any pair of equations, their graphs intersect at the point (4.5,0.5).

- What could have been determined if the total amount or 5 ounces was not given?

The total amount could be expressed as x + y.

The number 5 would be replaced by x + y in the second and third equations.

The amount of Acid would be expressed as 0.7x + 0.2y = 0.65(x + y).

Similarly, the amount of distilled water in the Mixture A plus the amount of distilled water in the Mixture B must be 65 percent of x + y ounces.

$0.3x + 0.8y = 0.35(x + y)$

Simplify the equations:

$$\begin{aligned} 0.7x + 0.2y &= 0.65x + 0.65y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

$$\begin{aligned} 0.3x + 0.8y &= 0.35x + 0.35y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

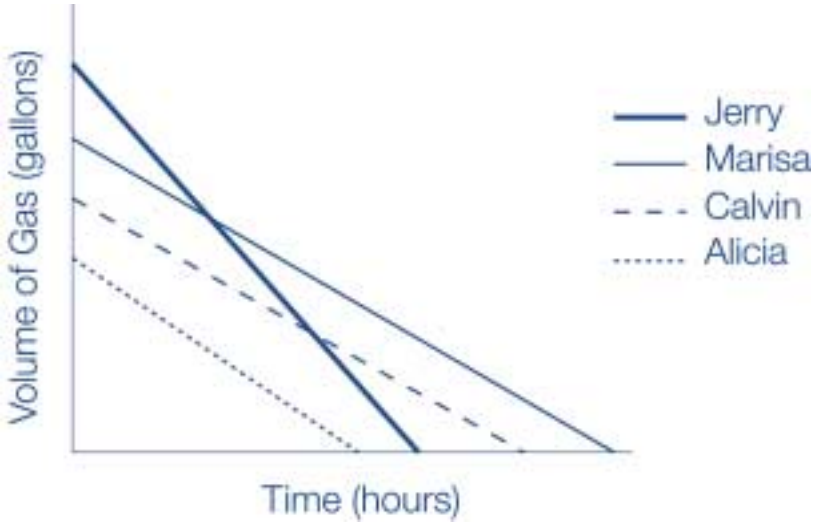
The system could not be solved for specific values of x and y, but one would know what the ratio of x to y must be for any solution.

$$\begin{aligned} 0.05x - 0.45y &= 0 \\ 0.05x &= 0.45y \\ \frac{x}{y} &= \frac{0.45}{0.05} = \frac{9}{1} \end{aligned}$$

To meet the conditions of the problem, the amount of Mixture A must be 9 times the amount of Mixture B.

Four Cars

Jerry, Alicia, Calvin, and Marisa wanted to test their cars' gas mileage. Each person filled their car to the maximum capacity and drove on a test track at 65 miles per hour until they each ran out of gas. The graphs given below show how the amount of gas in their cars changed over time.



Justify your answers to each of the following questions:

1. Whose car had the largest gas tank?

2. Whose car ran out of gas first?

3. Whose car went the greatest distance?

4. Whose car gets the worst gas mileage? What about the graph helped you decide?

5. How would parameters in the function for Calvin's graph compare to the function for Marisa's graph?

6. How would the equation for Marisa's graph compare to the equation for Jerry's graph?

7. How would the equation for Jerry's graph compare to the equation for Alicia's graph?

Teacher Notes

Scaffolding Questions:

- Which person's graph has the greatest *y*-intercept?
- What does that mean in this situation?
- Which person's graph has the greatest *x*-intercept?
- What does that mean in this situation?
- How does knowing that they all traveled at 65 miles per hour help you know who traveled the greatest distance?
- Which lines appear to be parallel?
- If two lines are parallel, what will be the same in their equations?

Sample Solution:

1. The information given is that each person filled their tank to capacity. Jerry's car has the largest tank because at 0 hours his car has the greatest volume.

2. Alicia's car ran out of gas first because her car reaches a volume of 0 in the shortest amount of time.

3. Marisa's car traveled the farthest because it took her car the most time to get to a volume of 0. Each person was traveling at 65 miles per hour. Since distance traveled is the rate multiplied by the time in hours, and her time was the greatest, she traveled the greatest distance.

4. Jerry's car gets the worst gas mileage because his graph is the steepest. His rate of change is decreasing at a faster rate. The absolute value of his rate of change is the greatest. The rate of change represents the number of gallons used per hour of travel.

5. It appears that Calvin's and Marisa's lines are almost parallel, so their slopes would be about the same in the equations, but their lines have different *y*-intercepts.

6. Marisa's and Jerry's equations would be very different. Jerry's gas tank is larger than Marisa's, so the equation that represents Jerry's situation would have a larger *y*-intercept. His car is also using gas at a faster rate, so the absolute value of his rate of change is greater.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

7. Jerry's gas tank holds more gas than Alicia's gas tank so the equation that represents Jerry's situation would have a greater *y*-intercept. Jerry's car is using gas at a faster rate than Alicia's car; therefore, the absolute value of his rate of change is greater.

Extension Questions:

- What does the point of intersection of Jerry and Marisa's lines mean?

It represents the point in time when their gas tanks contain the same amount of gas.

- If a fifth line was added to the graph and the line was parallel to Alicia's line, what would you know about that person's car?

It used gasoline at the same rate as Alicia's car, but it would have a different tank capacity.

- Suppose everyone traveled at 55 miles per hour instead of 65 miles per hour. How would this affect their graphs?

If they were traveling at a slower rate, the amount of gas used per hour would decrease. The inclines of all the lines would be less and the x-intercepts would be greater.

- After some mechanical work, Jerry is now getting better gas mileage. How would that affect his graph?

The x-intercept on his graph would be a larger number. The graph would decline more slowly.

- Suppose Calvin's graph could be represented by the rule $V = 30 - 6t$. What information do you now know about Calvin's car? When did his car run of gas? What would be a reasonable rule for Alicia's travel?

The capacity of his tank is 30 gallons because the y-intercept of V = 30 – 6t is 30. The x-intercept is 5 because if 0 = 30 – 6t, then x is 5. This means that it takes him 5 hours to run out of gas. If he is traveling at 65 miles per hour, he would have traveled 65 times 5 or 325 miles.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

The rules for the other drivers could be estimated using the intercepts.

Alicia's y-intercept is about half of Jerry's or 15 gallons. If Jerry's x-intercept is 5, her x-intercept is about 4.

$$
\begin{aligned}
y &= 15 - ax \\
0 &= 15 - 4a \\
a &= 3.75 \\
y &= 15 - 3.75x
\end{aligned}
$$

would be a possible rule for Alicia's travel.

Graph It

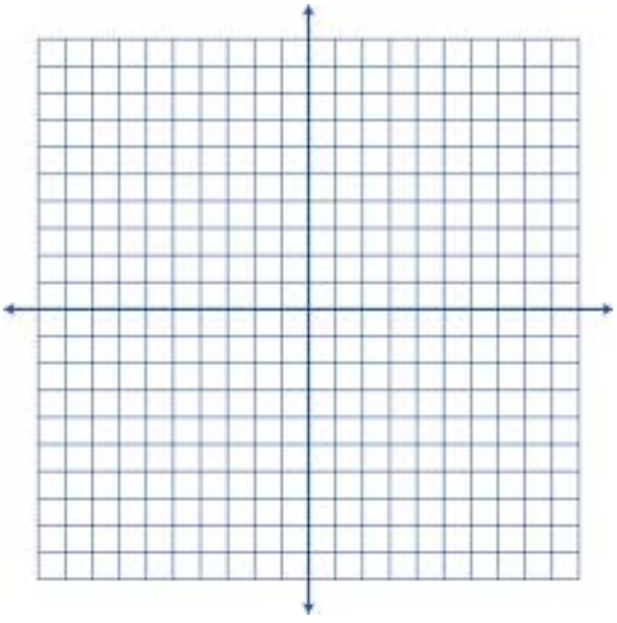
Create a graph, make a table, and write a function rule for the linear functions that meet the following conditions. Use one set of axes to graph all three lines.

1. The line has slope $-\frac{1}{2}$ and a *y*-intercept of 3.

2. The line is parallel to the line in problem 1.

3. The line has a *y*-intercept of 3 but a steeper slope than the line described in problem 1.

4. Compare and contrast the 3 lines.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

Scaffolding Questions:

- What point can be determined if you know that the *y*-intercept is 3?
- How will knowing the slope help you create a graph?
- How will knowing the slope help you create a table?
- What part of the equation must stay constant to produce a parallel line?

Sample Solution:

1. To draw the graph mark the *y*-intercept of 3 and use the slope to get another point on the graph. The slope $-\frac{1}{2}$ means that for every change in -1 of *y*, there is a change of 2 in *x*. Another point is (2,2).

   The equation of a line is of the form $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept.

   For this line the equation is $y = -\frac{1}{2}x + 3$.

x	*y*
-4	5
-2	4
0	3
2	2
4	1
6	0

2. If two lines are parallel, they have the same slope but a different *y*-intercept. The equation for one possible parallel line is $y = -\frac{1}{2}x + 6$.

   Any line of the form $y = -\frac{1}{2}x + b$ where *b* is any number would be correct.

x	y
-4	8
-2	7
0	6
2	5
4	4
6	3

3. The intercept is 3, but the slope must be different. Increase the absolute value of the slope to get a steeper line. For example, let the slope be -3.

   $y = -3x + 3$.

x	y
-4	15
-2	9
0	3
2	-3
4	-9
6	-15

   A positive number could also be selected. Let the slope be 3. $y = 3x + 3$.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(C) investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept
1.3 Exploring Rates of Change

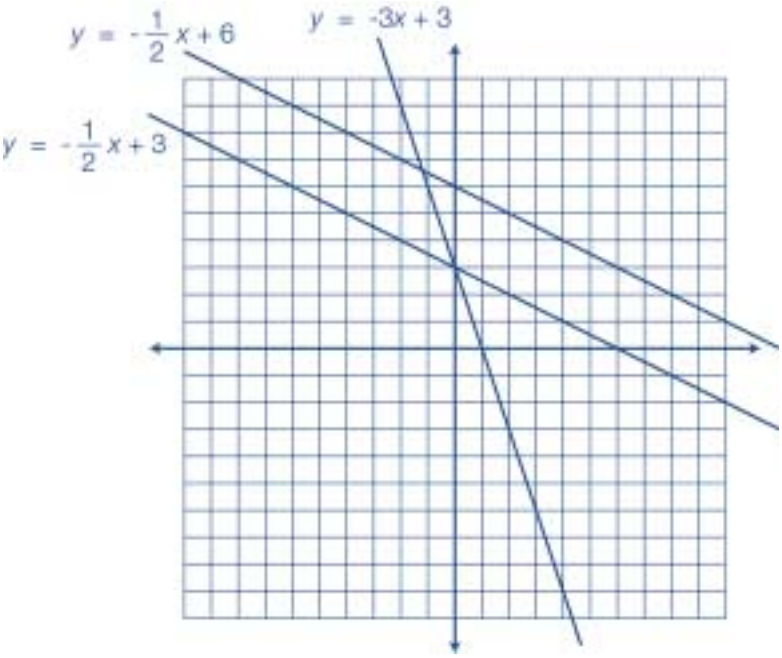
III. Nonlinear Functions
1 Quadratic Functions
1.3 Lines Do It Too

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions equations of lines to model problems involving real-world and mathematical situations.



4. The lines all have negative slopes and positive *y*-intercepts. The line for number 1 and number 2 have the same slope. They have different *y*-intercepts. They are the same distance apart. The third line intersects the other two; it has a slope of -3 and the same *y*-intercept as the line for number one.

Extension Questions:

- If another line is parallel to the first line and down 4 units, what is the equation of the new line?

The new line would have the same slope, but its y-intercept would be changed to 3 – 4 or -1. The equation would be $y = -\frac{1}{2}x - 1$.

- How are the table values affected by lowering the line 4 units?

All the y values would be decreased by 4.

x	$y = -\frac{1}{2}x + 3$	$y = -\frac{1}{2}x - 1$
-4	5	1
-2	4	0
0	3	-1
2	2	-2
4	1	-3
6	0	-4

- What is the equation of a line perpendicular to the first line and with the same *y*-intercept?

A perpendicular line has the opposite, reciprocal slope. The slope of the new line is +2.

The equation of the line is y = 2x + 3.

Summer Money

Debbie and Joey decided to earn money during the summer. Each student receives a weekly allowance and has taken a job. The graphs were used to model the weekly income including the allowance as a function of the number of hours worked.

1. Write a function rule that can be used to calculate the amount of money each student will have earned in terms of the number of hours worked in a week.

2. How will an increase in each of their allowances affect the table? The graph? The function rule? Give an example to justify your thinking.

3. How will an increase in hourly wages affect the table? The graph? The function rule ?

4. If Debbie's weekly allowance is doubled, will the new income be more or less than twice the original amount?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- What are the variables?
- What do you need to know to determine the equation of a line?
- What is the *y*-intercept for each situation?
- What does the *y*-intercept mean in the context of this situation?
- Explain how to find the slope from a graph.
- Describe what the slope represents for each line.
- Describe in words how much money Debbie will earn per week.

Sample Solution:

1. Use the fact that each person has a starting amount that will be the *y*-intercept of the function rule or the *b* in $y = mx + b$. This will be the amount of allowance that each person receives. The slope of the function rule is the rate of change per hour, which is the amount of money each person will get paid per hour. The slope will also be *m* in $y = mx + b$.

   Debbie's starting amount is \$8, and the rate of change from the point (0,8) to the point (1,16) is \$8 for one hour. The equation for this line is $y = 8x + 8$.

   Joey's starting amount is \$16, and the rate of change from the point (0,16) to the point (2,28) is \$12 for 2 hours or \$6 for one hour. The equation for this line is $y = 6x + 16$.

2. If a person's allowance is increased, the table will show that the corresponding income earned will increase by that amount. For example, if Debbie's allowance was increased by \$4.00, each of the *y* values in the table is increased by \$4.00.

Debbie's Income

Hours Worked	Income (dollars)
0	8
1	16
2	24
3	32
4	40
5	48
6	56
7	64
8	72

Allowance Increased by $4

Hours Worked	Income (dollars)
0	12
1	20
2	28
3	36
4	44
5	52
6	60
7	68
8	76

There is a constant difference of 4 in the *y*-values for the same *x* in the table. For example, the difference between the *y*-values for 8 hours in the two tables is 76 – 72 or 4.

The graph of the new situation will be a straight line parallel to the original line with a *y*-intercept of 12.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

In the function rule the constant term will change. For example, if Debbie's allowance is increased by $4.00, the new allowance will be $12.00, and the equation will be $y = 8x + 12$.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

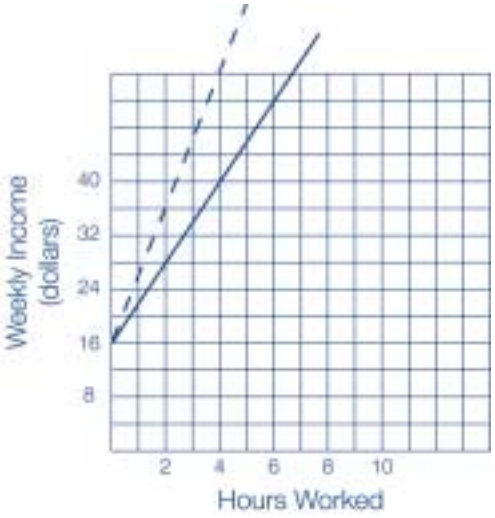
Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

3. If you increase the hourly wages in each situation, the constant rate of change per hour worked will increase. The slope of each line will get steeper, and in the function rule the coefficient of x will increase. For example, if Joey's hourly rate is increased by $4.00 per hour, the hourly rate becomes $10 per hour. His new equation becomes $y = 10x + 16$. This line will have the same y-intercept but a different slope.



Joey's Income

Hours Worked	Income (dollars)
0	16
1	22
2	28
3	34
4	40
5	46
6	52
7	58
8	64

Joey's Income with Increased Rate

Hours Worked	Income (dollars)
0	16
1	26
2	36
3	46
4	56
5	66
6	76
7	86
8	96

4. If Debbie's weekly allowance is doubled, then her function will change from $y = 8x + 8$ to $y = 8x + (2)8$.

   $y = 8x + 16$

   Her hourly wage remains the same.

   To get twice her original income we must double her original income. Since her original income is $y = 8x + 8$, then twice her original income would be $2y = 2(8x + 8) = 16x + 16$.

   The function rule for when her allowance doubled is $y = 8x + 16$. The function rule for twice her original income is $y = 16x + 16$.

   For any positive number x, $8x + 16 < 16x + 16$.

   The new income from doubling her allowance would be less than twice her original income because doubling her allowance does not affect her hourly wage, but doubling her original income increases her hourly wage.

Extension Questions:

- Which student will earn more money per week?

It depends on how many hours they work. If they work more than 4 hours, Debbie will earn more money. The y-values on Debbie's line are greater when x is greater than 4.

- What are reasonable domain values for this function?

It depends on whether the students may work portions of an hour. If they must work only whole number hours, the domain values will be whole numbers.

- The line is used to model the situation. Will all points on this graph represent the problem situation?

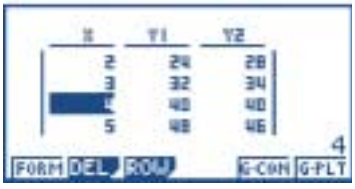
A person is usually paid for whole numbers of hours or perhaps half hours worked. The graph of the problem situation will really be sets of points and not the whole line.

- Describe the significance of the point of intersection of the two lines.

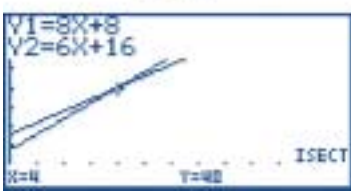
The point of intersection appears to be the point (4,40). This can be verified by examining the table or graph on the calculator or by substituting into the given functions.

Symbolic

Table



Graph



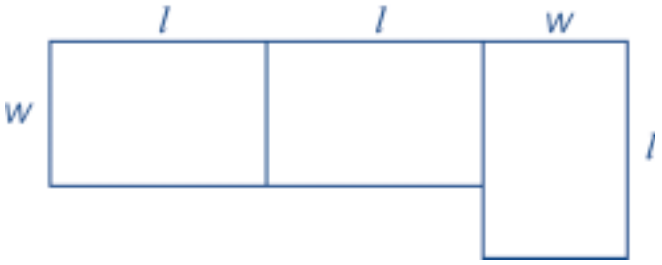
$$y = 6x + 16$$
$$40 = 6(4) + 16$$

$$y = 8x + 8$$
$$40 = 8(4) + 8$$

The point of intersection is (4,40). This means that when Debbie and Joey work for 4 hours they will both earn $40.

The Exercise Pen

Devin is planning to build exercise pens for his three horses. Because of the space that he has available, he has decided to create three rectangular pens as shown in the diagram. The three rectangular pens have the same size and shape. He has been advised that the perimeter of each of the three small pens should be 440 feet. He decides to use 1200 feet of fencing, because that is all he can afford. Use two different representations (tables, symbols, or graphs) to determine how he should construct the pen.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

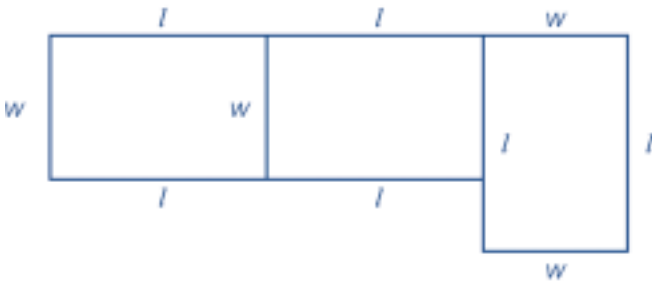
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the unknown quantities in this situation?
- What are some possible dimensions of the pens?
- How many different relationships are described in the problem?

Sample Solution:

The unknown dimensions are the length and width of the pen. In the diagram, the width of each small pen is represented by w, and the length of each small pen is represented by l.



The perimeter of each of the three smaller pens must be 440 feet. The perimeter is twice the length plus twice the width.

$2l + 2w = 440$ Equation 1

Divide by 2:
$l + w = 220$

The amount of fencing must be equal to 1200 feet. There are six lengths and four widths required to fence the pens.

$6l + 4w = 1200$ Equation 2

Divide by 2:
$3l + 2w = 600$

Subtract the first equation from the second equation.

$$3l + 2w = 600$$
$$2l + 2w = 440$$
$$l = 160$$

Substitute in equation 1 to solve for w.

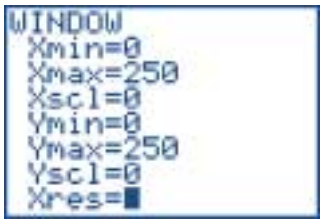
$$l + w = 220$$
$$160 + w = 220$$
$$w = 60$$

The pen should have measurements of approximately 60 feet and 160 feet.

Another method would be to solve each equation for the length, graph the equations, and find the point of intersection.

$$l + w = 220$$
$$w = 220 - l$$
$$3l + 2w = 600$$
$$w = \frac{600 - 3l}{2}$$

Enter the equations into the graphing calculator. Let the width be the y-value and the length be the x-value. Draw the graphs on the graphing calculator, and find the point of intersection.



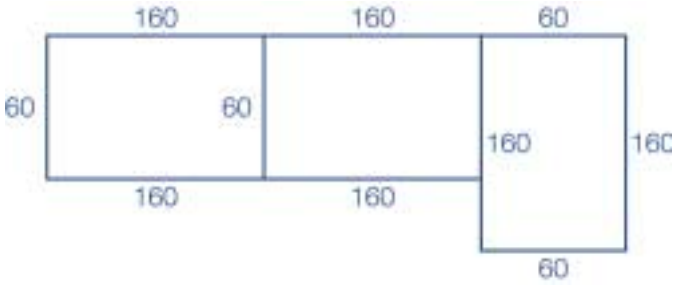
(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

The point of intersection is approximately (160,60). The *x*-value is the length of 160 feet and the *y*-value is the width of 60 feet. The fenced area would look like this diagram.



The perimeter of each small pen is 2(60) + 2(160) = 440 feet.
The total perimeter is 4(60) + 6(160) = 1200 feet.

Extension Questions:

- Two functions rules were used to create lines to represent the situation. Describe the domains and ranges for the functions and the domains and ranges for the problem situation.

The domain and range of each line function is the set of all real numbers. However, for the problem situation, the domain and range values are restricted to first quadrant values.

$$y = 220 - x \qquad 0 < x < 220 \qquad 0 < y < 220$$
$$y = \frac{600 - 3x}{2} \qquad 0 < x < 200 \qquad 0 < y < 300$$

For the two functions together, the domain is restricted to the intersection of the two domains $0 < x < 200$ and the range is restricted to $0 < y < 220$.

- How would your equations change if the total amount of fencing was 800 feet?

The first equation would not be different, but the total perimeter equation would become $6l + 4w = 800$ or $3l + 2w = 400$.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

- Solve this system and explain the solution.

Subtract the first equation from the second equation.

$$\begin{aligned} 3l + 2w &= 400 \\ 2l + 2w &= 440 \\ l &= -40 \end{aligned}$$

Since l represents the dimension of a rectangle, it may not be negative. The limit of 800 feet is not enough fence for the smaller pens to have a perimeter of 440 feet.

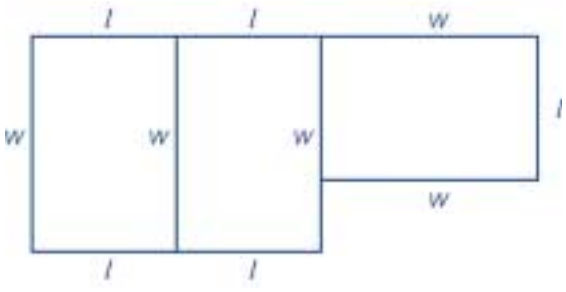
- Solve the problem if the amount of fencing is 1000 feet.

The second equation becomes $6l + 4w = 1000$ or $3l + 2w = 500$. Subtract the first equation from the second equation.

$$\begin{aligned} 3l + 2w &= 500 \\ 2l + 2w &= 440 \\ l &= 60 \\ w &= 220 - 60 = 160 \end{aligned}$$

The original diagram indicates that the longer side is the horizontal side in the diagram. The length , l , must be longer than the width, w. Thus, the values of $l = 60$ and $w = 160$ do not satisfy the conditions of the problem. The amount of fencing may not be 1000 feet.

- If the vertical measure in the diagram is longer than the horizontal measure, the diagram becomes



If the amount of fencing is 1000, what are the dimensions of the pen?

The perimeter of the smaller pen is still represented by the equation $2w + 2l = 440$ *or* $w + l = 220$.

The amount of fencing for this figure is represented by $5w + 5l$. *The amount of fencing is 1200.* $5w + 5l = 1200$ *or* $w + l = 240$.

The system of equations is

$w + l = 220$

$w + l = 240$.

This system has no solution. In the diagram above, the restriction on the perimeter $w + l = 220$ *means that* $5w + 5l = 5(220) = 1100$ *feet. He would purchase 1100 feet of fencing.*

The Run

The graph below gives information about two of the people who participated in a race. The over-seventy runner left first and the wheelchair rider left second. The line segments graphed represent the relationship between the distance traveled in miles from the starting gate and the number of minutes since the over-seventy runner began the race.

1. Use the graph to describe the situation for each runner.

2. Give the function rule that models each racer's trip.

3. What does the slope of each line represent?

4. What is the point of intersection of the two lines, and what does it mean?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

Scaffolding Questions:

- Which line segment represents the wheelchair rider? How do you know?
- Explain how to determine the rate at which the wheelchair rider is traveling.
- Which segment represents the over-seventy runner and what is his rate?
- What is the rate for the wheelchair rider?
- Which runner took the least amount of time to complete the race?

Sample Solution:

1. The over-seventy runner is the person who started when the time is 0. That first point is the point (0,1). He started one mile ahead of the starting line. The points (0,1) and (10,2) may be used to find the rate at which he ran. The rate of change from the point (0,1) to the point (10,2) is

   1 mile for 10 minutes. His rate is $\frac{1}{10}$ of a mile per minute.

   The equation for his run is the starting value plus the rate times the number of minutes.

   $y = 1 + \frac{1}{10}x$

   Each of the line segments stop when y is 13; therefore, the race length is 13 miles from the starting line. The y-value is 13 when x is 60 minutes. The over-seventy runner started the race at the 1-mile mark, so he traveled 12 miles in 120 minutes.

   The second person to leave is the wheelchair rider who left 20 minutes after the over-seventy runner. The wheelchair rider started at the starting line. The first point on the graph is the point (20,0). Another point on the graph is the point (80,13). The rate of change from (20,0) to (80,13) is

   13 miles for 60 minutes. His rate is $\frac{13}{60}$ miles per minute.

   He started at 0 miles at 20 minutes and ran at $\frac{13}{60}$ miles per minute. The equation of his run is

   $y = \frac{13}{60}(x - 20) + 0.$

2. The slopes of the lines represent the rate at which each person traveled. The wheelchair rider traveled 13 miles per 60 minutes. The over-seventy runner traveled at 1 mile per 10 minutes.

3. The graphs of $y = \frac{1}{10}x + 1$ (over-seventy runner) and $y = \frac{13}{60}(x - 20) + 0$ (wheelchair rider) intersect at a point, about (45,5.5).

   The wheelchair rider was traveling at a faster rate and was ahead of the over-seventy runner for the rest of the race.

   To determine the exact values, the equation may be solved.

   $$\frac{1}{10}x + 1 = \frac{13}{60}(x - 20) + 0$$

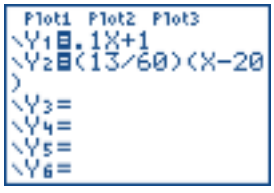
   Multiply both sides of the equation by 60.

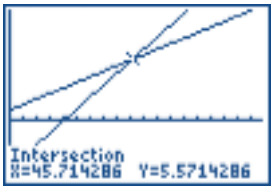
   $$\begin{aligned} 6x + 60 &= 13(x - 20) \\ 6x + 60 &= 13x - 260 \\ -7x &= -320 \\ x &= \frac{-320}{-7} = \frac{320}{7} = 45\frac{5}{7} \end{aligned}$$

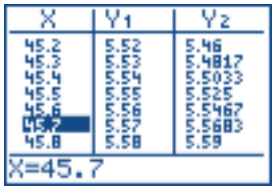
   $$y = \frac{1}{10}x + 1 = \frac{1}{10}\left(\frac{320}{7}\right) + 1 = \frac{39}{7} = 5\frac{4}{7}$$

   At $45\frac{5}{7}$ minutes the wheelchair rider overtakes the over-seventy runner at $5\frac{4}{7}$ miles from the starting gate.

   The calculator may also be used to determine the point of intersection.







The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and a y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

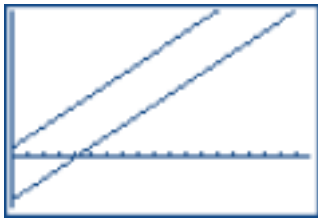
Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Extension Questions:

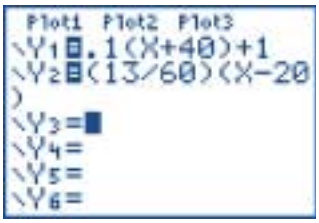
- Describe a way in which the over-seventy runner could have won the race.

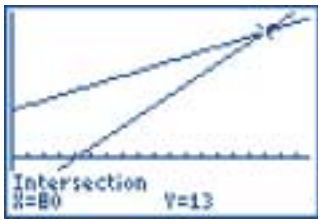
If he had traveled at the same rate as the wheelchair racer, the wheelchair racer would never catch up with him since he left first.



- If both racers traveled at their original rates, what factors or variables could be changed so that the over-seventy runner wins the race?

From the given graph it can be seen that the over-seventy runner ended 40 minutes after the wheelchair rider. If the over-seventy runner's graph is moved 40 units to the left, (x + 40) replaces x, they would end at the same time. That is the over-seventy runner needed an additional head start of 40 minutes to tie the wheelchair racer.





Thus, to win the race, the over seventy runner's rule is changed to reflect an added amount greater than 40. Then he would win. For example, replace x with x + 41.

$y = 0\ .1(x + 41) + 1$
$y = 0.1x + 4.1 + 1$
$y = 0.1x + 5.2$

He would need to start 5.2 miles from the starting line to beat the wheelchair rider.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundatons for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns
3 Interpreting Graphs
3.1 Interpret Distance vs. Time

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch
3 Linear Equations and Inequalities
3.3 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

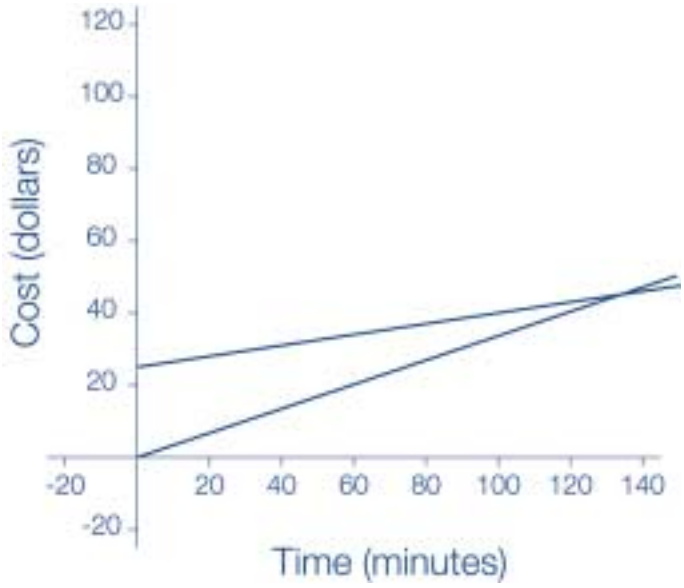
Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Which Plan is Best?

Students were given two cellular phone options to compare.

Plan 1: $C = \$0.35m$
Plan 2: $C = \$0.15m + \$25$

C represents the cost in dollars, and m represents the time in minutes. One group graphed the two plans.



1. Explain the differences between the two plans.

2. Explain the meaning of the slope for each plan.

3. Explain the meaning of the *y*-intercept for each plan.

4. Which plan do you think is a better deal? Explain your thinking.

5. If the second plan was changed to 20 cents per minute, what would be different about the graph?

6. If the first plan was changed so that the base fee was changed to $10, how would the graph have changed?

Teacher Notes

Scaffolding Questions:

- What does the slope represent for each plan?
- What does the *y*-intercept represent for each plan?
- What does the point of intersection mean in the context of this problem?

Sample Solution:

1. In Plan 1 the charge is \$0.35 per minute. In Plan 2 the charge is only \$0.15 per minute, but there is another fee of \$25.

2. In this situation the slope is the rate of change per minute or the amount of money charged per minute in each plan. In Plan 1 the slope is \$0.35. In Plan 2 the slope is \$0.15.

3. In this situation the y-intercept is the charge at zero minutes. In Plan 1 the *y*-intercept is zero because there is no charge for the plan itself. In Plan 2 the *y*-intercept is 25 because there is a \$25 charge to have this plan.

4. The best deal will be determined by the number of minutes you plan to use. Examine the table for the two plans:

If you are going to use less than 125 minutes, you should go with Plan 1. If you were going to use more than 125 minutes, the best plan for you would be Plan 2.

5. The slope of the graph would change. The new graph would have a steeper slope.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

6. The *y*-intercept of the original Plan 1 is 0. If a base fee of $10 was added, all points would rise up 10 units. The *y*-intercept would be changed to 10.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Extension Questions:

- Suppose the cellular phone companies that are offering these plans decided to merge. Together they have come up with a new plan. The new plan charges the customer $0.25 per minute but also gives the first 40 minutes free. The customer doesn't start paying until he has used 40 minutes of airtime. What is the function rule for this new plan?

First make a chart.

Minutes	Cost
40	$0.00
41	$0.25
42	$0.50
43	$0.75
44	$1.00

The change is 25 cents per minute or $2.50 for 10 minutes. Go back $2.50 for every 10 minutes. There are negative costs for less than 40 minutes. This negative cost represents the money the customer is not paying or the money the customer is saving. Continuing to backtrack in the chart tells us that at 0 minutes the cost is -$10.00. Therefore, the customer saves $10.00 when using the 40 free minutes.

Minutes	Cost
0	-$10.00
10	-$7.50
20	-$5.00
30	-$2.50
40	$0.00
41	$0.25
42	$0.50
43	$0.75
44	$1.00

The intercept value is -$10.00. Our function rule is C = $0.25x – $10.00, where x is the number of minutes and x must be greater than or equal to 40.

- How much money does the company lose per customer by giving away 40 minutes of airtime?

The company loses $10.00 per customer.

- Describe ways to change the method of charging in Plan 1 so that it would also be a better deal than the second plan.

If they charged the same base rate as the second plan but decreased the slope, their fee would always be less. For example, they could charge 14 cents per minute with a base fee of $25.
C = 0.14m + 25

Another plan would be to charge the same rate per minute but decrease the base fee of $25. Their price would be less for any number of minutes. For example, they could charge 15 cents per minute with a base fee of $20.
C = 0.15m + 20

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.3 Lines Do It Too

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.