Texas Version

Algebra I Assessments

The Charles A. Dana Center
at the University of Texas at Austin

With funding from
the Texas Education Agency and
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The Texas Education Agency

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The Charles A. Dana Center at the University of Texas at Austin works to support education leaders and policymakers in strengthening Texas education. As a research unit of UT Austin's College of Natural Sciences, the Dana Center maintains a special emphasis on mathematics and science education. We offer professional development institutes and produce research-based mathematics and science resources for educators to use in helping all students achieve academic success. For more information, visit the Dana Center website at www.utdanacenter.org.

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TEKS and TAKS Resources

The mathematics Texas Essential Knowledge and Skills (TEKS) were developed by the state of Texas to clarify what all students should know and be able to do in mathematics in Kindergarten through Grade 12. Districts are required to provide instruction to all students in the mathematics TEKS, which were adopted by the State Board of Education in 1997 and implemented statewide in 1998. The mathematics TEKS also form the objectives and student expectations for the mathematics portion of the Texas Assessment of Knowledge and Skills (TAKS), which will be implemented in spring 2003 for Grades 3 through 10 and for the Grade 11 Exit Level assessment.

The TEKS for mathematics can be downloaded in printable format, free of charge, from the Texas Education Agency website (www.tea.state.tx.us/teks). Bound versions of the mathematics and science TEKS are available from the Charles A. Dana Center at The University of Texas at Austin (www.utdanacenter.org).

Resources for implementing the mathematics TEKS, including professional development opportunities, are available through the Texas Education Agency and the Charles A. Dana Center, the state-designated Mathematics Center for Educator Development. Online resources can be found in the Mathematics TEKS Toolkit at www.tenet.edu/teks/math.

Additional products and services that may be of interest are available from the Dana Center at www.utdanacenter.org. These include the following:

- TEKS, TAAS, and TAKS: What's Tested at Grades 3–8? charts
- Mathematics Abridged TEKS charts
- Mathematics TEKS "Big Picture" posters
- Mathematics Standards in Action (available summer 2002)
- Geometry Assessments (available fall 2002) and corresponding professional development
- TEXTEAMS professional development mathematics institutes
- TEKS for Leaders professional development modules for principals and other administrators

Introduction

The Dana Center has developed *Algebra I Assessments* as a resource for teachers to use to provide ongoing assessment integrated with algebra instruction.

The National Council of Teachers of Mathematics (2000) has identified the following six standards to guide classroom assessment:

- Assessment should reflect the mathematics that all students need to know and be able to do.
- Assessment should enhance mathematics learning.
- Assessment should promote equity.
- Assessment should be an open process.
- Assessment should promote valid inferences about mathematics learning.
- Assessment should be a coherent process.

Implementing these assessment standards may require significant changes in how teachers view and use assessment in the mathematics classroom. Teachers should assess frequently to monitor individual performance and guide instruction.

What are the Algebra Assessments?

The assessments are algebra problems that reflect what all students need to know and be able to do in first-year Algebra. These assessments may be formative, summative, or ongoing. The problems focus on students' understanding as well as their procedural knowledge. The tasks require more than right or wrong answers; they focus on how students are thinking about a situation.

What is the purpose of the Algebra Assessments?

The purpose of these assessments is to make clear to teachers, students, and parents what is being taught and learned. Teachers should use evidence of student insight, student misconceptions, and problem-solving strategies to guide their instruction.
Teachers may also use the questions included with the assessments to guide learning and to assess student understanding. The use of these assessments should help teachers enhance student learning and provide them with a source of evidence on which they may base their instructional decisions.

What is the format of the Algebra Assessments?

This book contains 75 problems divided into two groups: the Core group and the Supplemental group. Collectively the Core group of problems addresses all the student expectations for Algebra I. The Supplemental group of problems is provided for practice or as substitution for problems in the Core group.

The problems have been divided into five categories:

Function Fundamentals
Linear Functions
Related Linear Functions and System of Equations
Quadratic Functions
Inverse Variation, Exponential Functions, and Other Functions

Each problem:

- Includes an algebra task;
- Is aligned with the Algebra I Texas Essential Knowledge and Skills (TEKS) Student Expectations;
- Is aligned with the Grade 11 Exit Level Texas Assessment of Knowledge and Skills (TAKS) objectives;
- Is aligned with the Algebra End-of-Course (EOC) Exam objectives;
- Is aligned with the TEXTEAMS Algebra I: 2000 and Beyond Professional Development Institute;
- Includes "scaffolding" questions that the teacher may use to help the student to analyze the problem;
- Provides a sample solution*; and
- Includes extension questions to bring out additional mathematical concepts in a summative discussion of solutions to the problem.

*The sample solution is only one way that a problem may be approached. There are other approaches that may provide a correct analysis of the problem. The authors have attempted to illustrate a variety of methods in the different problem solutions. To illustrate another approach, a sample student solution is included with some problems.

Following this introduction are alignments of all the problems to the TEKS and to the TAKS Grade 11 Exit Objectives. A solution guide is included that the student may use to help understand what is necessary for a complete problem solution.

TEXTEAMS Practice-Based Professional Development: Algebra I Assessments

The Dana Center offers a three-day TEXTEAMS institute to allow participants to experience selected assessments, examine the assessments for alignment with the TEKS and TAKS, analyze student work to evaluate student understanding, consider methods for evaluating student work, view a video of students working on the assessments, develop strategies for classroom implementation, and consider how the assessments support the TAKS. Teachers should contact their local school district or regional service center to determine when this institute is offered.

Mathematics TEKS Alignment

This chart indicates the Texas Essential Knowledge and Skills (TEKS) student expectations addressed by each problem. The student expectation has been included only if the problem specifically asks a question that requires mastery of that student expectation.

		FOUNDATIONS FOR FUNCTIONS				LINEAR FUNCTIONS				QUADRATIC AND NONLINEAR		
	Page Number	(b.1) A-E	(b.2) A-D	(b.3) A,B	(b.4) A,B	(c.1) A,B,C	(c.2) A-G	(c.3) A,B,C	(c.4) A,B,C	(d.1) A-D	(d.2) A,B	(d.3) A,B,C
A Ring Around the Posies	329	C,D,E		A,B	A,B						A	
Analysis of a Function	209	D	B,C			C	E,F			A,C		
Bathing the Dog	181	A,E				A						
Bears's Band Booster Club	99	C,D,E	B	A	A	B,C		A	A,B,C			
Block That Kick	333	C,D,E			A					A,D	A,B	
Bonnie's Dilemma	285			A,B					A,B,C			
Bright Lights	161	B,C,E	C,D	A,B	A							A,C
BRRR!	339	D,E			A					A,D	A,B	
Calculating Cost	343	D,E			A					D	A	
CDs for the Band	41	C,D		A	B	B		A,B,C				
College Tuition	413	A.C.D.E		A,B	A							A
Constructing Houses	419	A,B,C		A,B	A							B
Cost and Profit	105	C,D,E	C	A		C	A,B	A	A,B,C			
Create a Situation	217	D,E	C				B,E					
Distance and Time	185	D	C			C	B					

		FOUNDATIONS FOR FUNCTIONS				LINEAR FUNCTIONS				QUADRATIC AND NONLINEAR		
	Page Number	(b.1) A-E	(b.2) A-D	(b.3) A,B	(b.4) A,B	(c.1) A,B,C	(c.2) A-G	(c.3) A,B,C	(c.4) A,B,C	(d.1) A-D	(d.2) A,B	(d.3) A,B,C
Explorer's Glide	45	A,B,C,D,E		A,B	A	B						
Exploring Exponentional Functions	425	B,C,D,E	B,D	A,B	A						A,C	
Extracurricular Activities	189	C	B			B					A,B	
Finding Pairs	221	C,E		A	A,B	C						
Fireworks Celebration	133	C,D,E	B	A	A					A,B,C,D	A,B	
First Aid Supplies	225	C,D,E		A	A,B			A,B,C				
Four Cars	291	E	C			C	B					
Function Families	431		B			B				A		
Gas Tank	229	D,E	C			C	B					
Geothermal Energy	51	C,D,E			A	A,B,C		B				
Golfing	141	C,D								A,D	A,B	
Graph It	297	C,D		A	C	C,D						
Greetings	233	C		A				A,B,C				
Grocery Carts	239	B,C,D,E		A,B	A	A,B,C						
Home Improvements	145	C,D,E	B		A					D	A,B	
Hot-Air Balloon	55	A,C,D,E				A	B,C,D,E,F	A,B				
How Much Paint?	149	C,D,E	B	A	A					A,D	A,B	
Hull Pressure	243	A,C,D,E	B				A,B,E,F,G	B,C				
Insects in the Water	153	C,D,E	B	A,B						A,D	A,B	
Investigating the Effect of *a* and *c* on the Graph of $y = ax^2 + c$	347	D	A							B,C		
Making Pizzas, Making Money	61	C,D		A		C	A,B,F					

	Page Number	FOUNDATIONS FOR FUNCTIONS				LINEAR FUNCTIONS				QUADRATIC AND NONLINEAR		
		(b.1) A-E	(b.2) A-D	(b.3) A,B	(b.4) A,B	(c.1) A,B,C	(c.2) A-G	(c.3) A,B,C	(c.4) A,B,C	(d.1) A-D	(d.2) A,B	(d.3) A,B,C
Making Stuffed Animals	195	C,E		A,B	A	A,C		A,B,C				
Math-a-thon	247	C,D		A		C	B,F					
Mathematical Domain and Range of Nonlinear Functions	437	D	B							A		
Mosaics	25	C,D		A,B		C		A				
Motion Dectector Problem	111	A,C,D,E	C									
Music and Mathematics	169	B,C,D,E	D	A,B	A							B
Nested Rectangles	201	A,B,C,D,E		A		A,B,C	G					
Ostrich Pen	359	D,E	B	A						A,D		
Paper Boxes	447	A,C,D,E		A,B	B							
Pool Problem	65			A,B	A		A,B					
Rebound Height	453	A,B,C,D,E	D									
Recycling	251	D			A			B,C				
Seeing the Horizon	365	A,C,D,E	B	B	A					A	A	
Shopping	255	D,E		A	A,B			A,B				
Sky Diving	373	C,D								A,D	A,B	
Sound Travel	259			A				A,B				
Speeding Cars	117	B	AB			B	A,B,D,G					
Stacking Paper Cups	73	B,C,D,E		A,B	A			A,B				
Stretched Spring	81	B,C,D,E	D	A.B	A	A,C	A,B,E	A				
Summer Money	303	C,D,E	C	A		C	A,B,C,D,E,F					
Supply and Demand	379	C,D,E		A,B						A,D	A,B	

	Page Number	FOUNDATIONS FOR FUNCTIONS (b.1) A-E	(b.2) A-D	(b.3) A,B	(b.4) A,B	LINEAR FUNCTIONS (c.1) A,B,C	(c.2) A-G	(c.3) A,B,C	(c.4) A,B,C	QUADRATIC AND NONLINEAR (d.1) A-D	(d.2) A,B	(d.3) A,B,C
Swimming Pools	33	A,D,E				A,C	F					
Taxi Ride	263	E		A,B	A	A,B,C		A,B,C				
The 600-Meter Race	29	D,E	C			C	B					B,C
The Contractor	269	C,D,E		AB	A	C		A,B,C				
The Dog Run	385	A,D,E								A.D		
The Exercise Pen	311			A,B	A				A,B,C			
The Garden	273	A,C,D,E		A,B		A,B,C						
The Marvel of Medicine	173	C,D,E	B.D	A,B	A							A,C
The Run	317	C,D,E	C			C	A,B,D,E		A,B,C			
The Submarine	277	A,C,D,E		A		C	A,B,C,F	A,B				
The Walk	123	A,B,C,D,E	B,C	A		B	A,B,D	A	A,B,C			
Transformations of Quadratic Functions	391		A							B,C,D		
T-Shirts	87	D	B			B,C						
What is Reasonable?	457	A,C,D	B							A		A,B
What is the Best Price?	399	B,C,D,E	D	A,B		A				D		
Which is Linear?	93	C,E		A,B		A,C	A,B					
Which Plan is Best?	321	C.E				C	A,B,E.F					
Window Panes	407			A	A,B							

Mathematics TAKS Alignment

This chart shows the problems that have been aligned to the Grade 11 Exit Level TAKS based on the alignment to the Algebra I TEKS. A teacher may use the problem to assess the indicated objective.

	PROBLEM NAME	TEKS ADDRESSED
TAKS Objective 1:	Bathing the Dog	(b.1)AE,(c.1)A
	Bears's Band Booster Club	(b.1)CDE,(b.3)AB,(b.4)A,(c.1)BC,(c.3)A,(c.4)ABC
	Bonnie's Dilemma	(b.3)AB,(c.4)ABC
	Bright Lights	(b.1)BCE,(b.2)CD,(b.3)AB,(b.4)A,(d.3)AC
	College Tuition	(b.1)ACDE,(b.3)AB, (b.4)A, (d.3)A
	Constructing Houses	(b.1)ABC,(b.3)AB,(b.4)A,(d.3)B
	Exploring Exponentional Functions	(b.1)BCDE,(b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	Four Cars	(b.1)E,(b.2)C,(c.1)C,(c.2)B
	Gas Tank Problem	(b.1)DE,(b.2)C,(c.1)C,(c.2)B
	Golfing	(b.1)CD,(d.1)AD,(d.2)AB
	Hot Air Balloon	(b.1)ACDE,(c.1)A,(c.2)BCDEF,(c.3)AB
	How Much Paint?	(b.1)CDE,(b.2)B,(b.3)A,(b.4)A,(d.1)AD,(d.2)AB
	Hull Pressure	(b.1)ACDE,(c.2)ABEFG,(c.3)BC
	Insects in the Water	(b.1)CDE,(b.2)B,(b.3)AB,(d.1)ABCD, (d.2)AB
	Investigating the Effects of a and c on the Graph	(b.1)D,(b.2)A,(d.1)BC
	Mathematical Domains and Ranges of Functions	(b.1)D, (b.2)BC, (d.1)A
	Motion Detector Problem	(b.1)ACDE,(b.2)C
	Music and Math	(b.1)BCDE,(b.2)D,(b.3)AB, (b.4)A, (d.3)B
	Paper Boxes	(b.1)ACDE,(b.3)AB,(b.4)B
	Rebound Height	(b.1)ABCDE,(b.2)D,(d.3)C
	Seeing the Horizon	(b.1)ACDE,(b.2)B,(b.3)B,(b.4)A,(d.1)A,(d.2)A
	Speeding Cars	(b.1)B,(b.3)AB,(c.1)ABC,(c.2)ABDG
	Supply and Demand	(b.1)CDE,(b.3)AB,(c.4)A,(d.2)AB
	Swimming Pools	(b.1)ADE,(c.1)AC,(c.2)F
	The Dog Run	(b.1)ADE,(b.2)A, (b.3)AB,(d.1)AD
	The Garden	(b.1)ACDE,(b.3)AB,(c.1)ABC
	The Marvel of Medicine	(b.1)CDE, (b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	The Run	(b.1)CDE,(b.2)C,(c.1)C,(c.2)ABDE,(c.4)ABC
	The Submarine	(b.1)ACDE,(b.3)A,(c.1)C,(c.2)ABCF,(c.3)AB
	What is the Best Price?	(b.1)BCDE,(b.2)D,(b.3)AB,(c.1)A,(d.1)D

Mathematics TAKS Alignment (continued)

	PROBLEM NAME	TEKS ADDRESSED
TAKS Objective 2:	Analysis of a Function	(b.1)D,(b.2)BC,(c.1)C,(c.2)EF
	Bears's Band Booster Club	(b.1)CDE,(b.3)AB,(b.4)A,(c.1)BC,(c.3)A,(c.4)ABC
	Bonnie's Dilemma	(b.3)AB,(c.4)ABC
	Calculating Cost	(b.1)DE,(b.4)A,(d.1)D,(d.2)A
	College Tuition	(b.1)ACDE,(b.3)AB, (b.4)A, (d.3)A
	Create a Situation	(b.1)DE,(b.2)C,(c.2)BE
	Distance and Time	(b.1)D,(b.2)C,(c.1)C,(c.2)B
	Explorer's Glide	(b.1)ABCDE,(b.3)AB,(b.4)A,(c.1)B
	Exploring Exponentional Functions	(b.1)BCDE,(b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	Extracurricular Activities	(b.1)C,(b.2)B,(c.1)B
	First Aid Supplies	(b.1)CDE,(b.3)A,(b.4)AB,(c.3)ABC
	Four Cars	(b.1)E,(b.2)C,(c.1)C,(c.2)B
	Function Families	(b.2)B,(c.1)B,(d.1)A
	Gas Tank Problem	(b.1)DE,(b.2)C,(c.1)C,(c.2)B
	Geothermal Energy	(b.1)ACDE,(b.4)A,(c.1)ABC,(c.3)B
	Grocery Carts	(b.1)BCDE,(b.3)AB,(b.4)A,(c.1)ABC
	Insects in the Water	(b.1)CDE,(b.2)B,(b.3)AB,(d.1)ABCD, (d.2)AB
	Investigating the Effects of a and c on the Graph	(b.1)D,(b.2)A,(d.1)BC
	Mathematical Domains and Ranges of Functions	(b.1)D, (b.2)BC, (d.1)A
	Mosaics	(b.1)CD,(b.3)AB,(c.1)C,(c.3)A
	Motion Detector Problem	(b.1)ACDE,(b.2)C
	Music and Math	(b.1)BCDE,(b.2)D,(b.3)AB, (b.4)A, (d.3)B
	Paper Boxes	(b.1)ACDE,(b.3)AB,(b.4)B
	Shopping	(b.1)DE,(b.3)A,(b.4)AB,(c.3)AB
	Sound Travel	(b.3)A,(b.4)A,(c.2)BG,(c.3)AB
	Speeding Cars	(b.1)B,(b.3)AB,(c.1)ABC,(c.2)ABDG
	Stacking Paper Cups	(b.1)BCDE,(b.3)AB,(b.4)A,(c.3)AB
	Stretched Spring	(b.1)BCDE,(b.2)D,(b.3)AB,(b.4)A,(c.1)AC,(c.2)ABE,(c.3)A
	Supply and Demand	(b.1)CDE,(b.3)AB,(c.4)A,(d.2)AB
	The 600-Meter Race	(b.1)DE,(b.2)C,(c.1)C,(c.2)B
	The Garden	(b.1)ACDE,(b.3)AB,(c.1)ABC
	The Marvel of Medicine	(b.1)CDE, (b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	The Run	(b.1)CDE,(b.2)C,(c.1)C,(c.2)ABDE,(c.4)ABC
	What is the Best Price?	(b.1)BCDE,(b.2)D,(b.3)AB,(c.1)A,(d.1)D
	Window Panes	(b.3)A,(b.4)AB

Mathematics TAKS Alignment (continued)

TAKS Objective 3:

PROBLEM NAME	TEKS ADDRESSED
Bathing the Dog	(b.1)AE,(c.1)A
Bonnie's Dilemma	(b.3)AB,(c.4)ABC
Create a Situation	(b.1)DE,(b.2)C,(c.2)BE
Distance and Time	(b.1)D,(b.2)C,(c.1)C,(c.2)B
Explorer's Glide	(b.1)ABCDE,(b.3)AB,(b.4)A,(c.1)B
Finding Pairs	(b.1)CE,(b.3)A,(b.4)AB,(c.1)C
Four Cars	(b.1)E,(b.2)C,(c.1)C,(c.2)B
Gas Tank Problem	(b.1)DE,(b.2)C,(c.1)C,(c.2)B
Graph It	(b.1)CD,(b.3)A,(c.1)C,(c.2)CD
Grocery Carts	(b.1)BCDE,(b.3)AB,(b.4)A,(c.1)ABC
Hot Air Balloon	(b.1)ACDE,(c.1)A,(c.2)BCDEF,(c.3)AB
Hull Pressure	(b.1)ACDE,(c.2)ABEFG,(c.3)BC
Making Pizzas, Making Money	(b.1)CD,(b.3)A,(c.1)C,(c.2)ABF
Making Stuffed Animals	(b.1)CE,(b.3)AB,(b.4)A,(c.1)AC,(c.3)ABC
Math-a-thon	(b.1)CD,(b.3)A,(c.1)C,(c.2)BF
Mosaics	(b.1)CD,(b.3)AB,(c.1)C,(c.3)A
Nested Rectangles	(b.1)ABCDE,(b.3)A,(c.1)ABC,(c.2)G
Pool Problem	(b.3)AB,(b.4)A,(c.2)AB
Sound Travel	(b.3)A,(b.4)A,(c.2)BG,(c.3)AB
Speeding Cars	(b.1)B,(b.3)AB,(c.1)ABC,(c.2)ABDG
Stacking Paper Cups	(b.1)BCDE,(b.3)AB,(b.4)A,(c.3)AB
Stretched Spring	(b.1)BCDE,(b.2)D,(b.3)AB,(b.4)A,(c.1)AC,(c.2)ABE,(c.3)A
Summer Money	(b.1)DE,(b.2)C,(b.3)A,(c.1)C,(c.2)ABCDEF
The Garden	(b.1)ACDE,(b.3)AB,(c.1)ABC
The Run	(b.1)CDE,(b.2)C,(c.1)C,(c.2)ABDE,(c.4)ABC
The Submarine	(b.1)ACDE,(b.3)A,(c.1)C,(c.2)ABCF,(c.3)AB
Which is Linear?	(b.1)CE,(b.3)AB,(c.1)A,(c.2)AB
Which Plan is Best?	(b.1)CE,(c.1)C,(c.2)ABEF

Mathematics TAKS Alignment (continued)

	PROBLEM NAME	TEKS ADDRESSED
TAKS Objective 4:	Bears's Band Booster Club	(b.1)CDE,(b.3)AB,(b.4)A,(c.1)BC,(c.3)A,(c.4)ABC
	Bonnie's Dilemma	(b.3)AB,(c.4)ABC
	CDs for the Band	(b.1)CD,(b.3)A,(b.4)B,(c.1)B,(c.3)ABC
	Cost and Profit	(b.1)CDE,(b.2)C,(b.3)A,(c.1)C,(c.2)AB,(c.3)A,(c.4)ABC
	First Aid Supplies	(b.1)CDE,(b.3)A,(b.4)AB,(c.3)ABC
	Geothermal Energy	(b.1)ACDE,(b.4)A,(c.1)ABC,(c.3)B
	Greetings	(b.1)C,(b.3)A,(c.3)ABC
	Recycling	(b.1)D,(b.4)A,(c.3)BC
	Sound Travel	(b.3)A,(b.4)A,(c.2)BG,(c.3)AB
	Stretched Spring	(b.1)BCDE,(b.2)D,(b.3)AB,(b.4)A,(c.1)AC,(c.2)ABE,(c.3)A
	Taxi Ride	(b.1)CE,(b.3)AB,(b.4)A,(c.1)ABC,(c.3)ABC
	The Contractor	(b.1)CDE, (b.3)AB, (b.4)A,(c.1)C,(c.3)ABC
	The Exercise Pen	(b.2)AB(b.4)A,(c.4)ABC
	The Run	(b.1)CDE,(b.2)C,(c.1)C,(c.2)ABDE,(c.4)ABC
	The Submarine	(b.1)ACDE,(b.3)A,(c.1)C,(c.2)ABCF,(c.3)AB
	The Walk	(b.1)ABCDE,(b.2)BC,(b.3)A,(c.1)BC,(c.2)ABD,(c.3)A,(c.4)ABC
	T-Shirts	(b.1)D,(b.2)B,(c.1)BC

Mathematics TAKS Alignment (continued)

	PROBLEM NAME	TEKS ADDRESSED
TAKS Objective 5:	A Ring Around the Posies	(b.1)CDE,(b.3)AB,(b.4)AB,(d.2)A
	Block That Kick!!!	(b.1)CDE,(b.4)A,(d.1)AD,(d.2)AB
	Bright Lights	(b.1)BCE,(b.2)CD,(b.3)AB,(b.4)A,(d.3)AC
	BRRRR!!	(b.1)DE,(b.4)A,(d.1)AD,(d.2)AB
	Calculating Cost	(b.1)DE,(b.4)A,(d.1)D,(d.2)A
	College Tuition	(b.1)ACDE,(b.3)AB, (b.4)A, (d.3)A
	Constructing Houses	(b.1)ABC,(b.3)AB,(b.4)A,(d.3)B
	Exploring Exponentional Functions	(b.1)BCDE,(b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	Fireworks Celebration	(b.1)CDE,(b.2)B,(b.3)A,(b.4)A,(d.1)ABCD,(d.2)AB
	Function Families	(b.2)B,(c.1)B,(d.1)A
	Golfing	(b.1)CD,(d.1)AD,(d.2)AB
	Home Improvements	(b.1)CDE,(b.2)B,(b.4)A,(d.1)D,(d.2)AB
	How Much Paint?	(b.1)CDE,(b.2)B,(b.3)A,(b.4)A,(d.1)AD,(d.2)AB
	Investigating the Effects of a and c on the Graph	(b.1)D,(b.2)A,(d.1)BC
	Ostrich Pen	(b.1)DE,(b.2)B,(b.3)A,(d.1)AD
	Paper Boxes	(b.1)ACDE,(b.3)AB,(b.4)B
	Sky Diving	(b.1)CD,(d.1)AD,(d.2)AB
	Supply and Demand	(b.1)CDE,(b.3)AB,(c.4)A,(d.2)AB
	The Dog Run	(b.1)ADE,(b.2)A, (b.3)AB,(d.1)AD
	The Marvel of Medicine	(b.1)CDE, (b.2)BD, (b.3)AB, (b.4)A, (d.3)AC
	Transformations of Quadratic Functions	(b.2)A,(d.1)BCD
	What is Reasonable?	(b.1)ACD,(b.2)B,(d.1)A,(d.3)BC
	What is the Best Price?	1(b.1)BCDE,(b.2)D,(b.3)AB,(c.1)A,(d.1)D

Criteria Checklist Solution Guide

Name of Student: ________________________________

Name of Problem: ________________________________

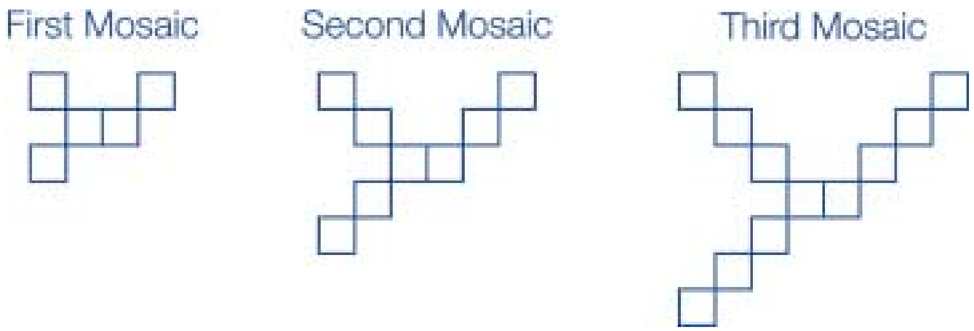
The teacher will mark the criteria to be considered in the solution of this particular problem:	CRITERIA	Solution Satisfies This Criteria
	Describes functional relationships.	❑
	Defines variables appropriately using correct units.	❑
	Interprets functional relationships correctly.	❑
	Uses multiple representations (such as tables, graphs, symbols, verbal description, and/or concrete models) and makes connections among them.	❑
	Demonstrates algebra concepts, processes, and skills.	❑
	Interprets the reasonableness of answers in context of the problem.	❑
	Communicates a clear, detailed, and organized solution strategy.	❑
	States a clear and accurate solution using correct units.	❑
	Uses correct terminology and notation.	❑
	Uses appropriate tools.	❑

CORE

Chapter 1:

Function Fundamentals

Mosaics



Reuben created the first mosaic. He decided on a pattern and created the second and third mosaics.

1. Develop a rule to determine the number of tiles in the *n*th mosaic. Write a description of how your rule is related to the tile picture including a description of what is constant and what is changing as tiles are added.

2. How many tiles would be in the tenth mosaic?

3. Would there be a mosaic in his set that uses 57 tiles? Explain your reasoning.

Teacher Notes

Materials:
30 tiles per student (optional).
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- How many tiles would be in the next mosaic?
- What is remaining constant in the set of mosaics?
- What is the rate of change for tiles with respect to the mosaic number?

Sample Solution:

1. There are two tiles in the middle of the mosaic. For each mosaic there are three outside tiles added. The number of tiles, T, is two plus three times the mosaic number, n, or

   $T = 2 + 3n$

2. Evaluate the function for $n = 10$.

   $T = 2 + 3(10)$
   or 32 tiles

3. To ask when there are 57 tiles is to ask when is T equal to 57.

   $57 = 2 + 3n$
   $55 = 3n$
   $n = \frac{55}{3} = 18\frac{1}{3}$

   Since n must be a whole number, there would not be a mosaic with 57 tiles.

Extension Questions:

- How would the function rule have been changed if the middle of the mosaic contained four tiles?

The constant would be 4. The rule would be $T = 4 + 3n$.

- If the function rule had been $T = 2 + 4n$, describe the first two mosaics and the general rule.

There would be two tiles in the middle and one tile on each corner for the first mosaic. The second mosaic would have two tiles in the middle and two tiles at each of four corners. The general rule means that there are two tiles in the middle and four tiles added for each new mosaic.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns
  2.2 Identifying More Patterns

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Student Work

(1) $2 + 3n = m$

2 in the equation represents the two blocks that are always there. The 3 shows the number of blocks connected to the other 2 blocks. The "n" tells you what mosaic number to put in. Then you would multiply 3 by the mosaic number. The "m" represents the total number of blocks in the mosaic. The constant is the two blocks that are always there. The "n" is changing. The total is also changing while the "n" changes.

mosaic number	2+3n	Total # of blocks
1	2+3(1)	5
2	2+3(2)	8
3	2+3(3)	11
4	2+3(4)	14
5	2+3(5)	17
6	2+3(6)	20
7	2+3(7)	23
8	2+3(8)	26
9	2+3(9)	29
10	2+3(10)	32

(2)

$2 + 3(n) = m$

$2 + 3(10) = m$

$2 + 30 = m$

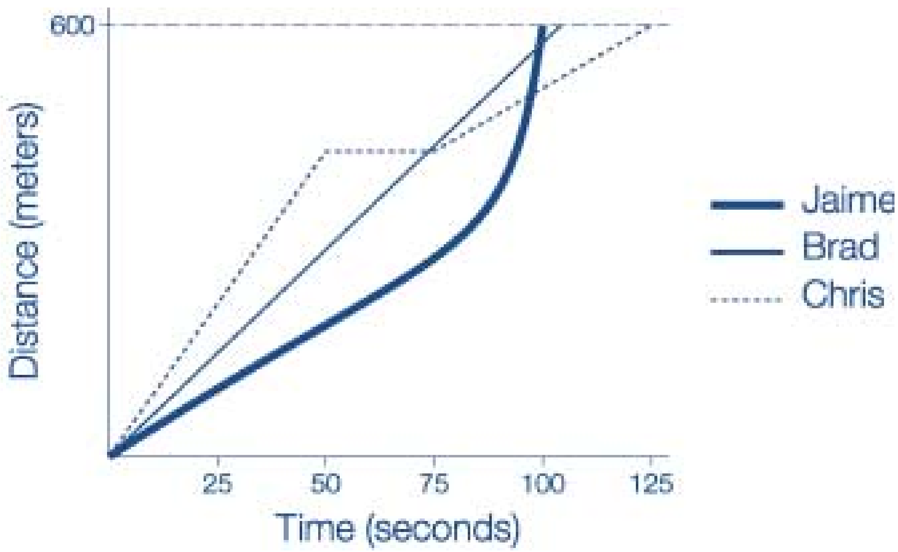
$32 = m$

There are 32 tiles in the tenth mosaic.

(3) No, because there isn't a mosaic that has exactly 57 tiles. The 18th mosaic has 56 tiles and the 19th mosaic has 59 tiles.

The 600-Meter Race

The graph below describes what happens when three athletes, Jaime, Brad and Chris, enter a 600-meter race.



Give a detailed interpretation of each athlete's experience like sprints, slowdowns, and speed throughout the race, including estimates of time and distance.

Teacher Notes

Materials:
None required.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Can the race be broken up into phases? What would these phases be?
- What does the graph show you about each of the runners? Who leads when?
- How do you know? How do their speeds compare? How do you know?
- Where are the runners at the end of each phase?
- Who finishes first? Last? How do you know? What can you say about the runners' times?

Sample Solution:

Let t = the time ran in seconds and d = the distance ran in meters. The graph indicates the race has four phases; the estimated time intervals of these phases are [0,50], [50,75], [50,100], and [100,125].

From 0 to 50 seconds:
During the first phase, Chris is in the lead, followed by Brad and then Jaime, since Chris has the greatest distance values for this interval, Brad has the second greatest distance values, and Jaime has the least distance values for this interval. Chris and Brad are running at constant rates since they traveled farther in the 50-second time interval. Chris is running faster than Brad since his graph has the greater slope.

From 50 to 75 seconds:
During the second phase, Chris stops suddenly (sharp turn in graph) and remains still for about 25 seconds. His graph is horizontal, showing no change in distance. Brad gains the lead and travels faster than Jaime and surpasses Chris at about 75 seconds. Jaime remains behind the other two runners because his distance values are less than Brad and Chris for any given time in the interval.

From 75 seconds to about 100 seconds:
Chris starts to run again and he runs at a constant but slower rate because the linear graph shows less slope. Now Brad is in the lead, followed by Chris and then Jaime. Jaime is increasing his speed because the curve is increasing faster. At about 100 seconds, the graphs of Brad and Jaime intersect; Jaime catches up with Brad and wins the race.

From 100 seconds to about 125 seconds:
Brad finishes the race just after 100 seconds. Chris completes the race at just about 125 seconds.

Extension Questions:

- What type of function could represent the distance ran by each athlete as a function of time?

Jaime's graph might be modeled with a quadratic function. Brad's graph appears to be linear and increasing, except at the very end of the race. There it appears to curve upwards. Chris's graph would have to be defined in three pieces: first, linear and increasing, then constant, and, finally, linear and increasing with less slope than the first piece.

- For the functions described above, how do the mathematical and situation domains compare?

The mathematical domain for each function described would be the set of all real numbers. The situation domains are limited to a finite time interval starting with a time of zero seconds and ending with the time it takes the last athlete to complete the race.

- Describe the range for the problem situation.

The range for the situation would be between 0 and the total distance in the race, 600 meters.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations of Functions
2 Using Patterns to Identify Relationships
 2.1 Identifying Patterns
3 Interpreting Graphs
 3.1 Interpreting Distance versus Time Graphs

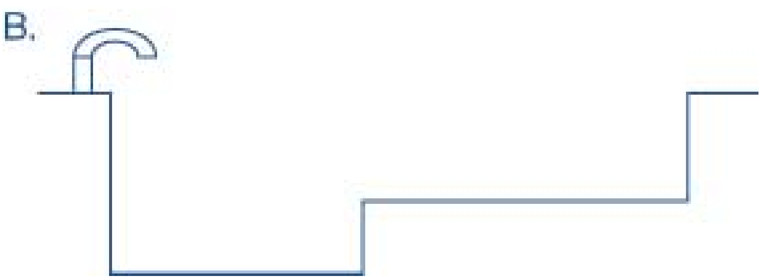
II. Linear Functions
1 Linear Functions
 1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Swimming Pools

The cross sections of two swimming pools that are being filled at a constant rate are shown below.



1. For each pool, write a description of how the depth in meters, *d*, of the water in the pool varies with time in minutes, *t*, from the moment the empty pool begins to fill.

2. Sketch a graph to show how the depth of the water in each pool varies with time from the moment the empty pool begins to fill.

Teacher Notes

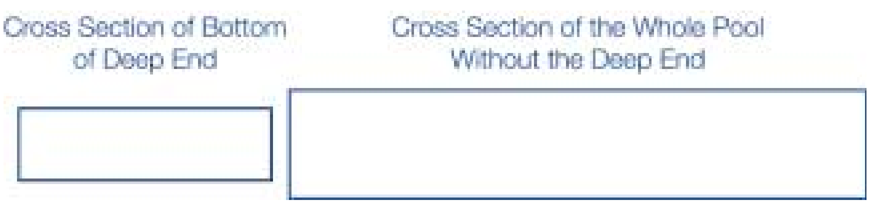
Scaffolding Questions:

- What section of the pool will be filled first?
- How are these sections different in each pool?
- How are the pools different from each other?
- How are the pools the same?
- What should the graph look like?
- How will the graphs be different?
- How will the graphs be the same?

Sample Solution:

1. For Pool A, the cross section of the pool is a rectangle. If water is flowing into the pool at a constant rate, the height of the pool will rise at a constant rate.

   For Pool B, the two sections to consider are the bottom of the deep end of the pool up to where the shallow end starts and then the rest of the pool. Both of these portions are rectangular prisms.



   The water depth will increase at a certain constant rate until the water level reaches the deep end edge of the shallow end of the pool. Then, as the water begins to fill the shallow end, the water depth increases at a slower but still constant rate until the pool is full. This is because the deep end of the pool is a prism with a smaller base than the base of the prism that is the whole pool.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

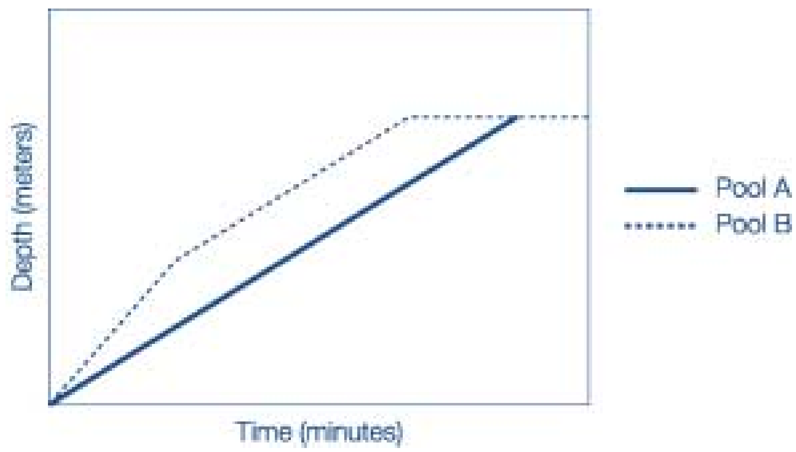
The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

2. Possible graphs:



Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.1 Variables and Functions

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Extension Questions:

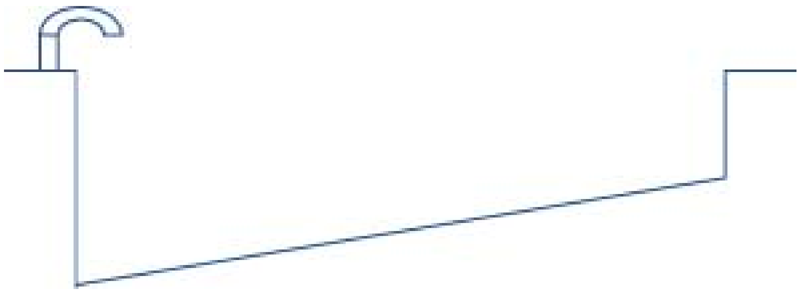
- How are the graphs of the pools related?

If the length, depth, and width of the pools are the same and water is being poured into the pools at the same rate, when the deep section of the Pool B is full, the last portion of the graph will be parallel to the graph representing Pool A.

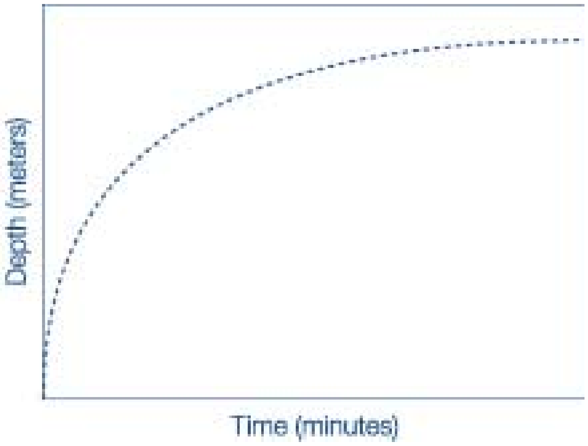
- Describe the portion of the graphs after the time when the pools are filled.

Since Pool B will hold less water, it will fill up sooner than Pool A. When a pool is full, the depth becomes constant and the graph is horizontal after the point in time when the pool is full. If the depths of the pools are the same, the graph representing Pool B will become horizontal before the graph of Pool A.

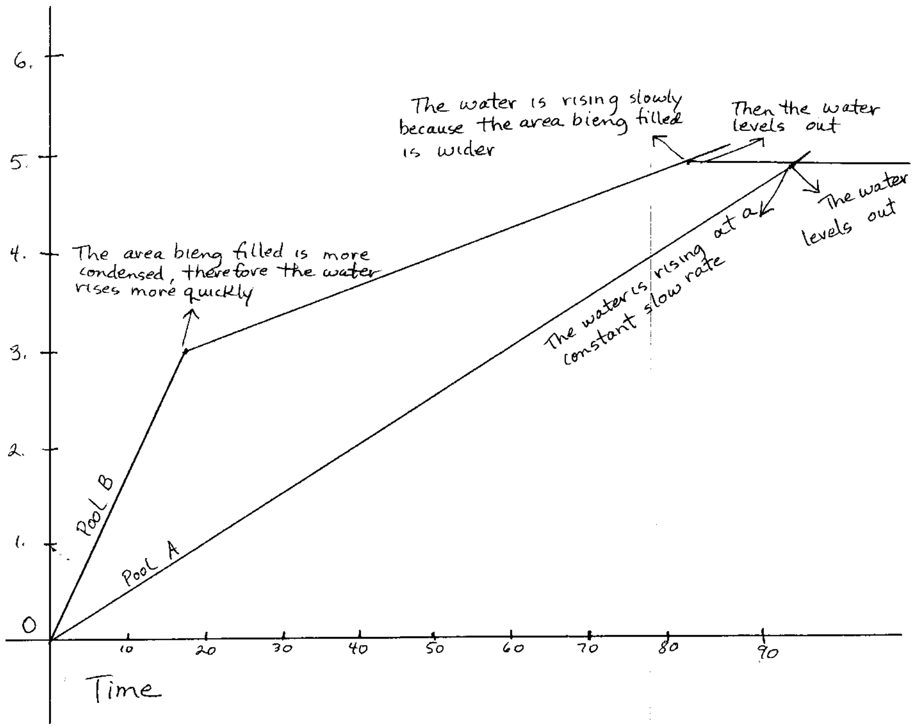
- Describe the graph if the cross section of the swimming pool had been a trapezoid.



The graph would be a curve. The height on the deep end would rise quickly. As the deep end of the pool fills, the volume of water "widens." The height rises more slowly as the surface area of the water in the pool increases.



Student Work



Chapter 2:

Linear Functions, Equations, and Inequalities

CDs for the Band

Bryan and his band want to record and sell CDs. There will be an initial set-up fee of $250, and each CD will cost $5.50 to burn. The recording studio requires bands to make a minimum purchase of $850, which includes the set-up fee and cost of burning CDs.

1. Write a function relating the total cost and the number of CDs burned.

2. Write and solve an inequality to determine the minimum number of CDs the band can burn to meet the minimum purchase of $850.

3. If the initial set-up fee is reduced by 50%, will the total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Scaffolding Questions:

- What will the cost be if they purchase only one CD? Two CDs? Ten CDs?
- What are the constants for this situation?
- What are the variables?
- Describe in words the dependency relationship between the variables.
- What does the $850 represent in this situation?

Sample Solution:

1. The total cost of recording CDs is a $250 set-up fee plus $5.50 times the number of CDs you want to purchase.

   $C = 250 + 5.50n$ where C represents the total cost and n represents the number of CDs

2. The total cost must be less than or equal to $850.
   Use the rule and put the values in a table.

Number of CDs	Total Cost
1	$255.50
10	$305.00
100	$800.00
110	$855.00

You know from the table that 110 CDs cost $855.00, which was just a little over the minimum fee of $850. Next calculate the cost of 109 CDs using the rule and find that the total cost equals

$250.00 + $5.50(109) = $849.50

109 CDs cost less than $850. They must purchase at least 110 CDs.

Another approach is to use the inequality to solve the problem.

$$\begin{aligned} 250 + 5.50x &\geq 850 \\ 5.50x &\geq 850 - 250 \\ 5.50x &\geq 600 \\ x &\geq 109.09 \end{aligned}$$

CDs must be purchased in whole number quantities. Therefore, the band can purchase 110 CDs.

3. If the set-up fee is reduced by 50%, it will be 0.50(250) or $125. The cost function becomes $C = 125 + 5.50n$. 50% of the original cost is

   $0.50(250 + 5.50n) = 0.50(250) + 0.50(5.50)n = 125 + 2.75n.$

   $125 + 2.75n \leq 125 + 5.50n$

   The new cost is more than 50% of the original cost.

Extension Questions:

- Suppose Bryan has found another company that charges a set-up fee of $200 and charges $6.00 per CD. Would this be a better company from which to purchase CDs if they have $850?

The cost function under these conditions is $C = 200 + 6n$

$$\begin{aligned} 200 + 6x &= 850 \\ 6x &= 850 - 200 \\ 6x &= 650 \\ x &= 108.33 \end{aligned}$$

They could purchase 108 CDs. This is not a better company to purchase from if they have $850.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

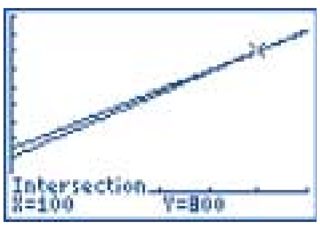
The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

- Under what circumstances would the second company be a better choice for the band to use for producing their CDs?

The tables and graphs of the two functions may be compared to determine when they are equal in cost.



The functions have the same value when x is 100. The first company's cost is greater for values of x less than 100. The second company's cost is more for values of x more than 100.

If they are going to purchase less than 100 CDs, they should buy from the second company. If they are going to purchase more than 100 CDs, they should buy from the first company.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Explorer's Glide

When the space shuttle Explorer returns back to earth for landing, it travels in a long glide. The observer begins to record the height at a time when it is 100 km above the earth's surface.

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

1. Using symbols and words describe the relationship between the time in minutes and the height in kilometers.

2. Where was the shuttle two minutes before the observer began timing?

3. How long before the observer began timing was the shuttle at 102 km above the earth?

Teacher Notes

Scaffolding Questions:

- What is changing in this situation?
- What would you expect the height to be in 5 minutes? Explain your reasoning.
- Is the relationship linear? How can you tell?
- What is the rate of change?
- What is the *y*-intercept?

Sample Solution:

1. Determine the rates of change from the table.

10 5 5 2

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

-2.8 -1.4 -1.4 -0.56

The rate of change is -0.28 km per 1 minute.

The height, *h*, is the starting height plus the rate of change times the number of minutes, *m*. The height of the Explorer in kilometers is 100 kilometers minus 0.28 kilometers per hour times the number of minutes.

$h = 100 – 0.28m$

2. If the descent had begun at this constant rate at least two minutes before the timing began, the time would be represented by $m = -2$, it would have been at 100 – 0.28(-2) or 100.56 kilometers above the earth.

We could also use the table or the graph.

Enter the rule $y = 100 – 0.28x$.
Look at the table of values to find the value of *y* when $x = -2$.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

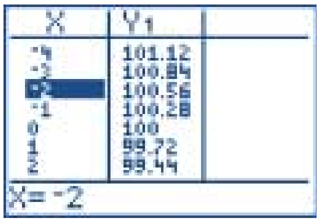
The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

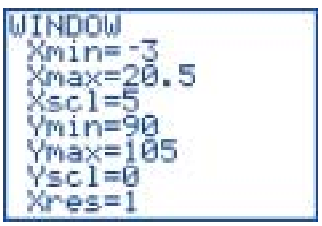
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

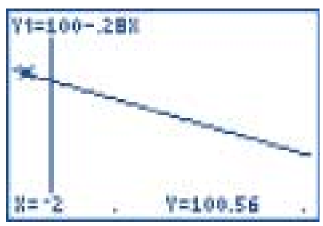
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.



The window may also be adjusted so that the graph may be traced to find the value when x = -2.

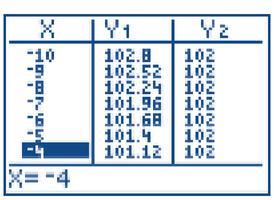


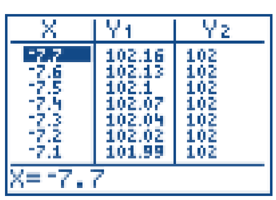


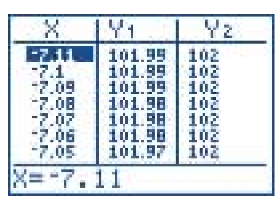
The height of the object when the time is two minutes before the observer began timing would be 100.56 km.

3. The question is asking when is 102 = 100 – 0.28x. Solving for x gives a value of about -7.14. This means that if the glide had begun at this rate at least 7.14 minutes before timing began, then the shuttle would have been at 102 km above earth.

To determine the time when y = 102, one could also examine a table of the function y_1 = 100 – 0.28x and the function y_2 = 102. Continue to increment the x value to smaller increments until y is 102.







The value is between -7.16 and -7.13. The table rounds the values to the nearest hundredth.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

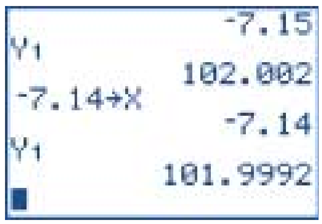
(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

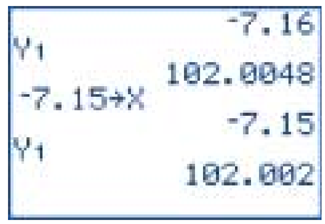
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors necessary in problem situations; and

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations.

To determine more precisely one may compute the values.





The time is between 7.15 minutes and 7.14 minutes before the observer began timing.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 The Linear Parent Function
1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Extension Questions:

- What does the *y*-intercept mean in this situation?

The y-intercept represents the height at time zero.

- What is the *x*-intercept, and what does it mean for this problem situation?

The x-intercept is approximately 367.14. It represents the number of minutes after the observer began recording time that the height is 0; that is the time when the shuttle would have hit the earth if it had continued to descend at the same rate.

- Is this a realistic model for the descent of a shuttle?

This is not a realistic situation. In reality the shuttle must decrease its speed as it gets closer to landing.

Student Work

1. The shuttle is descending at a rate of 0.28 km per minute.

$\frac{y_2 - y_1}{x_2 - x_1}$ Use the slope formula

$\frac{95.8 - 97.2}{15 - 10}$ Set up the equation inserting the values of any two points.

$\frac{-1.4}{5} = {}^{-}0.28$ Simplify the equation.

The rate of increase is ⁻0.28 km per minute, or it is descending at 0.28 km per minute.

2. 100.56 km

$y = mx + b$ Use the slope-intercept formula

$y = {}^{-}0.28x + 100$ You got you slope from number 1 and you have the point (0, 100), so you know the y intercept.

$y = {}^{-}0.28(^{-}2) + 100$ Set x to ⁻2 (two minutes ago) and solve

$y = 100.56$

Pg 1 of 2

3. 7½ minutes

$y = {}^{-}0.28x + 100$ Use the equation from number 2:

$102 = {}^{-}0.28x + 100$ Insert 102 for y

$2 = {}^{-}0.28x$ Solve

$\frac{200}{-28} = x$

$-7\frac{1}{2} = x$

He was at 102 km 7½ minutes ago.

Geothermal Energy

Whenever water comes into contact with heated underground rocks, geothermal energy is generated. The underground temperature of the rocks depends on their depth below the surface. The temperature t in degrees Celsius is estimated by the function $t(d) = 35d + 20$, where d is the depth of the rocks in kilometers.

1. Describe the graph of the function. Explain how the constants in the function are related to the graph and to the problem situation.
2. Identify the domains and ranges for this function and for the problem situation.
3. Find the temperature of the rocks at a depth of 3 kilometers.
4. Find the depth if the temperature of the rocks is 195 degrees Celsius.

Teacher Notes

Scaffolding Questions:

- If the depth of the rocks is 1 kilometer, what is the temperature?
- Find the temperature if the depth of the rocks is 2 kilometers; 3 kilometers.
- What is the relationship between the temperature at 1 kilometer, and at 2 kilometers? At 2 kilometers and 3 kilometers?
- Why can you NOT have negative values for the domain of the problem situation?
- What does the 35 mean in the equation?
- What does the 20 represent in the equation?
- What is the relationship between the depth and the temperature?

Sample Solution:

1. The temperature of the rocks depends on the depth of the rocks. For every kilometer of depth, the temperature increases 35 degrees Celsius. Because the rate of change is constant, the function is linear.

   The graph of the function will be a line with a positive slope and with a *y*-intercept value of 20. The intercept value of 20 means that at a depth of 0 kilometers the temperature is 20 degrees Celsius. The slope is the rate of change in the temperature per kilometer of depth. The rate of change is 35 degrees Celsius from every 1 kilometer of depth.

2. The domain represents the depth of the rocks. The problem states that geothermal energy is generated wherever water comes into contact with heated underground rocks. The positive values greater than zero would indicate depths of rocks in kilometers. A value of zero would represent the surface, not underground, and would not be included. The negative values would actually be representing kilometers above the surface and would not qualify either. The domain would be values greater than zero, or $x > 0$.

   The range represents the temperature in Celsius. At the surface (0 kilometers), the temperature of rocks is 20 degrees Celsius. Because the rocks must be underground, values greater than 20 are the only ones that satisfy the condition for the geothermal energy or $y > 20$.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations

3. Substitute the value $d = 3$ into the equation $t = 35d + 20$ to find the temperature of the rocks at 3 kilometers.

   $t = 35(3) + 20$
   $t = 105 + 20$
   $t = 125$ degrees C.

   At the surface (0 kilometers), the temperature of the rocks was 20 degrees Celsius because

   $t = 35(0) + 20$
   $t = 0 + 20$
   $t = 20$ degrees C.

4. If the temperature of the rocks is 195 degrees Celsius, the depth can be found algebraically:

$$\begin{aligned} t &= 35d + 20 \\ 195 &= 35d + 20 \\ 175 &= 35d \\ 5 &= d \end{aligned}$$

   The depth of the rocks at 195 degrees Celsius is 5 kilometers.

Extension Questions:

- Express the temperature of 195 degrees in Fahrenheit, given the formula $F = \frac{9}{5}C + 32$, where C = temperature in Celsius, and F = the temperature in Fahrenheit.

The temperature 195°C is converted to Fahrenheit using the formula

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(195) + 32$$

$$F = 351 + 32$$

$$F = 383$$

195 degrees C is equivalent to 383 degrees F.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

- How would the original equation, $t = 35d + 20$, change if the temperature t must be expressed in degrees Fahrenheit?

Convert from Celsius to Fahrenheit and substitute the Fahrenheit expression for Celsius in the function rule.

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$C = \frac{5}{9}(F - 32)$$

$t = 35d + 20$, t in Celsius

$$\frac{5}{9}(F - 32) = 35d + 20$$

$$F - 32 = \frac{9}{5}(35d + 20)$$

$$F = \frac{9}{5}(35d + 20) + 32$$

This rule defines the relationship between the distance, d, and the temperature in Fahrenheit, F.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
  1.3 Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Hot-Air Balloon

At the West Texas Balloon Festival, a hot-air balloon is sighted at an altitude of 800 feet and appears to be descending at a steady rate of 20 feet per minute. Spectators are wondering how the altitude of the balloon is changing as time passes.

1. What function relating the variables best describes this situation?
2. Make a table of values and/or graph to show the balloon's altitude every 5 minutes beginning at 5 minutes before the balloon was sighted until the balloon lands.
3. How high was the balloon 5 minutes before it was sighted?
4. How long does it take the balloon to reach an altitude of 20 feet? How long does it take the balloon to land?
5. A second balloon is first sighted at an altitude of 1200 feet and is descending at 20 feet per minute. How does the descent and landing time of the second balloon compare with that of the first balloon? What does this mean graphically?
6. A third balloon is first sighted at an altitude of 800 feet but is descending at 30 feet per minute. How does the descent and landing time of the third balloon compare with that of the first balloon? What does this mean graphically?
7. At the instant the first balloon is sighted, a fourth balloon is launched from the ground rising at a rate of 30 feet per minute. When will the first and fourth balloon be at the same altitude? What is that altitude? What does this mean graphically?

Teacher Notes

Scaffolding Questions:

- What are the constants in the problem? What quantities vary?
- What quantity will be the dependent variable? The independent variable?
- What kind of function models the situation? How do you know?
- What decisions must you make to build a table for the function?
- What decision must you make to graph the function?
- How can you find the balloon's height at any given time?
- How can you find the time it takes the balloon to reach a given height?

Sample Solution:

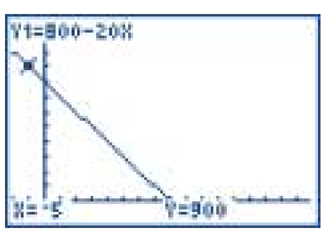
1. The starting height, 800 feet, is decreased by 20 feet per minute. The height, h, equals 800 minus 20 times the number of minutes, m.

   $h = 800 - 20m$

2. The time 5 minutes before it was sighted is represented by -5.

m	$800 - 20m$	h
-5	800 – 20(-5)	900
0	800 – 20(0)	800
5	800 – 20(5)	700
10	800 – 20(10)	600
15	800 – 20(15)	500
20	800 – 20(20)	400

The graph may also be used to examine the situation.



Materials:
One graphing calculator per student

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

3. The value of *y* is 900 when *x* is -5. Therefore, the balloon was at 900 feet 5 minutes before it was first sighted.

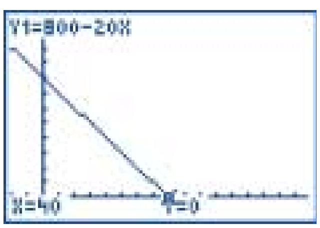
4. Solve for *m*:

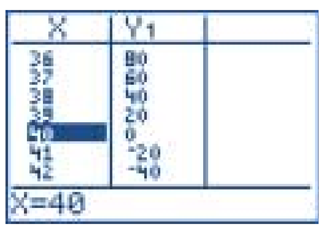
$$\begin{aligned} 800 - 20m &= 20 \\ -20m &= -780 \\ m &= 39 \end{aligned}$$

It takes the balloon 39 minutes to descend to 20 feet above the ground.

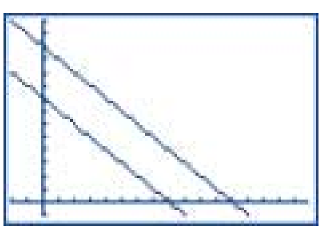
Solve $800 - 20m = 0$ for *m* to get $m = 40$. The balloon lands in 40 minutes.

The graph or table may also be examined to determine when the height is 0.





5. The balloon is at a higher altitude but descending at the same rate. It will take longer to land. The second function is $y = 1200 - 20x$. The graphs have different *y*-intercepts and *x*-intercepts. The graphs will be parallel lines since they have the same slope.



(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of y = mx + b;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept
1.3 Exploring Rates of Change

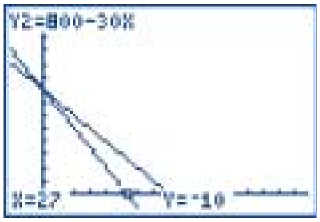
Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

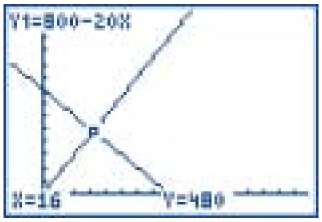
Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

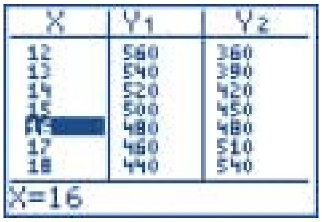
6. The third balloon starts at the same height as the first but is descending faster. Therefore, the third balloon will land sooner. The third function rule is $y = 800 - 30x$.

   The graphs have the same y-intercept but different x-intercepts. The x-intercept for the third balloon is less than that of the first balloon. The graph for the third balloon's descent will be steeper than that for the first balloon.



7. The function for the fourth balloon is $y = 30x$. To see if they are ever at the same altitude, explore with tables or graphs, or solve $800 - 20x = 30x$ to get $x = 16$. Sixteen minutes into descent/launch, both balloons will be at the same height, 480 feet.





X	Y1	Y2
12	560	360
13	540	390
14	520	420
15	500	450
16	480	480
17	460	510
18	440	540

X=16

Extension Questions:

- If the function of the motion of a fifth balloon had been $y = 700 - 20x$, how would the movement of the balloon have been different from the first?

The balloon would have been sighted at a height of 700 feet instead of 800 feet. The rate of descent would have been the same as the rate of descent of the first balloon.

- Would the fifth balloon have landed sooner or later than the first balloon? Explain how you know.

If it started at a lower altitude and descended at the same rate, it would land sooner. The x-intercept would be 700 divided by 20, or 35 seconds.

Student Work

1) y = 800 - 20x
starting height · 20 feet per minute

2)

feet y	minutes x
900	-5 ← 5 minutes before sited at 900
800	0 ← when it was sited
700	5 ← 5 minutes after it was sited
600	10 ← 10 minutes after sited
500	15 ← 15 minutes after sited
400	20 ← 20 minutes after sited

3) 900 feet in the air from the table
I made a graph on the calculator and traced

4) 39 minutes at x = 20
40 minutes at x = 0

5) The second balloon is going later because its starting at 1200 and the first one was sited at 800 There is a (400 feet difference)

6) The Third balloon is descending at 30 ~~feet~~ per minute and the first one was going 20 feet per minute so the Third balloon is going to get to the ground faster

7) The 4th and 1st balloon are going to meet at 16 ~~feet~~ minutes at 480 feet. I graphed y = 800 - 20x and y = 30x and found the intersection

Making Pizzas, Making Money

The CTW Pizza Company is planning to produce small square pizzas. It will cost them \$2.00 to make each pizza, and they will sell them for \$5.00 a piece.

1. Express the profit earned as a function of the number of pizzas sold.
2. Graph the function rule and describe the relationship between the two variables.
3. What is the slope of the graph, and what does it mean in the context of the situation?
4. Discuss at least two methods for finding the number of pizzas that need to be sold to make a profit of \$180.
5. CTW found a cheaper supplier, and now it costs \$0.50 less to make each pizza. Describe how this will change the function rule, the graph, and the table, and explain how you know.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

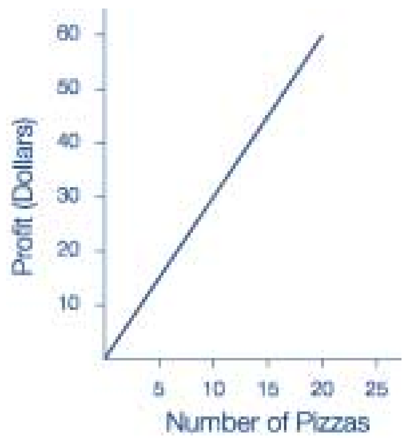
Scaffolding Questions:

- What is the profit for one pizza? For two pizzas? For three pizzas?
- What are the variables in this situation?
- What is the profit per pizza?
- What does the table look like?
- How much profit can be made selling 50 pizzas?
- If your goal is to make a profit of $300 a day, how many pizzas must you sell each day?

Sample Solution:

1. The profit from making pizzas can be determined by subtracting the cost to make each pizza from the selling price. Therefore, the function rule for the profit, p, will be $p = 5x - 2x$ or $p = 3x$ where x represents the number of pizzas.

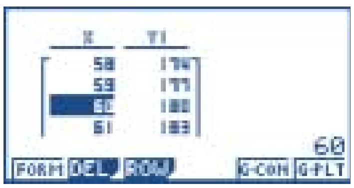
2.

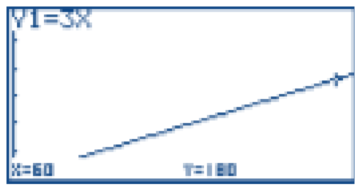


The profit in dollars is the number of pizzas multiplied by $3. There will be a $3.00 profit per pizza. The more pizzas sold, the more profit made.

3. The slope is the profit in dollars per number of pizzas. It can be determined from the graph by looking for the rate of change; the profit goes up $3.00 for every pizza sold.

4. Use the function $p = 3x$. When p is 180, the rule becomes $180 = 3x$. Since 3 times 60 is 180, they must sell 60 pizzas to make a profit of $180. The table or graph may also be used to answer the question.





5. The new cost is $1.50 per pizza. The profit for each pizza would be $5x - 1.50x$ or $3.50x$. The profit has now increased by $0.50 per pizza, therefore, the profit per pizza is now $3.50. The b value in the function rule $y = mx + b$ is still 0, but the m will increase by $0.50. The slope of the graph will now be greater, because for every pizza you sell you now make $3.50 instead of $3.00. The table will also show an increase of $3.50 in the y value for every increase in 1 of the x value.

Extension Questions:

- Describe how to determine the slope from the table.

Calculate the rates of changes by finding the difference of two y values, divide by the corresponding differences in the x values, and look for a constant rate of change.

- Describe how to determine the slope from the graph.

You can determine the slope from the graph by finding the ratio of the vertical change to the horizontal change between any two points on the line.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

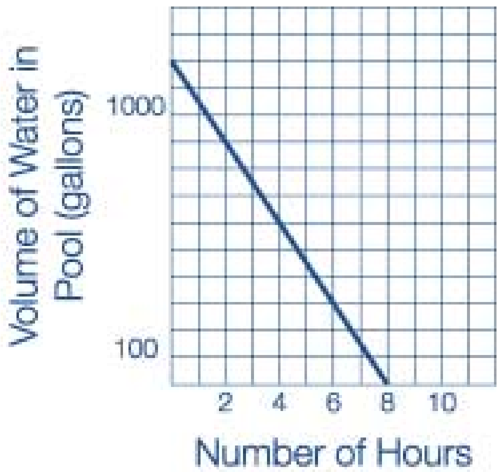
II. Linear Functions
1 Linear Functions

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Pool Problem



1. The graph shows the relationship between the amount of water in a pool and the number of hours that have elapsed since a pump began to drain the pool. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.

2. How much water would be in the pool after 4 hours and 20 minutes?

3. How many hours after they began draining the pool would it contain 720 gallons of water?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Define the independent variable and the dependent variable for this problem situation.
- What type of relationship is described by the graph?
- How much water was in the pool when the pumping started? What part will this number play in the function rule?
- How much water was in the pool after two hours? Four hours? Six hours? Organize your response in a table.
- At what rate is the amount of water decreasing per hour?
- Use the rate of change and the starting volume in the pool to write a function rule.

Sample Solution:

1. The amount of water in the pool at time zero is 1200 gallons.

   The water is being drained at a constant rate, because the graph is the graph of a straight line. It takes 8 hours to drain the pool. The rate per hour would be 1200 gallons divided by 8 hours or 150 gallons per hour. Because the water is draining, the rate of change is -150 gallons per hour.

   The amount of water in the pool is the starting value plus the rate times the number of hours.

   Let w be the amount of water in the pool at time t in hours,

   $w = 1200 + (-150)t$
   $w = 1200 - 150t$

   where t is any number between 0 and 8, inclusive.

2. The time is 4 hours and 20 minutes or $4\frac{1}{3}$ hours.

   $$w = 1200 - 150\left(4\frac{1}{3}\right) = 1200 - 650 = 550$$

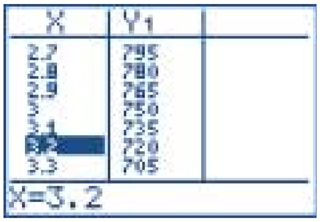
   The amount of water in the pool after 4 hours and 20 minutes is 550 gallons.

3. The amount of the water in the pool will be 720 gallons when $y = 720$.

$$720 = 1200 - 150t$$
$$t = 3.2$$

There will be 720 gallons of water in the pool after 3.2 hours.
0.2 (60) = 12
The time is 3 hours and 12 minutes.

A table or graph could also be used to determine when the amount of water is 720 gallons. Set the table minimum at 1 and increments at 0.1, and scroll down the table to find the value when $y = 720$ at $x = 3.2$. In 3.2 hours the amount in the pool would be 720 gallons.



Extension Questions:

- What is the domain for the function rule you have written?

The domain is the set of all real numbers.

- Describe the domain for this problem situation and explain why you selected this domain.

The domain is the set of all real numbers from 0 to 8 inclusive. The domain values must be a non-negative number and must give non-negative range values. The pool is empty after 8 hours.

- How much time would have elapsed if the pool is half-empty?

The original amount of water in the pool was 1200 gallons. The amount of water is 600 gallons at 4 hours. Note that this is one-half the time it takes to empty the pool.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Developing Mathematical Models
1.2 The Y-intercept
1.3 Exploring Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

- Will this relationship work if you are asked about how long it takes to empty one-third of the water? Explain your reasoning.

It would take one-third of the time it takes to drain the pool. There is a proportional relationship between the time and the portion of the water that has been drained.

The time it takes to drain the pool is $\frac{\text{1200 gallons}}{\text{150 gallons per hour}}$ *or 8 hours.*

If one-third of the pool is drained, two-thirds of the pool volume remains.

$$\frac{2}{3}(1200)1 = 1200 - 150x$$
$$150x = \frac{1}{3}(1200)$$
$$x = \frac{1}{3} \bullet \frac{1200}{150} = \frac{1}{3}(8)$$

Let f be the fractional part of the pool drained. The part remaining is 1-f.

$$\begin{aligned}(1-f)1200 &= 1200 - 150x \\ 150x &= 1200 - (1-f)1200 \\ 150x &= f\,1200 \\ x &= f\left(\frac{1200}{150}\right) \quad \text{or} \quad 8f\end{aligned}$$

Thus, if the amount drained is (f)(1200), the time it takes is (f)(8) or f times the amount of time it takes to drain the pool.

- If the pool started at 1500 gallons, but emptied at the same rate, how would that affect your graph?

The only value changed in the function is the y-intercept.

$y = 1500 – 150\ x$

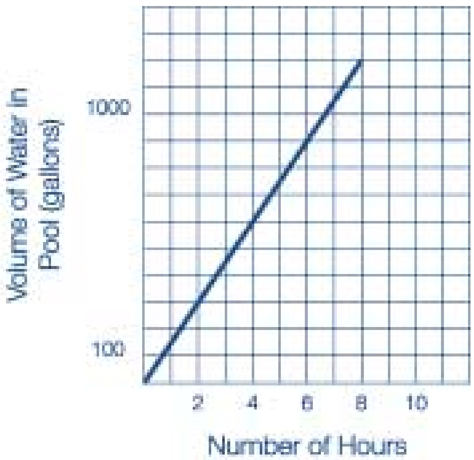
The graph would be a straight line parallel to the original line, but with a y-intercept of 1500.

- If the pool started at the same amount, but emptied at 100 gallons per hour, how would the graph be changed?

The rate of change or slope is -100.
The function would be $y = 1200 – 100x$

- Suppose an empty pool was being filled at the same rate and with the same capacity of 1200 gallons. Sketch the graph to represent this situation and write the function to represent this new situation.

The function would be $y = 150x$ where x varies from 0 to 8. Since the capacity of the pool is 1200 gallons, the graph terminates at the point (8,1200); the graph is a line segment.



Student Work

Pool Problem

1. The graph shows that for every two hours that pass, 300 gallons are drained out of the pool. This is an inverse relationship, meaning that as time (or the x-variable) goes on, the y-variable, or the gallons of water, decreases. An equation that could go with the graph is $y = -150x + 1200$. The 1200 is how much water the pool starts out with, and -150 is the rate at which the water is decreasing each hour. As you can also see, the graph is a linear relation, because it moves in a straight line and changes at a constant rate. X is the variable that stands for the number of hours passed and y is the amount of water remaining in the pool.

2. First, figure out what 4 hours and 20 minutes is in fraction or decimal form. 20 minutes is out of a whole of 60 minutes in one hour, therefore, $\frac{20}{60} = \frac{1}{3}$. This means 4 hours and 20 minutes = $4\frac{1}{3}$. Now, place the hours $4\frac{1}{3}$ into the equation $y = -150x + 1200$ as the x-variable. -150 is the rate at which the water is decreasing and 1200 is the water that was there to begin with. 1200 is also the y-intercept.

$y = -150(4\frac{1}{3}) + 1200$

$y = -650 + 1200$

$y = 550$ gallons of water remaining after 4 hours and 20 minutes.

3. Because 720 is how many gallons is remaining, you replace the y-variable in the equation with 720, because that is the number you're solving for. So, $720 = -150x + 1200$. You then solve the equation by subtracting 1200 from both sides.

$$720 = -150x + 1200$$
$$-1200 \qquad -1200$$
$$\frac{-480}{-150} = \frac{-150x}{-150}$$
$$3.2 = x$$
hours draining

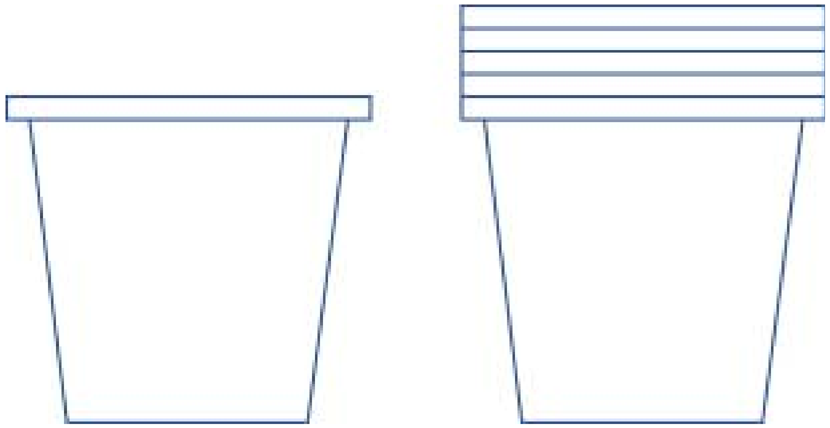
Then, you divide both sides by -150. To find how many minutes .2 equals, put it into decimal form: $\frac{2}{10}$ and set it equal to $\frac{x}{60}$. The solve by cross-multiplying: multiply 10 w/ x and 60 with 2. Divide each side by 10.

$$\frac{2}{10} = \frac{x}{60}$$
$$\frac{120}{10} = \frac{10x}{10}$$
$$12 = x$$
minutes

Now, you know that it takes 3 hours and 12 minutes for the pool to contain only 720 gallons.

Stacking Paper Cups

The figure shows drawings of one paper cup and five paper cups that have been stacked together. The cups are shown at one-half of the real size. Use a centimeter ruler to help answer the questions.



1. Create a function rule that gives the actual height of a stack of cups in terms of the number of cups in the stack. Define the variables. Explain in detail how you created your rule.

2. What would be the total height of 12 stacked cups? Justify your solution.

3. How many cups would fit stacked in a space 1 meter in height? Justify your solution.

4. Find a function that gives the number n of cups in a stack in terms of the height h of the stack.

5. If a new stack is created with the base of the cups remaining the same but the height of the lip of the cup doubled, will the new stack be more than, less than, or equal to twice the height of the original stack?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

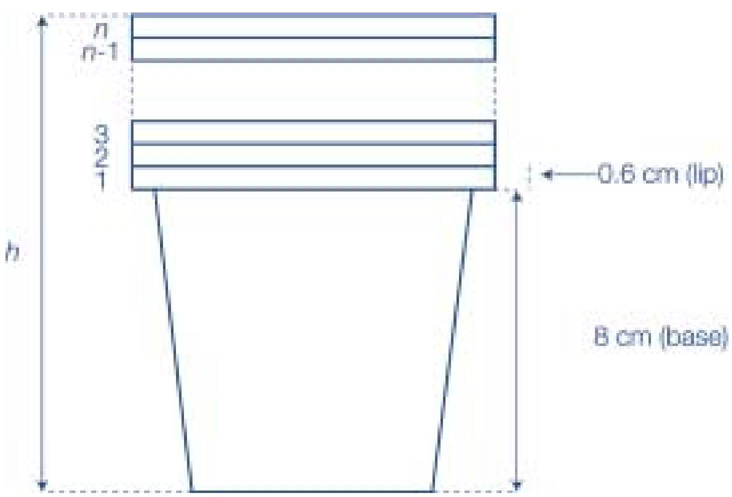
(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- How will you start?
- What is the measurement of one cup?
- What is the measurement of two cups?
- What is the measurement of one lip?
- What are the variables?
- What is the dependent variable?
- What is the independent variable?
- Describe how you might create a table to help you determine the function rule.

Sample Solution:

This solution is based on the following diagram of a stack of cups with all measurements doubled.



1. The height *h* of a stack of *n* cups can be broken down into the 8 cm "base" of the bottom cup plus *n* times the 0.6 cm "lip" of each cup. We can therefore represent the height *h* as a function of the number *n* of cups as

   $h(n) = 8 + 0.6n$

This is a "height function" that gives the height *h* in terms of the number *n*. The slope of the function is 0.6 cm per cup.

2. To find the height of a stack of 12 cups we can evaluate the height function at $n = 12$:

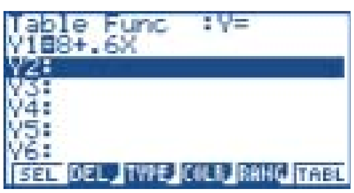
   $h(12) = 8 + 0.6(12) = 15.2$ cm

3. To find how many cups fit into a space 100 centimeters high we can set the height function equal to 100:

   $100 = 8 + 0.6n$

   This is an equation that we can solve for *n*:

   $$n = \frac{100 - 8}{0.6} = 153\ \frac{1}{3}$$



Since we can't have $\frac{1}{3}$ of a cup, we see that 153 whole cups will fit into this 100 centimeters space. As a check one can see how much space 153 cups occupy by evaluating the function at $n = 153$:

$h(153) = 8 + 0.6(153) = 99.8$ cm

There is a space of 0.2 cm left over when 153 cups are put into a 100 cm space.

One may also enter the function rule $y = 8 + 0.6x$ into the graphing calculator. Examine the table and look for the *y* column value of 100. When *x* is 153, $y = 99.8$. When *x* is 154, the *y* value is 100.4. There would be 153 cups in the stack.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

4. To express the number of cups as a function of the height, solve the rule for *n.*

$$h = 8 + 0.6n$$
$$h - 8 = 0.6n$$
$$\frac{h-8}{0.6} = n \quad \text{or}$$
$$n = \frac{h-8}{0.6}$$

This is a "count function" that gives the number n in terms of the height h.

5. If the thickness of the lip of each cup is doubled, the rule for the height of a stack of cups becomes

$h(n) = 8 + 2(0.6)n$ or
$h(n) = 8 + 1.2n$

Twice the height of the original stack would be $2(8 + 0.2n)$ or $16 + 1.2n$. $8 + 1.2n$ is less than $16 + 1.2n$. The height is less than the height of the doubled stack.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Extension Questions:

- What is the rate of change (slope) of the function rule and what does it mean in this situation?

The rate of change is 0.6cm per one cup. The rate of change is the change in the height of the stack per one cup.

- What limits the domain in this situation?

The domain values must be counting numbers. There may not be a fractional number of cups or a negative number of cups.

- What are the y-intercept and x-intercept and what do they mean in this situation?

The y-intercept of the graph of the line $y = 8 + 0.6x$ is 8, but this point (0,8) would not be a point plotted in this problem situation because it means a cup without a lip. The measurement of the cup without the lip is 8 centimeters.

The x-intercept is $\frac{-8}{0.6} = -\frac{40}{3} = -13\frac{1}{3}$, but the point $\left(-\frac{40}{3}, 0\right)$ has no meaning in this problem situation, because there cannot be a negative number of cups.

- Does the graph add any additional information?

The graph of the function is a line, but the graph of the problem situation is a set of points in the first quadrant. The domain of the problem situation is the set of counting numbers. Thus, the points must have first coordinates that are counting numbers. The graph on a graphing calculator may be used to answer questions about the situation, to evaluate the function at specific values, and to solve equations related to the function.

- If the height of the cup were 10 centimeters and the height of the lip stayed the same, how would it have changed the function for this situation?

The height of this new cup without the lip would be 10 – 0.6 or 9.4. The function rule would become $y = 9.4 + 0.6x$.

- If the problem had said the cup was drawn to the actual size of the cups, how would that have changed your function rule?

The values of the intercept and the slope would have been reduced by one-half. The function rule would have been $y = 4 + 0.3x$.

- If the problem had said the cup was one-third of the actual size, how would that have changed your function rule?

The measurements would have been multiplied by 3 instead of 2. The height of the cup would have been 12.9 centimeters. The lip would measure 0.9 cm. The function rule would become $y = 12 + 0.9x$.

- Another way of writing the count function is

$$n(h) = \frac{h}{0.6} - \frac{8}{0.6} = -13\frac{1}{3} + 1\frac{2}{3}h$$

$$n(h) = -13\frac{1}{3} + 1\frac{2}{3}h$$

- What does the slope of this rule mean in the context of the problem?

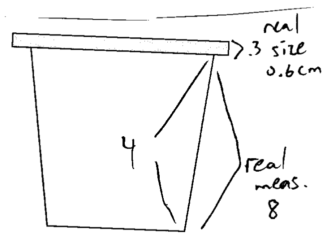
The slope of this function is $1\frac{2}{3}$ *cups per centimeter, which means that* $1\frac{2}{3}$ *cup lips fit in 1 centimeter.*

Student Work

① $y = 0.6x + 8$

Let y = the total height in centimeters

Let x = number of cups



The scale is ½ → Six millimeters are added each time a cup is added to the original base part of cup (8 cm)

② $y = 0.6 \times 12 + 8$

$y = 15$ cm

③ $100 = 0.6x + 8$

$92 = 0.6x$ 153 cups because you can't have ⅓ of a cup.

$153\frac{1}{3} = x$

④ $\frac{y-8}{0.6} = \frac{0.6n}{0.6}$

$\frac{5}{3}h - \frac{40}{3} = n$

⑤ Let $x = 10$

$0.6 \times 10 + 8 = 14$ cm   The new stack is less than twice the height of the original

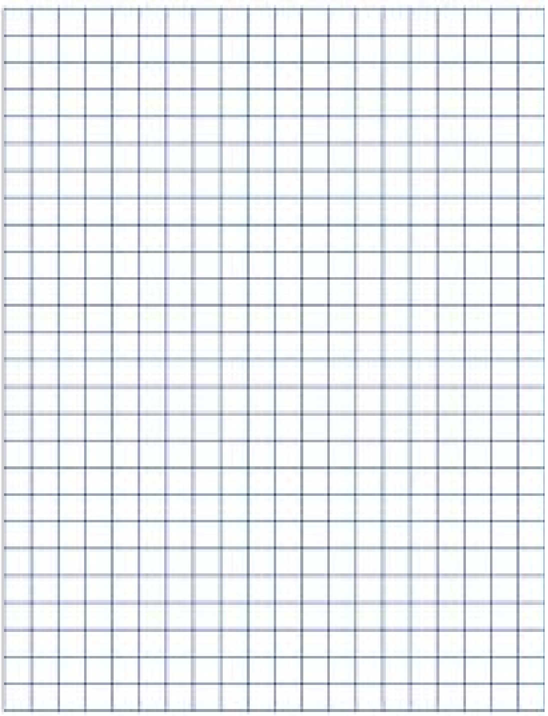
$1.2 \times 10 + 8 = 20$ cm

Stretched Spring

Data is collected in an experiment to determine the relationship between the length of a spring and the mass of an object hanging from it. The length of the spring depends on the mass of the object. The table below left gives a sample of the data.

Length versus Mass

Mass (kg)	Length (cm)
50	5.0
60	5.5
70	6.0
80	6.3
90	6.8
100	7.1
110	7.5
120	7.7
130	8.0
140	8.6
150	8.8
160	9.2
170	9.5
180	9.9
190	10.3



1. Construct a scatterplot of the data. Describe the functional relationship between the length of the spring and the mass suspended from it verbally and symbolically.

2. Predict the length of the spring for a suspended mass of 250 kilograms.

3. Predict the mass that stretches the spring to 15 centimeters.

Teacher Notes

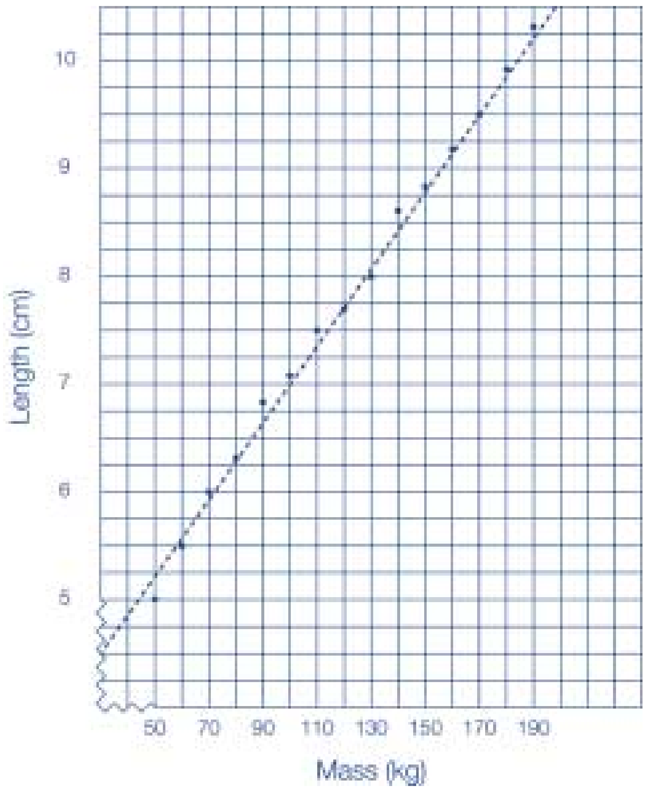
Scaffolding Questions:

- How will you organize the data that is collected?
- What will you need to consider to construct a scatterplot of the data?
- What will you need to consider to determine a reasonable interval of values and scale for each of the axes?
- What function type (linear, quadratic, exponential, inverse variation) appears to best represent your scatterplot?
- What do you need to know to determine a particular function model for your scatterplot?

Sample Solution:

1. Create the scatterplot.

   The scatterplot is nearly linear.



Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

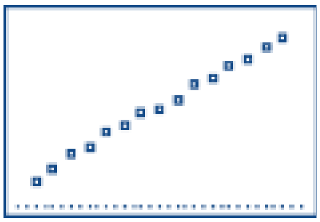
The student:
(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

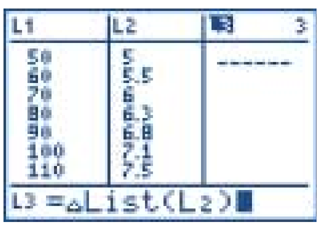
The student:
(A) uses symbols to represent unknowns and variables; and

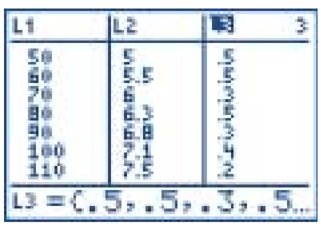
(B) given situations, looks for patterns and represents generalizations algebraically.

The data points may also be entered into a graphing calculator to create the scatterplot.



The consecutive difference in the length values may be computed using the list feature.





The average of these consecutive differences is approximately 0.38. The difference in the consecutive mass values is 10. The rate of change may be approximated as 0.38 divided by 10 or .038. Using 0.038 centimeters per kilogram as the rate of change, a trend line is of the form $y = 0.038x + b$. Use any other data point to find a possible value for b. If the point (50, 5) is used, the value is 3.1.

$$5 = 0.038(50) + b$$
$$b = 5 - 0.038(50) = 3.1$$

$$y = 0.038x + 3.1$$

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

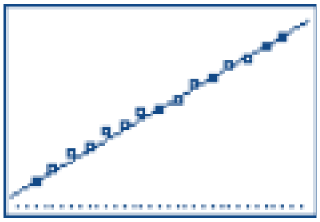
(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.

The graph of this line is an approximate trend line for the data.



(Note: Using the regression features of the calculator, the line of best fit is $y = 0.036x + 3.37$.)

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities

2. To determine the length of mass, evaluate the function for $x = 250$.

$$\begin{aligned} y &= 0.038\,(250) + 3.1 \\ &= 12.6 \text{ cm} \end{aligned}$$

A mass of 250 kilograms will stretch the spring to a length of 12.6 centimeters.

3. To predict the mass that stretches the spring to 15 centimeters, use the function and solve the resulting equation:

$$\begin{aligned} 0.038x + 3.1 &= 15 \\ 0.038x &= 11.9 \\ x &= 313.16 \text{ kg} \end{aligned}$$

A mass of about 313.16 kilograms stretches the spring to a length of 15 centimeters.

Extension Questions:

- In your experiment, how did the mass suspended from the spring change and, in general, how did this affect the length to which the spring stretched?

The mass suspended from the spring started at 50 kilograms and increased by 10 kilograms each time until we reached a mass of 190 kilograms. The initial amount of stretch (at 50 kilograms mass) was 5 centimeters and increased by small amounts (0.2 centimeters to 0.5 centimeters) with each additional 10 kilograms of mass added to the spring.

- Since the mass increased in increments of 10 kilograms and the "stretch length" increased each time in the range from 0.2 centimeters to 0.5 centimeters, what did this suggest the functional relationship between spring length and mass would be?

It should be a linear relationship. As the mass increases in constant amounts, the amount by which the spring's length increases is nearly constant. This suggests a constant rate of change.

- How long is the spring when no mass is suspended from it?

Use the model, $y = 0.038x + 3.1$. When the spring has no mass attached to it, the value of x is 0 and y is 3.1 centimeters long.

- Suppose the initial length of the spring is changed to 6.8 centimeters, and we suspend mass from the spring in increments of 20 kilograms instead of 10 kilograms. How will this change the function that models this situation?

The impact of increasing the weight increments to 20 kilograms will not affect the amount of stretch in the spring. If the spring is of the same stretching ability, the rate of change would still be 0.038 centimeters per kilogram of mass. Changing the initial length of the spring to 6.8 centimeters will change the y-intercept to 6.8. The model of the situation would become $y = 0.038x + 6.8$.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

T-Shirts

A school organization has found four different places from which they may place an order for t-shirts. Each function below could represent the cost of placing a t-shirt order as a function of the number of t-shirts purchased.

A) $c = 5t$
B) $c = 3.25t + 55$
C) $c = 3t + 100$
D) $c = 6t - 55$

1. Write a scenario for each function.
2. Do all four functions fit a t-shirt situation? Explain your answer.
3. Make a table for each function.
4. Graph each function.
5. Describe the differences in the domain for the function and the domain for your problem situation.
6. Describe the differences in the range for the function and the range for the problem situation.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- What is the dependent variable?
- What is the independent variable for each situation?
- In situation A what must 5 represent?
- In situation B which constant represents the cost per t-shirt?
- In situation B what might the constant 55 represent?
- In situation C what does the 3 represent?
- In situation C what might the constant 100 represent?
- In situation D which constant represents the cost per t-shirt?
- In situation D what might the constant -55 represent?
- In situation D think about someone selling you the shirts at a constant rate but giving you a set discount.

Sample Solution:

1. A) Juan made a great deal with the manager of T-Shirts Plus. If the Math Club places an order of shirts, the cost will be $5.00 per shirt.

   B) The Spanish Club feels they have a better deal because they will get their t-shirts for only $3.25 each. They do have to pay a $55.00 set-up fee.

   C) The Math Club found another better deal. They will only pay $3.00 per shirt with a $100.00 set-up fee.

   D) The President of the Freshman class thinks he has the best deal: his father's friend will sell them shirts for $6.00 each and give him a $55.00 discount.

2. Yes, function A is the cost per shirt with no set-up fee, B and C both show the cost per shirt plus a set-up fee, and D shows cost per shirt with a discount.

3.

A) $c = 5t$

t	c
0	0
10	50
20	100
30	150
40	200
50	250
60	300

B) $c = 3.2t + 55$

t	c
0	55
10	87.50
20	120
30	152.50
40	185
50	217.50
60	250

C) $c = 3t + 100$

t	c
0	100
10	130
20	160
30	190
40	220
50	250
60	280

D) $c = 6t - 55$

t	c
0	-55
10	5
20	65
30	125
40	185
50	245
60	305

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

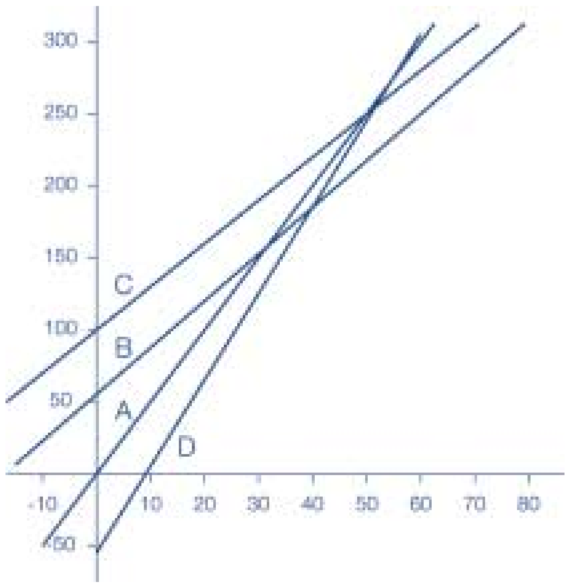
I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 Linear Functions
1.2 The Y- Intercept
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

4.



5. The domain of each function is all real numbers, because each function is a linear function. For the t-shirt situation the domain values must be whole numbers because shirts can not be purchased in fractions.

6. The range of each function is all real numbers. However, in the t-shirt scenario the amounts will be restricted to dollar values depending on the situation. For example, in situation B the amounts must be $55 plus a whole number multiple of $3.25.

Extension Questions:

- In situation A if the company had decided to give you a discount of $40, how would that change the equation?

The equation would become $c = 5t - 40$.

- From which t-shirt company should the group purchase the shirts if they are going to purchase 50 shirts?

By examining the table or the graph, one can see that the cost for 50 shirts is the least in situation B.

- Will situation B always give the least cost?

Situation C and B have the same cost at 180 shirts. After that number situation C has a smaller cost. This can be determined from examining the graph or table, or by solving symbolically.

Which Is Linear?

Four function rules were used to generate the following four tables:

I

x	y
-1	6
0	8
1	10
2	12
3	14

II

x	y
0	5
3	5
6	5
9	5
12	5

III

x	y
-2	-5
-1	-4.5
0	-4
3	-2.5
4	-2
5	-1.5

IV

x	y
-1	0.5
0	0
1	0.5
2	2
3	4.5
4	8
5	12.5

1. Which table(s) represent linear relationships? Explain how you decided.
2. Make a graph of the data in each table. Describe how the graphs are related.
3. Write a function for the linear relationships, and explain your thinking.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

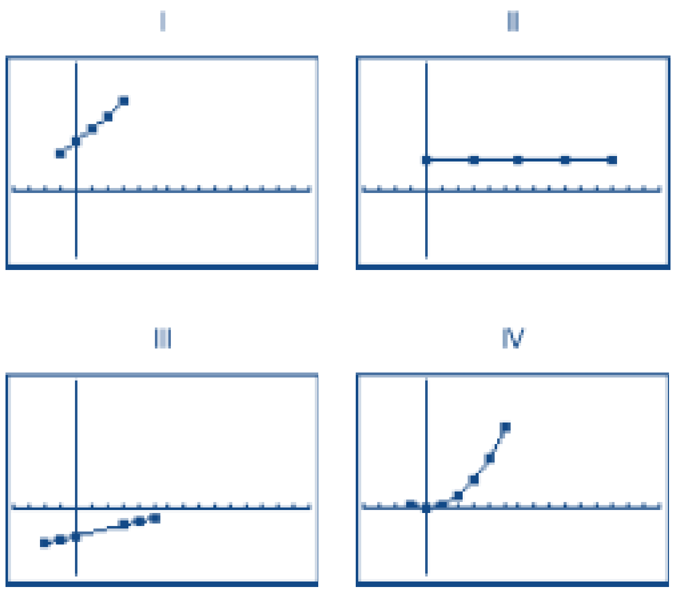
Scaffolding Questions:

- As the *x* values are increasing, what is happening to the *y* values?
- How are the patterns in the tables similar?
- How are the patterns in the tables different?
- What must be true about a function in order for it to be linear?
- How can you decide if a relationship is linear by looking at its table?
- What two numbers must you determine to write the linear function rule?

Sample Solution:

1. In Table I as *x* increases by 1, *y* increases by 2. In Table II *x* increases by 3, *y* stays constant. In Table III as *x* increases by 1, *y* increases by 0.5. In Table IV there is not a constant rate of change. Therefore, Tables I, II, and III represent linear relationships. The graphs of these sets of points form lines. As *x* increases by a constant number, *y* is also increasing by a constant number.

2. The scatter plots of the data are shown below in connected mode.



Three of the graphs show a linear relationship: I, II, and III. The graph of Table IV is not linear. The graph of Table I is the steepest linear graph. The graph of Table II has a slope of zero.

3. In Table I the rate of change is 2 because as x increases by 1, y increases by 2. The point (0,8) indicates that the line crosses the y-axis at 8, so 8 is the y-intercept.

   $y =$ starting value + the rate • x.
   $y = 8 + 2x$

   In Table II the rate of change is 0 because there is no change in y as x changes. The point (0,5) shows where the line crosses the y-axis. The function rule for Table II is $y = 5$.

   In Table III the rate of change is 0.5 because the ratio of the change in y to the change in x is 0.5. The point (0,-4) indicates that the line intersects the y-axis at -4.

   $y =$ starting value plus the rate times x
   $y = -4 + 0.5x$
   The function for Table III is $y = 0.5x – 4$.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

Extension Questions:

- How can you decide if a relationship is linear by looking at its graph?

The graph of the data points forms a line whose slope can be determined.

- How can you decide if a relationship is linear by looking at its symbolic representation?

The rule shows the constant rate of change as the coefficient of the independent variable.

- When might you use a table to answer a question about a linear relationship?

Tables can be used when the values needed are part of the data given. Tables might also be used when they can be easily extended to find an answer; however, it is not always time-effective to extend a table. You may also set up a table using smaller increments for the domain values.

- When might you use a symbolic representation to answer a question about a linear relationship?

Symbolic representation can be used when a specific input or specific output value is required.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Chapter 3:

Interacting Linear Functions, Linear Systems

Bears' Band Booster Club

The Bears' Band Booster Club has decided to sell calendars to the band members and their parents. The cost of the calendars will be $8 per calendar, but they must also pay an initial fee of $65 for designing the cover for the calendar. They decide to sell the calendars for $12 each. Investigate the situation and determine how many calendars they must sell to make a profit.

1. Write a function that describes the relationship between cost and number of calendars.

2. Write a function that describes the relationship between revenue and the number of calendars sold.

3. How many calendars must they sell to make a profit? Describe your process for answering the question.

4. If they want to make at least $400, how many calendars must they sell?

Teacher Notes

Scaffolding Questions:

- Describe how to compute the cost of 10 calendars.
- Explain how to compute the cost of 15 calendars.
- What are the constants in this situation?
- What are the variables in this situation?
- Explain how to compute the revenue from the sale of 10 calendars.
- Consider the sale of 15 calendars. What is the cost of the 15 calendars? What is the revenue from the sale of 15 calendars?
- Do you make a profit when you sell 15 calendars? Explain how you know.
- How can you tell when you start making a profit?

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Sample Solution:

1. The cost is 65 dollars plus 8 dollars per calendar.

   The equation is $y = 65 + 8x$ where y is the cost in dollars and x is the number of calendars.

2. The revenue is 12 dollars times the number of calendars.

   $y = 12x$ where y is the revenue in dollars and x is the number of calendars.

3. They will break even when the revenue equals the cost.

   Cost: $y = 8x + 65$.

   Revenue: $y = 12x$

   $$\begin{aligned} 12x &= 8x + 65 \\ 4x &= 65 \\ x &= 16.25 \end{aligned}$$

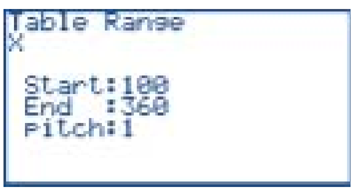
   The only reasonable values in this situation are whole numbers. You cannot sell a fraction of a calendar or a negative number of calendars.

   The first time the cost is less than the revenue is when 17 calendars are sold. They must sell at least 17 calendars to make a profit.

4. You may examine the table or the graph to see when the difference in the cost and revenue is at least $400.

   You may start with a guess of 100 calendars and examine the table.

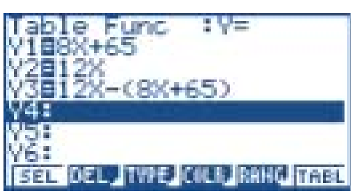
   At 110 the difference is less than 400. At 120 the difference is more than 400. The amount is between these two values. Reset the table at 110 with an increment of one.



   They must sell 117 calendars to make a profit of at least $400.

   Another approach is to write a general rule for profit.

   Profit = Revenue minus Cost
   Profit = $12x - (65 + 8x)$



   Examine the table to find the first value when the profit is at least 400. This occurs when x is 117.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Extension Questions:

- If the situation had been different and the equation for cost was written $y = 80 + 15x$, how would the situation have been described?

The set-up charge was $80, and the cost per calendar was $15.

- How could an equation be used to solve for the number of calendars when the profit is 400 dollars?

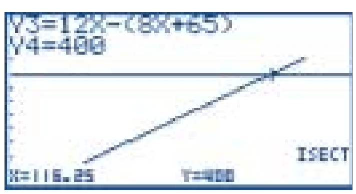
Profit = Revenue minus Cost

$$400 = 12x - (65 + 8x)$$
$$400 = 12x - 65 - 8x$$
$$465 = 4x$$
$$x = 116.25$$

They may not sell a fraction of a calendar. They must sell 117 calendars.

- Describe how the graph could be used to answer the question.

Graph the function y = 12x–(65+8x) and the function y = 400, and find the point of intersection.



Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
  1.2 Valentine's Day Idea

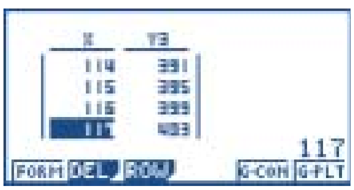
II. Linear Functions
1 Linear Functions
  1.2 The Y-Intercept
3 Linear Equations and Inequalities
  3.4 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

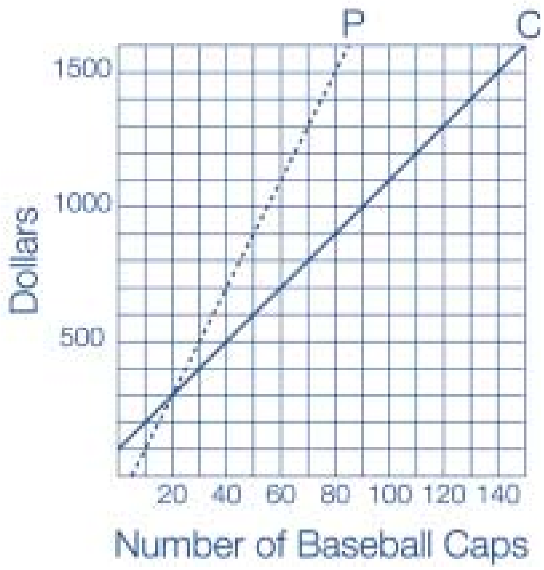
Another approach is to graph the function y = 12x–(65+8x) and the look for the value of x that gives a y-value close to 400.

The profit for 116 calendars is $399, and the profit for 118 calendars from the graph is $407. The table will show the profit for 117 calendars is $403.



Cost and Profit

The Bartlett Booster Club is purchasing baseball caps to sell at school. The graph shows the line C that models the cost of making the baseball caps in terms of the number of baseball caps and the line P that models profit in terms of the number of baseball caps. The lines are represented as dashed lines because not all points on the line would represent the problem situation.



1. Write a function rule for the cost in terms of the number of baseball caps.
2. Describe how to find the cost of 60 baseball caps.
3. How many baseball caps were purchased if the cost was $340?
4. Write the function for the profit in terms of the number of baseball caps.

5. Explain how to determine the profit from the sale of 200 caps.

6. If the profit is the revenue minus the cost, what is an expression for the revenue? Describe how to determine the revenue from the sale of 54 caps?

7. What does this function rule tell you about how they sold the baseball caps?

Teacher Notes

Scaffolding Questions:

- What points on the lines really could represent the situation?
- What is the *y*-intercept of the C line? Explain what it means in this situation.
- Describe the rate at which the cost of the baseball caps is increasing? Discuss how you found the rate.
- What is the *x*-intercept of the P line, and what does it mean in this problem situation? What is the rate of change for the P line, and what does it mean in this situation?
- Describe how to determine the *y*-intercept of the P line. Explain what it means.

Sample Solution:

1. The initial cost from the graph is $100, and the rate of change is $100 for every 10 caps or $10 for 1 cap. The rule for the cost is $100 plus $10 times the number of caps.

   $C(x) = 100 + 10x$, where C is the cost and *x* is the number of caps.

2. To determine the cost of 60 caps, evaluate the function at $x = 60$.

   $C(60) = 100 + 10(60)$ or $700

   The cost of 60 caps is $700.

3. To determine the number of caps that will cost $340, solve the equation.

   $$\begin{aligned} 340 &= 100 + 10x \\ 240 &= 10x \\ x &= 24 \end{aligned}$$

   24 caps would cost $340.

4. There are two points on the profit line that may be used to determine the equation of the line, (20,300) (30,500). The rate of change is $200 for 10 caps or $20 for every 1 cap.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

To determine the *y*-intercept of the P line, imagine extending the graph in the negative directions from the point (20,300). Reduce *y* by 200 for every reduction of 10 in *x* until *x* is 0.

(20–10, 300–200) or (10,100) is a point on the graph.
(10–10, 100–200) or (0,-100) is a point on the graph.
The *y*-intercept is -100.

The profit is -$100 plus 20 times the number of caps sold.

$P(x) = -100 + 20x$, where x is the number of caps and $P(x)$ is the profit.

5. Let $x = 200$
   $-100 + 20(200) = 3900$

   The profit from the sale of 200 caps is $3900.

6. If profit is revenue minus cost, then revenue is profit plus cost.

   $P = R - C$
   $R = P + C$

   The function rule for the revenue is the sum of the two rules, profit and cost.

   $R = (100 + 10x) + (-100 + 20x)$
   $R = 30x$

   The revenue from the sale of 54 caps is 30 times 54 or $1620.

7. The revenue is $30 times the number of caps, so they must have sold the caps for $30 per cap.

Extension Questions:

- Explain the significance of the point of intersection of the two lines that represent cost and profit.

The point of intersection of the two lines on the graph is the point (20,300). The cost and profit are both equal to $300 at this point.
$C(20) = 100 + 10(20) = 300$
$P(20) = -100 + 20(20) = 300$

Another way to look at it is cost equals profit.

$P = R - C$
$R = P + C$
If $P = C$, then $R = C + C = 2C$.
When 20 caps are sold, the revenue is twice the cost of 20 caps.

- Suppose that they were able to find someone who would sell them caps at the same price, but with an initial cost of $80. How would the graph of the cost line be affected?

The graph would be a line parallel to the original line, but with a y-intercept of 80. The function for cost would be $C = 10x + 80$.

- If they continue to sell the caps for $30, how is the profit affected by this new cost function? Describe the effect on the graph.

The revenue function would still be $R = 30x$.
Profit equals revenue minus cost.
$P = 30x - (10x + 80) = 20x - 80$
The previous profit function was $P = 20x - 100$. The profit would be increased by $20. The graph of the line would be raised 20 units, but would have the same slope.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.3 Exploring Rates of Change
3 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Student Work

1) C

# of T-shirts	Dollars
20	300
40	500
60	700
80	900
100	1100
120	1300
140	1500

$Y = 10x + 100$

2) $Y = 10(60) + 100$

$C = 700$

3) $20(22) - 100 = 340$

22 t-shirts

4) P

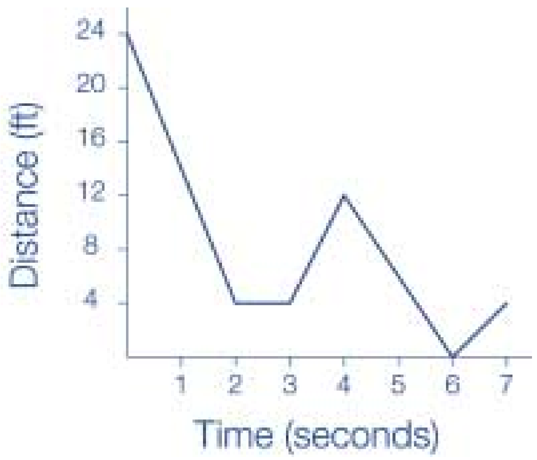
# of T-shirts	Dollars
20	300
40	700
60	1100
80	1500
100	1900
120	2300
140	2700

5) $Y = 200(20) - 100$

$Y = 3900$

Motion Detector Problem

The graph below shows how the distance between a person and a motion detector depends on the time that has elapsed since the person began walking.



1. Describe, in detail, how the person's position relative to the motion detector changes over the time interval from 0 seconds to 7 seconds. Include a description of the person's speed for each portion of the graph.

2. When is the person moving the fastest? Explain.

3. Write a function for each phase you described in problem 1.

Teacher Notes

Scaffolding Questions:

- How far from the motion detector is the person initially?
- Is he moving toward the detector or away? How fast is he moving?
- When is the person standing still? For how long?
- When is the person moving away from the motion detector?
- Does the person ever reach the sensor?

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

Sample Solution:

1. At the beginning, the person is 24 feet away from the motion detector. He walks for two seconds and stops 4 feet from the sensor. He is walking at a rate of 20 feet per two seconds, i.e., 10 feet per second.

   He stands still 4 feet from the sensor for one second.

   He turns and walks away for one second, stopping 12 feet from the motion detector. He is walking at a rate of 8 feet per second.

   Next, he turns and walks for two seconds back toward the motion detector, going all the way up to it. He is walking at a rate of 12 feet per two seconds, i.e., 6 feet per second.

   Finally, he turns and walks away from the motion detector for one second, stopping 4 feet away from the sensor. He is walking at a rate of 4 feet per second.

2. The person is moving the fastest during the first two seconds when he is walking toward the motion detector at 10 feet per second.

3. For the first phase the y-intercept is 24 and the rate of change is -10 feet per second. The rate is negative because the distance between the person and the motion detector is decreasing as time increases. The function is $y = 24 - 10x$, where $0 \le x \le 2$.

   Between 2 seconds and 3 seconds, the distance stays constant. There are 4 feet between the person and the motion detector, so the function rule is $y = 4$, where $2 \le x \le 3$.

   Between 3 and 4 seconds, the rate of change is 8 feet per second because the distance between the person and the motion detector is increasing. Substituting either data point (3,4) or (4,12) in $y = 8x + b$ gives the y-intercept.

$$\begin{aligned} 4 &= 8(3) + b \\ 4 - 8(3) &= b \\ b &= -20 \end{aligned}$$

The function is $y = 8x - 20$ where $3 \le x \le 4$.

Between 4 seconds and 6 seconds, the rate of change is -6 feet per second. Use (4,12) or (6,0) in $y = b - 6x$ to get the y-intercept.

$$\begin{aligned} y &= b - 6x \\ 0 &= b - 6(6) \\ 0 &= b - 36 \\ b &= 36 \end{aligned}$$

The function rule is $y = 36 - 6x$, where $4 \le x \le 6$.

Finally, between 6 seconds and 7 seconds, the rate of change is 4 feet per second. Use (6,0) or (7,4) in $y = 4x + b$ to get the y-intercept $b = -24$.

The function is $y = 4x - 24$, where $6 \le x \le 7$.

Extension Questions:

- How would the graph change for the time interval 0 to 2 seconds, if the person walked toward the motion detector with increasing speed?

The graph would be a curve opening downward. It would be shaped like part of a quadratic such as $y = x^2$, $x \le 0$.

- How would the graph change for the time interval 3 to 4 seconds if the person walked away from the motion detector with decreasing speed?

The graph would be a curve opening upward and shaped like part of a quadratic such as $y = -x^2$, $x \ge 0$.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
3 Interpreting Graphs
 3.1 Interpreting Distance Versus Time Graphs

II. Linear Functions
1 Linear Functions
 1.3 Exploring Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

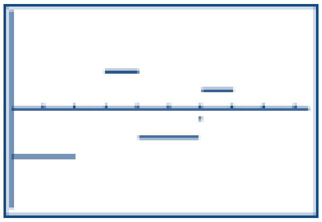
Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

- What would be the function rule for velocity versus time, and what would its graph look like?

The velocity is the rate of change for each section of the graph. In each section the slope is a constant. The values for y in each section would be constant. The function rules for the sections would be

If $0 \le x \le 2$, $y = -10$.
If $2 \le x \le 3$, $y = 0$.
If $3 \le x \le 4$, $y = 8$.
If $4 \le x \le 6$, $y = -6$.
If $6 \le x \le 7$, $y = 4$.

The graph would be



Student Work

Motion Detector Problem

① The person started 24 feet away at first and then within 2 minutes was within 4 feet of the motion detector. The person stayed 4 feet away for 1 minute then went 12 feet away from the motion detector in the next minute. It took 2 minutes for the person to arrive at the motion detector (0 feet away). Then, on the 7th minute, the person had travelled 4 feet.

② The person is moving the fastest between 0 and 2 minutes. We know this because in 2 minutes he moved at a steady pace and went 20 feet. This means 20 feet/2 minutes or 10 feet per minute during this period.

③

(0,24)	(1,14)	$-10/1$	$f(x) = -10x + 24$
(3,4)	(4,12)	$8/1$	$f(x) = 8x - 20$
(4,12)	(6,0)	$-12/2 = -6$	$f(x) = -6x + 36$
(6,0)	(7,4).	$4/1$	$F(x) = 4x - 24$

Speeding Cars

Four cars start from the same city at the same time. The following data was collected on the performance of the four different cars based on miles driven in terms of hours:

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

Car B

Hours	Miles
0	0
1	75
2	150
3	225
4	300

Car C

Hours	Miles
0	0
5	200
10	400
15	600
20	800

Car D

Hours	Miles
0	0
1	65
2	85
3	105
4	125

1. Which car was traveling the fastest? How do you know?
2. Which car was traveling the slowest? How do you know?
3. Compare and contrast the tables.
4. Write a function rule to model each car's travel.
5. Create a scatterplot for each table. Compare and contrast the graphs. Compare the domains for the functions and the domains for the problem situation.
6. Do any of the tables represent a direct variation? Explain how you know.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- How can you use the table to determine the speed at which each car traveled?
- How can you tell if the car is traveling at a constant rate?
- What are the similarities in the table values?
- What are the differences in the table values?
- How can you tell from a table if it represents a linear function?
- How can you tell from the graph that a function is linear?
- What must be true if a set of points represents a direct variation?

Sample Solution:

1. Determine the speed or rate of change for each car.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

Car B

Hours	Miles
0	0
1	75
2	150
3	225
4	300

Car C

Hours	Miles
0	0
5	200
10	400
15	600
20	800

Car D

Hours	Miles
0	0
1	65
2	85
3	105
4	125

To find the speed at which the car is traveling find the differences in the distances and the times from the starting point (0,0).

Car A:

$\frac{120-0}{2-0}=60$ $\frac{180-0}{3-0}=60$ $\frac{300-0}{5-0}=60$ $\frac{360-0}{6-0}=60$

Car A is traveling at 60 miles per hour.

Car B:

$\frac{75-0}{1-0}=75$ $\frac{150-0}{2-0}=75$ $\frac{225-0}{3-0}=75$ $\frac{300-0}{4-0}=75$

Car B is traveling at 75 miles per hour.

Car C:

$$\frac{200-0}{5-0}=40 \qquad \frac{400-0}{10-0}=40 \qquad \frac{600-0}{15-0}=40 \qquad \frac{800-0}{20-0}=40$$

Car C is traveling at 40 miles per hour.

Car D:

$$\frac{65-0}{1-0}=65 \qquad \frac{85-0}{2-0}=42.5 \qquad \frac{105-0}{3-0}=35 \qquad \frac{225-0}{4-0}=55$$

Car D is not traveling at a constant rate.

If the rates are examined for the one-hour time intervals, the differences are not the same.

$$\frac{65-0}{1-0}=65 \qquad \frac{85-65}{2-1}=20 \qquad \frac{105-85}{3-2}=20 \qquad \frac{125-105}{4-3}=20$$

Car D is traveling at a constant rate after the first hour.

Car B is traveling the fastest; for every hour it travels 75 miles.

2. Car C is traveling the slowest constant rate; for every hour it travels only 40 miles. However, Car D is traveling at a slower rate from the first to the fourth hours.

3. The tables are similar in several ways; all 4 tables start at (0,0). The tables all report miles and hours. All the tables show that as the hours increase the miles also increase. The tables are different because they don't all increase by the same hour values; the time interval is not one hour on all tables, and in table D there isn't a constant rate for the all values.

   Tables A, B, and C represent linear relationships because they indicate a constant rate of change. Table D is not a linear relationship because the first hour the car traveled 65 miles, but after the first hour it only covers 20 miles for each additional hour.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(G) relates direct variation to linear functions and solves problems involving proportional change.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

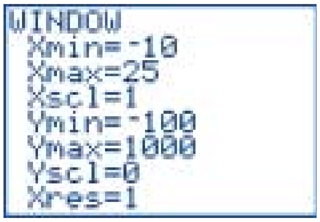
Objective 3:
The student will demonstrate an understanding of linear functions.

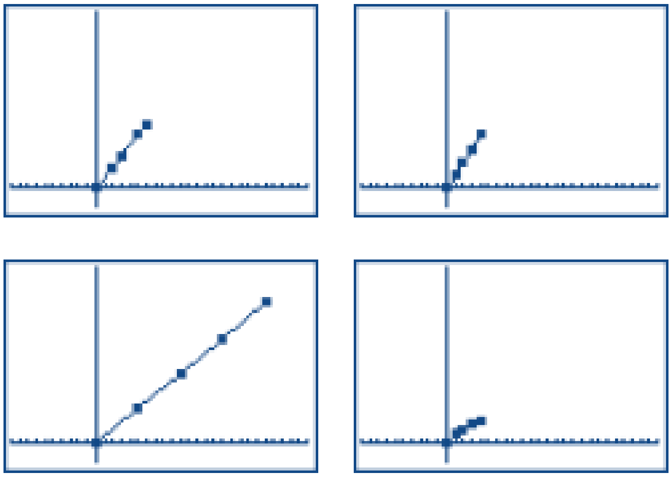
4. The first three could be represented by the equations of the form $y = mx + 0$ because the starting value is 0 where m is the slope or rate of change.

Table A:	$y = 60x$
Table B:	$y = 75x$
Table C:	$y = 40x$
Table D:	$y = 65x$ for $0 \leq x \leq 1$
	$y = 20x$ for $1 \leq x \leq 4$

For values of x between 0 and 1, there is not a constant rate, and the linear model $y = 65x$ would not be appropriate.

5. Each of the graphs is a set of points. All graphs start at (0,0). The lines all have a positive slope. Graph B has the greatest slope because it is the steepest and has the greatest rate of change. Graph D could not be modeled by a line. Graph A, B, and C represent linear relationships.





Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns
2.2 Identifying More Patterns

II. Linear Functions
1 Linear Functions
1.2 Y-Intercept
1.3 Exploring Rates of Change
1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

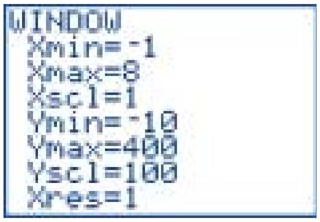
Objective 2:
The student will graph problems involving real-world and mathematical situations.

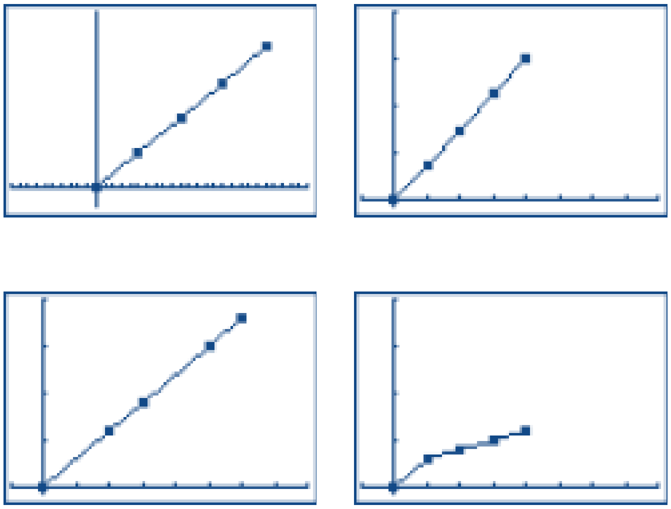
Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

The graphs may be examined more carefully with a smaller window.





The domains for the functions are all real numbers. The domain for the problem situation are numbers greater than or equal to zero. The graphs show connected points because the functions are continuous for the values of the number of hours. However, the upper limit on the domain values depends on how long each car travels.

6. Tables A, B, and C represent direct variations because there is a constant rate of change and the relationship contains the point (0,0). Table D does not represent a proportional relationship because there is not a constant rate of change.

Extension Questions:

- If another car had traveled at a speed that was twice the speed of Car C, how would the table values have been affected?

If the car is traveling at twice the speed, the equation for the distance as a function of the number of hours would be y = 80x. If the x-values are the same, the y-values would have been twice the original values of Car C.

- Suppose that another car has the same values as Car A, except that 20 is added to each of the *y*-values.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

New Car

Hours	Miles
0	20
2	140
3	200
5	320
6	380

Describe how this car's motion is the same or different from Car A.

The new car is traveling at the same rate as Car A , because 20 has been added to each y-value.

$$\frac{140-20}{2-0}=60 \quad \frac{200-20}{3-0}=60 \quad \frac{320-20}{5-0}=60 \quad \frac{380-20}{6-0}=60$$

One possible way to interpret the difference is to say that the new car started out 20 miles ahead of Car A.

- How is the graph of this new car's function different?

The y-intercept of this new graph would be at 20, but the graph would be parallel to the original graph of Car A.

The Walk

Two adjacent motion detectors have been set up in a room so that Pam and Abigail may walk in parallel paths in front of a motion detector. The tables below show the data that was collected for each walk. Assume the students each walked at a constant rate and started at the same time.

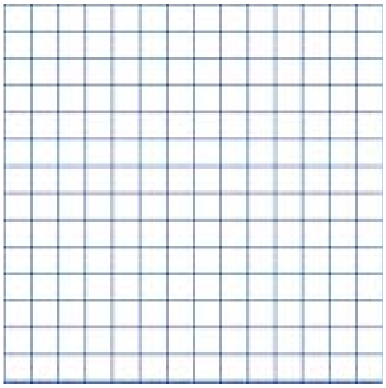
Pam's Walk

Time (seconds)	Distance (ft) from Motion Detector
1	7.9
3	5.3
6	1.4

Abigail's Walk

Time (seconds)	Distance (ft) from Motion Detector
2	3.6
4	5.2
7	7.6

1. Create a graph to model the students' walks. Label the axes.



2. Write a function rule that models each person's distance from the motion detector in terms of the number of seconds.

3. Determine when the two students will be next to each other when they're walking the path.

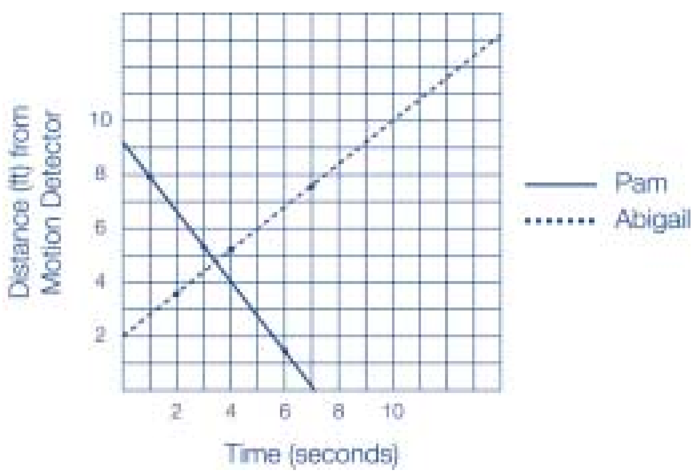
Teacher Notes

Scaffolding Questions:

- If the girls walked at a constant rate, describe the shape of the graph.
- How can you use the table to determine how fast each person was walking?
- If you plot the points, what pattern do you see?
- How is the way Pam was walking different from the way Abigail was walking?

Sample Solution:

1.



2. The differences may be used to determine how fast each person was walking.

Pam's Walk

	Time (seconds)	Distance (ft) from Motion Detector	
	1	7.9	
2			-2.6
	3	5.3	
3			-3.9
	6	1.4	

Abigail's Walk

	Time (seconds)	Distance (ft) from Motion Detector	
	2	3.6	
2			1.6
	4	5.2	
3			2.4
	7	7.6	

Pam was walking at a rate of -1.3 feet per second.
Abigail was walking at a rate of 0.8 feet per second.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(b.3) **Foundations for functions.**
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) **Linear functions.**
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) **Linear functions.**
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.

(c.3) **Linear functions.**
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The distance Pam was from the motion detector is decreasing, so she must be walking toward the detector. Abigail's distance was increasing, so she was walking away from the detector. The function rule for Pam's walk will be the starting point plus the rate times the number of seconds. Use the table and the rates to determine the starting points. Add an extra row for time 0.

Pam's Walk

Change in time	Time (seconds)	Distance (ft) from Motion Detector	Change in distance
	0	9.2	
1	1	7.9	-1.3
2	3	5.3	-2.6
3	6	1.4	-3.9

Abigail's Walk

Change in time	Time (seconds)	Distance (ft) from Motion Detector	Change in distance
	0	2	
2	2	3.6	1.6
2	4	5.2	1.6
3	7	7.6	2.4

To determine the starting point add the 1.3 feet for the one second. She started at 7.9 + 1.3 or 9.2 feet from the motion detector. The rule for Pam's walk is $y = 9.2 - 1.3x$. The function rule for Abigail's walk will be her starting point plus the rate times the number of seconds.

Her starting point is 3.6 minus the distance she travels in two minutes or 3.6 – 1.6 or 2 feet.

The function rule that describes her walk is $y = 2 + 0.8x$.

3. We want to know when the two people are the same distance away from the motion detector.

By examining the graphs one can see that at about 3 seconds they are both 5 feet away from the detector.

To check this solution solve the system

$$y = 9.2 - 1.3x$$
$$y = 2 + 0.8x$$

$$2 + 0.8x = 9.2 - 1.3x$$
$$2.1x = 7.2$$
$$x = 3.428571$$

$$y = 2 + 0.8(3.428571) = 4.742857$$

They will be next to each other when they are about 4.74 feet from the motion detector.

The domain values must be any number greater than or equal to zero, but in an actual situation there is a limit to the number of seconds the person can walk with the motion detector. This limit is set by the calculator operator. The range values represent distance and must be positive numbers.

Extension Questions:

- What are reasonable domain and range values for this problem situation?

The domain values must be any number greater than or equal to zero, but in an actual situation there is a limit to the number of seconds the person can walk with the motion detector. This limit is set by the calculator operator. The range values represent distance and must be positive numbers.

- Suppose the motion detectors had been set up on opposite sides of the room 10 feet apart and the girls had walked on parallel paths. If the same data is used, how would your answers have been different?

If the motion detectors had been 10 feet apart on opposite sides of the room, the two walkers would have been walking in the same direction. The equations describing their motion relative to each person's motion detector would still have been

Pam: $y = 9.2 - 1.3x$.

Abigail: $y = 2 + 0.8x$.

However to consider when they would be in the same horizontal position, one must write the equations in terms of distance from one of the motion detectors. Suppose the equations are written as distance from Abigail's motion detector with respect to time.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Abigail started 2 feet from her motion detector. Pam started 9.2 feet from her motion detector. Since the distance between the motion detectors is 10 feet, Pam would be on a horizontal distance of 10 – 9.2 or 0.8 feet from Abigail's motion detector. The equation of her movement relative to the motion detector is her starting point plus her rate times the number of minutes.

$y = 0.8 + 1.3x.$

The rate is positive because her distance is increasing from Abigail's motion detector.

Abigails's rule is $y = 2 + 0.8x.$

Solving this system of equations results in a solution of

$$0.8 + 1.3x = 2 + 0.8x$$
$$0.5x = 1.2$$
$$x = 2.4$$

$$y = 2 + 0.8(2.4) = 3.92$$

They will both be 3.92 feet from Abigail's motion detector 2.4 seconds after they started walking.

- How would the functions that describe the motion in this last situation have been changed if the motion detectors had been positioned 12 meters apart instead of 10 meters?

Pam would be on a horizontal distance of 12 – 9.2 or 2.8 feet from Abigail's motion detector. The equation of her movement relative to Abigail's motion detector is her starting point plus her rate times the number of minutes.

$y = 2.8 + 1.3x.$

Pam's equation would be $y = 2.8 + 1.3x.$
Abigail's rule is not changed. It would be $y = 2 + 0.8x.$

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.3 Exploring Rates of Change
3.0 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalites

Connections to Algebra End-of-Course Exam:

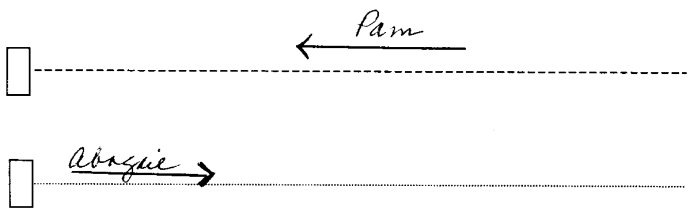
Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Student Work

The Walk

Two adjacent motion detectors have been set up in a room so that Pam and Abigail may walk in parallel paths in front of a motion detector. The table below shows the data that was collected for each walk. Assume the students each walked at a constant rate and started at the same time.



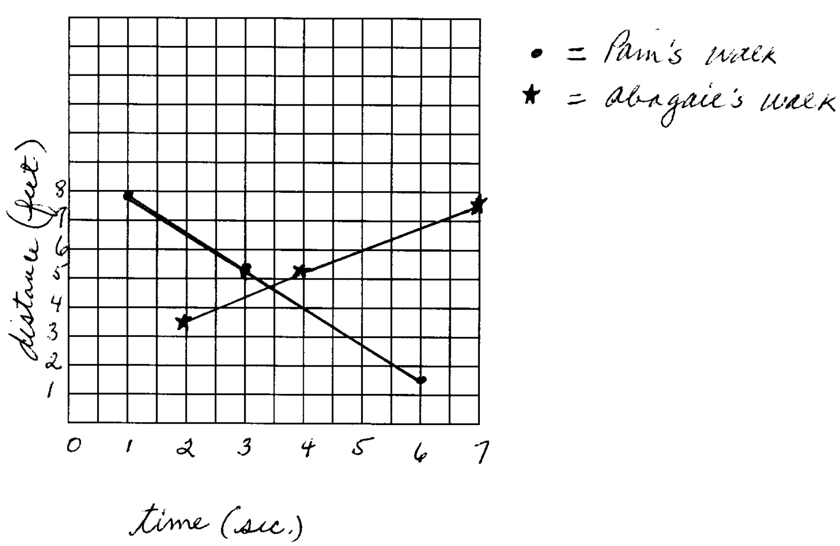
Pam's Walk

Time in seconds	Distance in feet from the motion detector
1	7.9
3	5.3
6	1.4

Abigail's Walk

Time in seconds	Distance in feet from the motion detector
2	3.6
4	5.2
7	7.6

1. Create a graph to model the students' walks. Label the axes.



2. x = time in seconds
y = distance from motion dietector in feet

Pam's walk

	x	y	
	0	9.2	
1+	1	7.9	−1.3
1+	2	6.6	−1.3
1+	3	5.3	−1.3
1+	4	4	−1.3
1+	5	2.7	−1.3
	6	1.4	−1.3

Abagaie's walk

	x	y	
	0	2	
1+	1	2.8	+.8
1+	2	3.6	+.8
1+	3	4.4	+.8
1+	4	5.2	+.8
1+	5	6	+.8
	6	6.8	
	7	7.6	

To get my tables, I divided the measurments by the time intervals so I could see the relationship when it had time intervals of one second. That showed me it was a linear relationship because x and y had a constant charge.

(Pam) $y = -1.3x + 9.2$
(abagaie) $y = .8x + 2$

3. $-1.3x + 9.2 = .8x + 2$
$-.8x \quad\quad -.8x$
$-2.1x + 9.2 = 2$

$-2.1x + 9.2 = 2$
$-9.2 \quad -9.2$
$\frac{-2.1x}{-2.1} = \frac{-7.2}{-2.1}$

x = 3.43 seconds

Chapter 4:

Quadratic Functions

Fireworks Celebration

At a fireworks display celebration, a fireworks rocket is launched upward from the ground with an initial velocity of 160 feet per second. Spectators watch and wonder how high the rocket will go before it begins to descend back to the ground.

The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v is the initial velocity, and s is the initial height. Time t is measured in seconds, and height h is measured in feet.

1. What function describes the height, h in feet, of the rocket t seconds into launch?

2. Sketch a graph of the position of the rocket as a function of time into launch, and give a verbal description of the graph.

3. How high is the rocket after 3 seconds into launch? When does it reach this height again?

4. For the safety of the audience, the rocket, as it descends, should be set to explode at least 250 feet off the ground. The operator has a choice of fuses to use to explode the rocket. Fuse A will detonate the rocket between 3 and 5 seconds, Fuse B will detonate it between 4 and 6 seconds, and Fuse C will detonate it between 6 and 8 seconds. Which fuse should be used? Why?

5. Suppose the rocket is launched from the top of a 200-foot tall building. How will this change the position function for the rocket? How will the graph of the new position function compare with the graph of the first position function? What does the new graph tell you about the situation?

6. Suppose you are the operator and want to have the rocket launched from the ground to stay in the air 3 seconds longer (13 seconds instead of 10 seconds). How would you accomplish this? What effect will it have on the maximum height the rocket reaches?

Teacher Notes

Scaffolding Questions:

- What is the initial velocity and height in this problem?
- How can you rewrite the function so that it is easier to determine an appropriate window for graphing?
- How can you use your graph to answer questions about time and height?

Sample Solution:

1. The vertical motion formula is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second. The initial velocity, v, is 160 ft per second, and s, the initial height, is 0 because the rocket is thrown from the ground.

   $h(t) = -16t^2 + 160t$

2. To determine a reasonable window, find the x-intercepts by solving

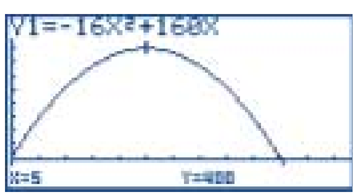
   $$h(t) = -16t^2 + 160t$$
   $$0 = -16t^2 + 160t$$
   $$0 = -16t(t + 10)$$

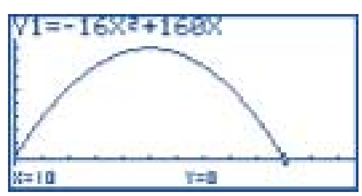
   $-16t = 0$ or $t - 10 = 0$
   $t = 0$ or $t = 10$

   The graph will be a parabola opening down, and the vertex is halfway between 0 and 10 at $t = 5$.

   $h(5) = -16(5)^2 + 160(5) = 400$

   The graph below shows the rocket rising to reach its maximum height of 400 feet in 5 seconds, then falling to hit the ground in 10 seconds.





Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

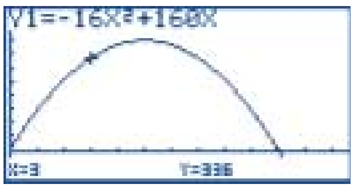
(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

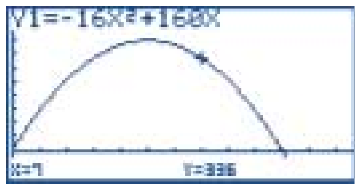
The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

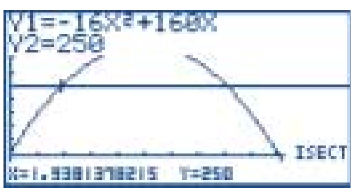
The student:
(A) uses symbols to represent unknowns and variables

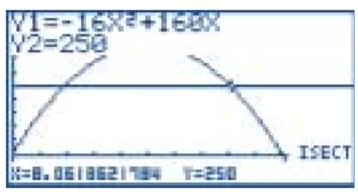
3. By tracing the graph, we see that the rocket reaches a height of 336 feet when $t = 3$ seconds and again when $t = 7$ seconds.





4. By drawing the line $y = 250$ and finding its intersection points with the graph of the parabola, we see that the rocket is at least 250 feet off the ground between 2 and 8 seconds into launch.





This value could also be found by solving the equation

$$250 = -16t^2 + 160t$$
$$0 = -16t^2 + 160t - 250$$
$$0 = 16t^2 - 160t + 250$$
$$t = \frac{160 \pm \sqrt{(160)^2 - 4(16)(250)}}{2(16)}$$
$$t = \frac{160 \pm \sqrt{9600}}{2(16)}$$
$$t = 8.062 \quad \text{or} \quad t = 1.938$$

It does not make sense to detonate it when the rocket is rising. Therefore, it should be detonated at 8 seconds. The operator should use Fuse C since it has the time interval that includes 8 seconds.

5. Since the initial height of the rocket is now 200 feet instead of zero feet (on the ground) and the initial velocity is still the same, the new position function will be

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$; and

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Functions
1.2 Transformations

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

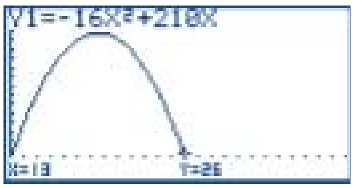
$h(t) = -16t^2 + 160t + 200$

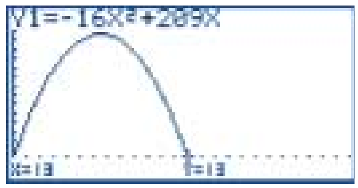
The new graph will be the original graph translated up 200 units. Both graphs have the same shape and orientation. The new graph and the original graph are shown below.

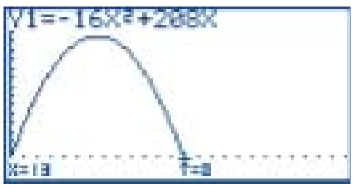
The graphs have the same axis of symmetry $x = 5$, but the vertex of the new graph is (5,600). This makes sense because the original vertex has been translated up 200 units. By the same reasoning, the new y-intercept is (0,200). The x-intercepts for the new graph cannot be found as easily. By tracing or using the “zero” function of the calculator, one x-intercept is between 11 and 12, and the other is between -1 and -2.

The part of the graph that makes sense for the situation is the portion from the y-intercept to the x-intercept that is approximately (11,0). The graph shows that the rocket starts at a height of 200 feet, reaches a maximum height of 600 feet in 5 seconds, and then lands after a little more than 11 seconds.

6. We know that in the original function the gravitational constant, -16 feet per square second, does not change. Therefore, we should experiment with the initial velocity of the rocket. By “Guess and Check” changing of 160 feet per second and graphing, we see that the initial velocity needs to increase to about 200 feet per second and that the rocket reaches its maximum height of about 676 feet in about 6.5 seconds.







Another method uses the roots of the function. We see that the original time interval of 0 to 10 minutes shows up when we look at the original position function in factored form:

$$h(t) = -16t(t - 10)$$

Therefore, change the function to

$$h(t) = -16t(t - 13)$$
$$h(t) = -16t^2 + 208t$$

Now, the x-intercepts are (0,0) and (13,0), so that the rocket is in the air 13 seconds. The rocket reaches its maximum height in 6.5 seconds, and by evaluating the new function, the maximum height is 676 feet. By increasing the velocity, you have kept the rocket in the air longer and shot it to a greater height!

Extension Questions:

- What decisions must be made to determine an appropriate window for the graph of the function $ht = -16t^2 + 160t$?

The domain and range for the situation must be determined. This helps determine minimum and maximum values to use for x and y in the window, so that a complete graph of the function is viewed.

- What do you know about the shape of the graph of the function that helps you determine the domain and range? How will you use this information?

The graph is a parabola opening downward. One x-intercept is 0. Therefore the vertex of the parabola is above the x-axis. By finding the other x-intercept of the graph, we know to have x range in value from a little less than 0 to a little more than the second x-intercept. We get this second x-intercept by solving $0 = -16t^2 + 160t$ *by factoring and using the Zero Product Property.*

Once we know the x-intercepts, we know the x-coordinate of the vertex is halfway between them, and we can evaluate the function at that x-coordinate to get the maximum height. Then we know to have a range in value from a little less than 0 to a little more than the maximum height.

- How does changing the original height from which the rocket is launched change the function and its graph? What part of the graph of the mathematical function will make sense in the situation?

Launching the rocket from a point higher than ground level, e.g., s feet, changes the y-intercept of the graph to correspond to point (0,s). It also will translate the point that is the original maximum (vertex) up s units. The new graph will now have a negative x-intercept and a positive x-intercept, greater than the one before. Thus, the rocket reaches a greater maximum height and stays in the air longer. The part of the graph that makes sense in the situation is from the y-intercept to the positive x-intercept, since this is when the rocket leaves its launch site and when it lands.

- What parameter in the function cannot change? What does this tell you must change in order to launch the rocket from the ground and have it stay in the air longer?

Since the gravitational constant and initial height are constant, we must vary the initial velocity? By experimenting with different values, one can see that the initial velocity should be increased.

- Is it possible for a rocket to be launched from the ground and land on the ground in the same amount of time but go higher than the first rocket? Explain.

Since the rocket launches from the ground, its initial height is 0 and the function has the form

$$h(t) = -\frac{1}{2}gt^2 + vt + 0$$
$$h(t) = t\left(-\frac{1}{2}gt + v\right)$$

The t-intercepts are those values that give a height of zero.

$$h(t) = t\left(-\frac{1}{2}gt + v\right)$$
$$t = 0 \quad \text{or} \quad -\frac{1}{2}gt + v = 0$$
$$t = 0 \quad \text{or} \quad t = \frac{2v}{g}$$

Changing the initial velocity, v, changes the positive t-intercept of the graph so that the rocket is in the air a different amount of time. The constant g is the gravitational constant, -32 feet per square second, and cannot change if the rocket is being launched on earth! The rocket cannot go higher and land on the ground at the same time as the original rocket.

Golfing

The height *h* (in feet) above the ground of a golf ball depends on the time, *t* (in seconds) it has been in the air. Ed hits a shot off the tee that has a height modeled by the velocity function $f(t) = -16t^2 + 80t$. Sketch a graph and create a table of values to represent this function.

1. How long is the golf ball in the air?

2. What is the maximum height of the ball?

3. How long after it is hit does the golf ball reach the maximum height?

4. What is the height of the ball at 3.5 seconds? Is there another time at which the ball is at this same height?

5. At approximately what time is the ball 65 feet in the air? Explain.

6. Suppose the same golfer hit a second ball from a tee that was elevated 20 feet above the fairway. What effect would that have on the values in your table? Write a function that describes the new path of the ball. Sketch the new relationship between height and time on your original graph. Compare and contrast the graphs.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

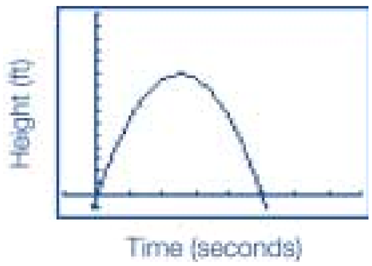
(B) relates the solutions of quadratic equations to the roots of their functions.

Scaffolding Questions:

- Describe your graph.
- Explain what values are reasonable for the domain and range in this situation.
- Describe the relationship between the height of the ball and the time.
- Describe the height of the ball over time.

Sample Solution:

1. The height h (in feet) above the ground of a golf ball depends on the time, t (in seconds) it has been in flight. Ed hits a shot off the tee that has a height modeled by the velocity function $f(h) = -16t^2 + 80t$. Sketch a graph and create a table of values to represent this function.



The graph of this function is a parabola that opens downward. It has a domain (representing the time in seconds) of $0 \le x \le 5$. The range (representing the height of the ball) is $0 \le y \le 100$.

The table of values is:

X	Y1
0	0
.5	36
1	64
1.5	84
2	96
2.5	100
3	96

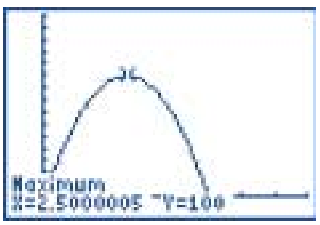
X=0

X	Y1
3.5	84
4	64
4.5	36
5	0
5.5	-44
6	-96
6.5	-156

X=6.5

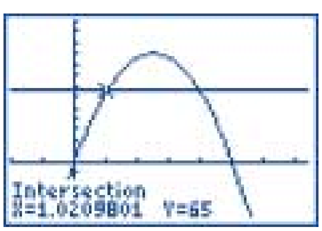
Using the table, it can be determined that the golf ball is in the air a total of 5 seconds.

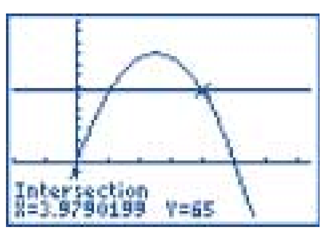
2. The table suggests that the golf ball reaches a maximum height of 100 feet (y) when the time (x) is 2.5 seconds. Another method of estimating the maximum height is to use the calculate function on the graphing calculator. Setting a lower and upper bound on the curve will yield a maximum height value of 100 feet and a corresponding time value that rounds to 2.5 seconds. (The trace feature may also be used to estimate the maximum height.)



3. The height of the ball at 3.5 seconds can be found in the table in the answer to question 1, when $x = 3.5$. At 3.5 seconds, the golf ball reaches a height of 84 feet.

4. The golf ball reaches a height of 65 feet at two different times. Because the graph is a parabola, the y value of the vertex is more than 65. The y value of 65 appears twice. The table above does not show an exact y value of 65 because the values are rounded. The value of 64 is found in the table at 1 second and 4 seconds. To find a more exact answer, the graphing calculator can be used. Results show very small differences in time from the table.





5. If the same golfer hit a second ball from a tee that was elevated 20 feet above the fairway, the values for the time remain the same in the table. The height values for the elevated golf shot are larger because the shot started 20 feet higher.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
  1.1 Quadratic Relationships
2 Quadratic Equations
  2.1 Connections
  2.2 Quadratic Formula

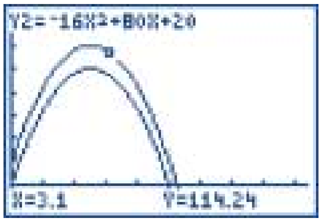
Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

The new function is $h = -16t^2 + 80t + 20$. The graph of the new function at time 0 seconds begins at 20 feet. This is 20 feet higher than in the original graph. The maximum for the new function is 20 feet higher than the original function. This can be found by tracing the function and looking in the table. For the first situation the golf ball hits the ground at 5 seconds. The second ball hits the ground at about 5.27 seconds.

The graph shows the original function and the new function.



Extension Questions:

- If the initial velocity had been 60 feet/second, how would the velocity function have been written?

Replacing the 80 in the original function with a 60 indicates the initial velocity is 60 feet per second. The function will be $f(t) = -16t^2 + 60t$.

- How would the graph of your function with an initial velocity of 60 compare to your original graph?

The new function has a maximum height of the ball at approximately 56.24 feet after about 1.9 seconds. The original function had a maximum at 100 feet after about 2.5 seconds.

- How does greater initial velocity appear to affect the flight of the ball?

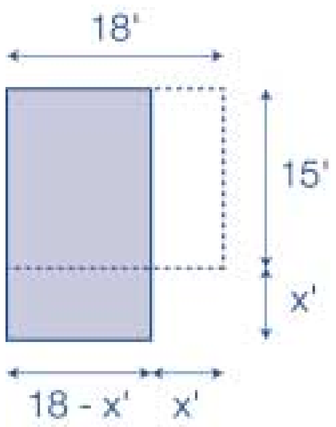
From the example it appears that the greater the initial velocity, the higher the maximum path of the ball over a longer time period.

- How do the x-intercepts of the two graphs compare?

If the velocity is increased, the x-intercept will increase, that is the ball will take longer to land on the ground.

Home Improvements

Ken's existing garden is 18 feet long and 15 feet wide. He wants to reduce the length and increase the width by the same amount, according to the diagram below.



1. Write a function that models the area of the new garden plot.

2. What value of *x* will produce a new area of 280 square feet? Justify your solution.

3. What value of *x* will produce a new area of 266 square feet? Justify your solution.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

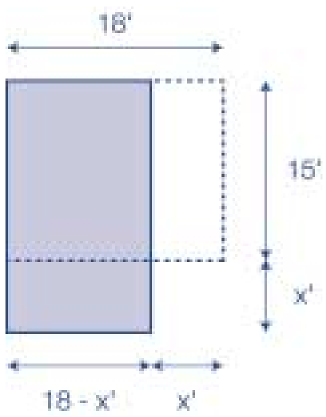
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- How can you find the area of any rectangle?
- What are the dimensions of the new garden plot?
- What methods might you use to solve your equation?
- How will the given values of the area relate to your algebraic area representation?

Sample Solution:

1. The new garden plot is a rectangle with dimensions of $(15 + x)$ by $(18 - x)$. The formula for finding the area of a rectangle is $A = l \bullet w$.



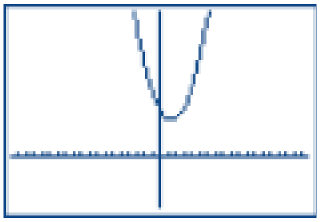
The function that models the area is:

$A = (15 + x)(18 - x)$
$A = 270 + 3x - x^2$

2. To find the value of x that will produce a new area of 280 square feet, substitute the new area for the variable A in the function above.

$$270 + 3x - x^2 = 280$$
$$0 = x^2 - 3x + 10$$

The graph of the function $y = x^2 - 3x + 10$ never crosses the x-axis, so there are no roots.



This means that there are no values for x that will produce a garden area of exactly 280 square feet.

Another way to solve this would be to check the discriminant. This shows that there are no solutions for this equation. The value of $b^2 - 4ac$ is less than zero.

$b^2 - 4ac = (-3)^2 - 4(1)(10) = 9 - 40 = -31$.

3. To determine values of x for which the garden has an area of 266 square feet, one can set the area function equal to 266 as follows.

$$\begin{aligned} 270 + 3x - x^2 &= 266 \\ 0 &= x^2 - 3x - 4 \\ 0 &= (x-4)(x+1) \\ x &= 4, -1 \end{aligned}$$

Both values will produce an area of 266 square feet when substituted back into the original equation.

$$\begin{aligned} (15+x)(18-x) &= 266 \\ (15+4)(18-4) &= 266 \\ (19)(14) &= 266 \end{aligned} \quad \text{and} \quad \begin{aligned} (15+-1)(18--1) &= 266 \\ (14)(19) &= 266 \end{aligned}$$

However, the value of x cannot be negative because the problem stated that he wanted to reduce the length and increase the width, so only the value of 4 makes sense for this situation. The dimension change that produces an area of 266 square feet is and increase of 4 feet.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

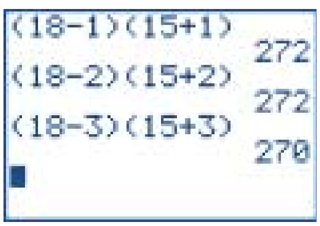
Extension Questions:

- What values of *x* will give Ken a garden with the dimensions of the original garden?

Ken's original garden was 18 feet by 15 feet. If the length and width are reduced and increased by the same amount (x), the value of x would have to be 3. If you reduce the 18 foot side by 3, it will result in a side of 15 feet. If you increase the 15 foot side by 3, it will produce a side of 18 feet. Therefore, the dimensions of the new garden will match that of the original garden.

- What values of *x* would produce a garden with maximum area?

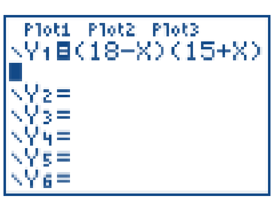
Using the area formula, the length and width can be decreased and increased by the same amount to find the largest area. Using the calculator, values of 1 foot and 2 feet produce the same and largest area.

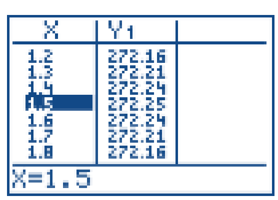


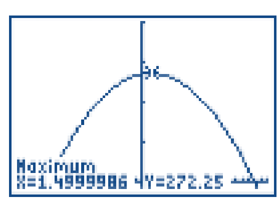
These values will produce dimensions of 16 feet by 17 feet. These whole number values are the closest to having the garden plot a square figure, which produces the maximum volume.

The area of the garden could be maximized more if decimal values are used. By increasing and decreasing the length by 1.5 feet, the area becomes 272.25 square feet.

The table and graph show this maximum value occurs when the increase is 1.5 feet.

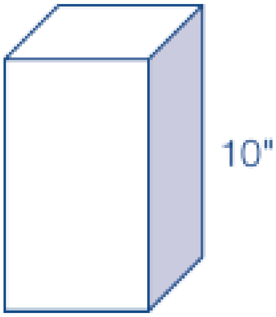






How Much Paint?

Emily has a can of paint that will cover 3800 square inches. She wants to build a small wooden box with a square base and a height of 10 inches. The paint will be used to finish the box.



1. Write a function to represent the total surface area of the box.

2. What equation will allow you to determine the dimensions of the box for which the surface area is 3800 square inches? Show how to solve the equation you wrote symbolically.

3. Describe how to solve the equation using a graph.

4. What is the measure of the base of the largest box Emily can build? Explain your answer.

Teacher Notes

Materials:
One graphing calculator per student.

Connection to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

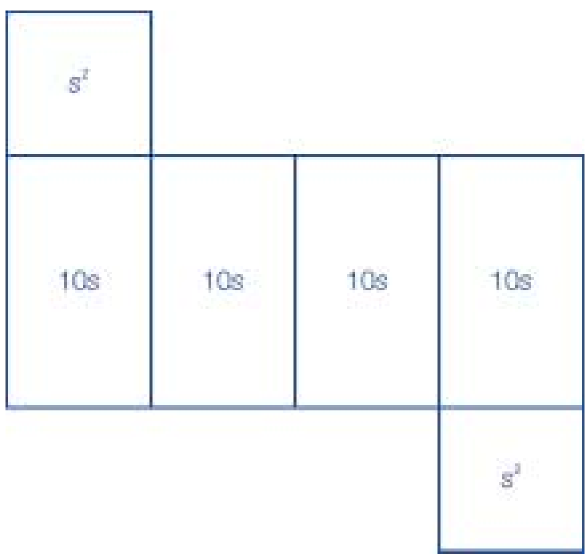
The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- How could you algebraically represent the area of each face of the box?
- How could you use the area of each face to help you represent the total area of the box?
- Describe how to use a graph to determine the number of roots.
- What are some different methods to solve for the side length?

Sample Solution:

1. Let s = the measure of a side of the square base. Then s^2 = the area of the square base. There are 2 square bases with area represented by $2s^2$.



The area of one rectangular side of the box is $10s$, using the formula of (base)(height). There are 4 sides on the box representing an area of $4(10s)$ or $40s$. The total area of the box equal $2s^2 + 40s$.
The function is $A(s) = 2s^2 + 40s$.

2. The total area of the box equal $2s^2 + 40s$. This is the area to be painted. The paint covers a total area of 3800 square feet. Solve the equation $2s^2 + 40s = 3800$.

Transform the equation into standard form:
$2s^2 + 40s - 3800 = 0$

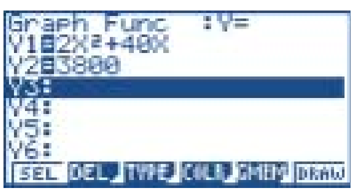
Use the quadratic formula with $a = 2$, $b = 40$, and $c = -3800$

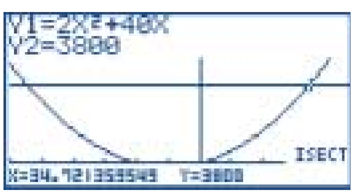
$$s = \frac{-40 \pm \sqrt{(40)^2 - 4(2)(-3800)}}{2(2)}$$

$s = 34.72$ inches or $s = -54.72$

There are two solutions, but only one makes sense for this situation. Since length cannot be negative, the measure of the side of the square base of the largest box should be about 34.72 inches.

3. Graph the function for the surface area, $y = 2x^2 + 40x$.
Graph the line $y = 3800$.





The graph shows the intersection points at approximately (–54.72,3800) and (34.72,3800). The *x*-value of the intersection points represents the possible lengths for the base edge on the box.

4. Since lengths cannot be negative, only *x*-values between 0 and 34.72 can be considered. The largest possible side length that will be within the paint coverage limit is 34.72 inches.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Extension Questions:

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

- Describe another graph that could be used to determine the measure of the base if 3800 square inches is to be covered.

The function $y = 2x^2 + 40x - 3800$ could be graphed. Determine for which values of x the function is 0.

- What is the meaning of this function $y = 2x^2 + 40x - 3800$ for the problem situation?

The rule represents the amount of the surface area less 3800 square feet. When the function value is zero, the surface area would be equal to 3800 square feet.

- How would the original function change if the height of the box had been 15 inches?

The function would be $A(s) = 2s^2 + 4(15)s$.

Insects in the Water

A biologist was interested in the number of insect larvae present in water samples as the temperature of the water varied. He collected the following data:

Temperature (C°)	0	10	20	30	40	50
Insect Population	20	620	950	920	670	75

1. Make a scatterplot of the data. Given that the value of b is 75, experiment with values for a and c in $y = ax^2 + bx + c$ to fit a quadratic function to your plot.

2. Write a verbal description of what the graph tells you about the insect population and the temperature of the water samples. What do the intercepts mean? When is the insect population greatest?

3. The water sample is considered to be mildly contaminated and does not need to be treated if the insect population is no more than 300. For what temperatures will this occur? Explain.

4. Suppose at 0° C, testing showed virtually no larvae present, and the model for this situation is the function $y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

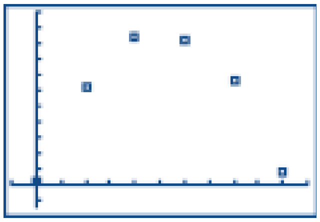
(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- How does the data in the table help you determine a reasonable window for your plot?
- What does the table tell you is a reasonable value for *c*? What does the shape of the graph tell you about the value of *a*?
- What is the function that most closely models this scatterplot?
- Experiment with values of *a* between -1 and -2.
- How can you use the graph of $y = 300$ to help you answer Question 3?

Sample Solution:

1. Scatterplot of population vs. water temperature:

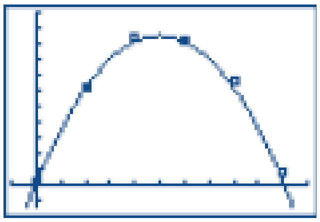


Since $y = 20$ when $x = 0$, the value for *c* in $y = ax^2 + 75x + c$ is 20. Since the scatterplot shows the larvae population increasing and decreasing, *a* must be negative.

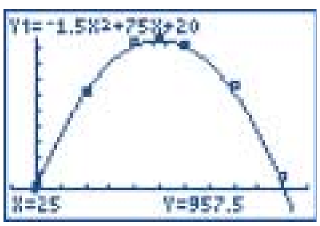
Graphing $y = -1x^2 + 75x + 20$ gives a parabola that opens wider than the plot appears and whose maximum value occurs at a greater temperature, 37.5º C, than the temperature that appears to give the maximum population for the scatterplot, between 10º C and 30º C.

Graphing $y = -2x^2 + 75x + 20$ gives a parabola that opens narrower than the plot appears and whose maximum value occurs at 18.75º C, which is too low.

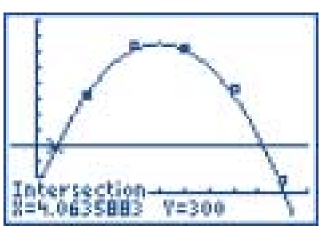
Trying a value close to $a = -1.5$, we find that a good fitting quadratic is $y = -1.5x^2 + 75x + 20$.

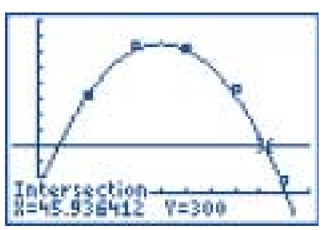


2. The *y*-intercept of the graph shows that at 0º C, there are 20 insect larvae in the water sample. As the temperature increases to 25º C, the population increases to 957 insect larvae in the water sample. This is shown by finding the coordinates of the vertex, (25,957.5), using the graph. Then the population decreases to 0 at about 50º C, the right *x*-intercept.



3. By graphing *y* = 300 along with the population graph and finding the points of intersection, we can determine the temperatures when the population is no more than 300.





(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$; and

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships
1.2 Transformations

III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

The graph shows that for temperatures up to 4º C, the population is no more than 300 insect larvae. Since this point is 21 units to the left of the axis of symmetry, $x = 25$, the other intersection point is 21 units to the right of the axis of symmetry, and, therefore, is (46,300). When the temperature is between 46º C and 50º C, the population will again be no more than 300 insect larvae.

4. Since the first model is in polynomial form and the second model is factored form, rewrite the second model in polynomial form:

$$\begin{aligned} y &= -1.5x(x - 50) \\ &= -1.5x^2 + 75x \end{aligned}$$

Finding the x-intercepts and vertex of the graph either by calculator or analytically, we find that the second model has x-intercepts (0,0) and (50,0) and vertex (25,937.5). It is a translation of the first model, $y = -1.5x^2 + 75x + 20$, down 20 units. The first model accounted for the 20 larvae observed to be present at 0º C. Compared to the first model, the second model underestimates the number of insect larvae present at any temperature by 20 larvae.

Extension Questions:

- What trends in the table and the graph tell you what is happening with the insect larvae in the water samples?

The table shows that as the temperature increases to 20º C, the larvae population increases to 950 and then decreases to 75 at 50º C. The graph gives a more accurate picture. Since it is a parabola opening downward, the maximum number of larvae occurs at the vertex, which is when the temperature of the water is 25º C. The table shows the larvae population decreasing to 75 at 50º C. The graph shows the population decreasing to 20 at 50º C and no larvae present at a fraction of a degree hotter than 50º C.

- How can you graphically investigate when the insect larvae population is no more than 300?

"No more than 300" means "less than or equal to 300." By graphing the line $y = 300$, we can find the portion of the larvae population graph that lies below the line graph, including the intersection points. We can do this using calculator features such as trace or intersect.

- Suppose y = -1.5x(x – 50) + 20 is considered a usable model for predicting the amount of insect larvae present in water samples, and a second round of experiments shows that the population at each of the previous temperatures in the table doubles. How will this affect the scatterplot and the function to model this new scatterplot?

The new table will be

Temperature (C)	0	10	20	30	40	50
Insect Population	40	1240	1900	1840	1340	150

The original scatterplot will be stretched vertically by a factor of 2 since the larvae population doubles. All of the y values from the original function are multiplied by 2.

The new function will be $y = 2[-1.5x(x - 50) + 20]$ *or* $y = -3x(x - 50) + 40$.

- How does the new function in polynomial form compare with the original one?

The coefficients and the constant in the new function are twice those of the original function.

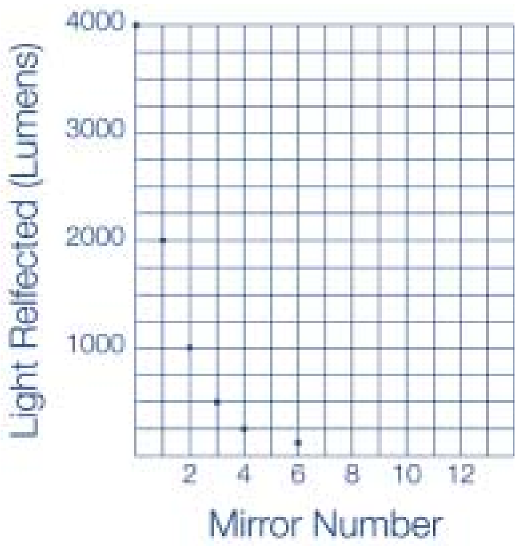
The original is $y = -1.5x(x - 50) + 20 = -1.5x^2 + 75x + 20$.
The new function is $y = -3x(x - 50) + 40 = -3x^2 + 150x + 40$.

Chapter 5:

Inverse Variations, Exponential Functions, and Other Functions

Bright Lights

The brightness of light can be described with a unit called a lumen. A light of 4000 lumens is shined on a series of mirrors. The resulting number of lumens of reflected light is recorded on this graph. The brightness of the light decreases in the same way after each reflection.



1. Describe the relationship between the mirror number and the lumens measurement. Give your description in words and symbolically. Identify the variables.

2. If this reflection continues, what would be the measurement of the sixth mirror? Explain.

3. If this reflection continues, which mirror would you expect to have a measurement of 50 lumens? Explain.

4. What mirror number might have a measurement of 3.9 lumens? Explain.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs; and

(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

Scaffolding Questions:

- Is this a linear relationship? Explain your reasoning.
- What is the relationship between the original measurement in lumens and the first measurement?
- What is the relationship between the first measurement and the second measurement?
- What fraction of light does this set of mirrors reflect?
- What would you have to do to the original measurement to get the second measurement?
- What would you have to do to the original measurement to get the third measurement?

Sample Solution:

1. The amount of light reflected each time is one-half of the previous mirror's measurement in lumens.

   The first mirror's lumens measurement is $4000\left(\frac{1}{2}\right)$.

   The second mirror's lumens measurement is $4000\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ or $4000\left(\frac{1}{2}\right)^2$.

   The third mirror's lumens measurement is $4000\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ or $4000\left(\frac{1}{2}\right)^3$.

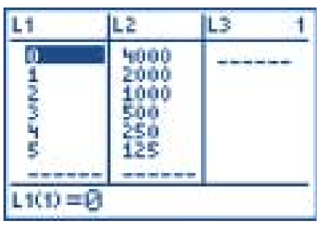
   The *n*th mirror's lumens measurement is $4000\left(\frac{1}{2}\right)^n$.

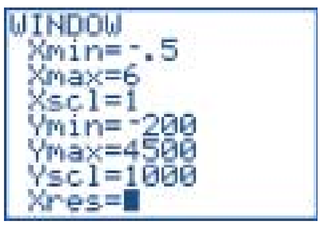
   The function that models this relationship is $L = 4000\left(\frac{1}{2}\right)^n$, where 4000 is the initial amount of light in lumens, $\left(\frac{1}{2}\right)$ is the factor by which the light decreases from mirror to mirror, and n is the mirror number.

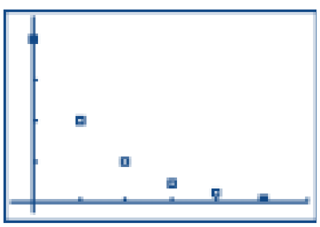
Mirror Number	0	1	2	3	4	5
Number of Lumens	4000	2000	1000	500	250	125

Successive ratios $\frac{2000}{4000} = \frac{1000}{2000} = \frac{500}{1000} = \frac{250}{500} = \frac{125}{250} = \frac{1}{2}$

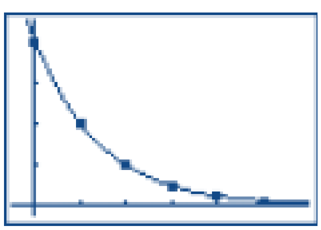
The values for the domain (the *x*-values) are based on the number of mirrors in the table. The *y* values in the table represent the number of lumens. The ordered pairs plot a curve.







The function $y = 4000\ (0.5)^x$ models the situation. The curve went through all of the graphed points.



2. The table feature on the calculator verifies the lumens for the sixth and seventh mirrors. The feature also tells us if any mirror would reflect exactly

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

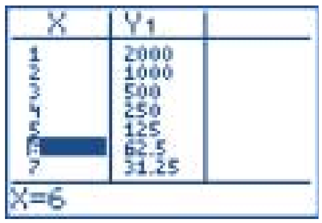
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

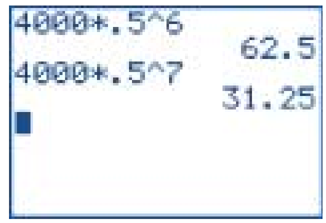
(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(A) uses patterns to generate the laws of exponents and applies them in problem-solving situations;

(C) analyzes data and represents situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

50 lumens. The lumen value may also be verified by substituting the values into the function.





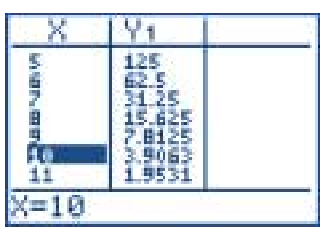
The sixth mirror would reflect $4000\left(\frac{1}{2}\right)^6$ lumens or 62.5 lumens.

The seventh mirror would reflect $4000\left(\frac{1}{2}\right)^7$ or 31.25 lumens.

3. No mirror would reflect exactly 50 lumens.

   Although the graph of the equation is a curve, not all of the points on the curve make sense for this situation. You can only consider mirrors that are whole numbers. Since 50 is between the values 62.5 and 37.5, there is no whole number that will give a function value of 50.

4. The table may be used to determine that the tenth mirror will have approximately 3.9 lumens.



Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
3 Exponential Functions and Equations
- 3.1 Exponential Relationships
- 3.2 Exponential Growth and Decay
- 3.3 Exponential Models

Connections to Algebra End-of-Course Exam:

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

Extension Questions:

- Describe the domain in this situation.

The domain as graphed is {0,1,2,3 …, n}, where n is the number of mirrors.

- Describe the range in this situation.

*The range is {4000, 2000, …, 4000 ** $\left(\frac{1}{2}\right)^n$*}*

- Describe the rate of change.

There is not a constant rate of change in this problem. The rates of change are:

2000 lumens per mirror after the first mirror.
1000 lumens per mirror between the second and first mirrors.
500 lumens per mirror between the second and third mirrors.
250 lumens per mirror between the third and fourth mirrors.
125 lumens per mirror between the fourth and fifth mirrors.

Because the rate of change is not constant, the equation is not linear. There is a constant ratio of successive terms: one-half.

- If the initial measurement had been 3500 lumens, how would the function have been written?

$$L = 3500\left(\frac{1}{2}\right)^n$$

- If the fraction of light reflected by the series of mirrors had been one fourth, how would the function be different?

The fraction would be affected and the function would become

$$L = 3500\left(\frac{1}{4}\right)^n.$$

- If you continued the reflection process, when will the amount of light reflected be zero or less than zero?

Theoretically, the value would never reach zero or below. It would continue to get closer to zero.

Student Work

Bright lights - Bonus

① The relationship between the mirror #, and the lumens reflected, is that for every time the mirror number goes up one, the lumens reflected is half the lumens from the time before.

M = mirror number

PL = Previous lumens

M + 1 = PL / 2

② 4th = 250

$5 = \frac{250}{2} = 125$

$6 = \frac{125}{2} =$ 62.5 lumens

I knew the 4th mirror, so using the relationship from above, I divided it in two to get the 5th mirror, and again to get the 6th, which equaled 62.5.

③ The measurement of 50 lumens would be in between the 6th and 7th mirror. The 6th mirror has a measure of 62.5 lumens, but the 7th mirror has a measure of 31.25 lumens, and 50 lumens is inbetween those two numbers.

④ 7th = 31.25 / 2 =

8th = 15.625 / 2 =

9th = 7.8125 / 2 =

10th = 3.90625

The 10th mirror ≈ 3.9

I just started with the 7th mirror and divided by two again and again until I got to 3.9.

Music and Mathematics

Stringed instruments, like violins and guitars, produce different pitches of a musical scale depending on the length of the string and the frequency of the vibrating string. When under equal tension, the frequency of the vibrating string varies inversely with the string length.

1. Complete the table to find the string lengths for a C-major scale. Round your answers to the nearest whole number.

Pitch	C	D	E	F	G	A	B	C
Frequency (cycles/sec)	523	587	659	698	784	880	988	1046
String Length (mm)	420	___	___	___	___	___	___	___

2. Find a function that models this variation.

3. Describe how the values of the frequency change in relation to the string length.

4. Make a scatterplot of your data. Describe the graph.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

Scaffolding Questions:

- What is true about the product of the frequency and string length for the first pitch, C?
- What does it mean to say the frequency and string length vary inversely?
- What is true about the product of the frequency and string length for the second pitch, D?
- How can you determine the value of the string length for the second pitch, D?

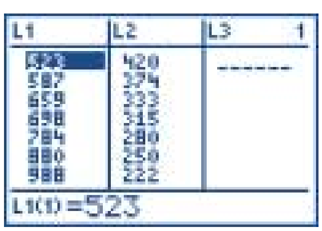
Sample Solution:

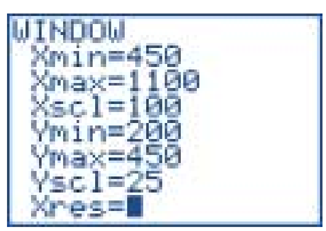
1. The problem stated the frequency of the vibrating string varies inversely with the string length. Two quantities vary inversely if their product remains constant. In this situation, the product of the frequency and the string length must remain constant. The product of the frequency for the pitch C (523) with the string length for that pitch (420) resulted in 219,660. This product is used to work backwards and obtain the string length. 219, 660 is divided by each given frequency to complete the table.

Pitch	C	D	E	F	G	A	B	C
Frequency (cycles/sec)	523	587	659	698	784	880	988	1046
String Length (mm)	420	*374*	*333*	*315*	*280*	*250*	*222*	*210*

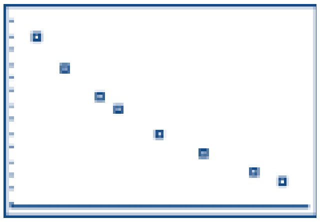
2. As the frequency increases, the string length decreases.

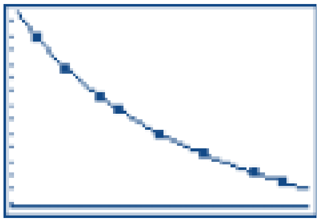
3. Let the *x*-values represent the frequency, and the *y*-values represent the string length.





The scatterplot would not be linear, because the rate of change was not constant. The graph is nonlinear.





4. Because the situation was described as inverse variation, the product of the quantities would remain constant, 219660. This was the pattern used to complete the table, and it is used to write the function.

$$xy = 219660$$
$$y = \frac{219660}{x}$$

where *x* represents the frequency and *y* represents the string length.

Extension Questions:

- Describe the domain and range for the function rule.

The domain and range for the function are all real numbers except zero. Zero is excluded because if one of the numbers, x or y, was zero, the product, xy, would have to be zero.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(B) analyzes data and represents situations involving inverse variation using concrete models, tables, graphs, or algebraic methods.

- Describe the domain and range for the problem situation.

The frequencies for the given pitches are constant as shown on the table. They may be extended in the positive direction. They may not be negative numbers. String length would approach zero but may not be less than zero.

- Compare the function $y = \frac{219660}{x}$ with the function $y = 219660x$.

The function $y = \frac{219660}{x}$ *represents an indirect variation. As the value of y increases, the value of x decreases. The domain and range are all numbers except 0. The graph of the function is not a line.*

The function y = 219660x represents a direct variation. As the value of y increases, the value of x increases. The domain and range of the function are all real numbers. The graph is a straight line.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

None.

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

The Marvel of Medicine

A doctor prescribes a dosage of 400 milligrams of medicine to treat an infection. Each hour following the initial dosage, 85% of the concentration remains in the body from the preceding hour.

1. Complete the table showing the amount of medicine remaining after each hour.

Number of Hours	Number of Milligrams Process	Number of Milligrams
0	400	400
1	400 (0.85)	340
2	400 (0.85)(0.85)	
3	400 (0.85)(0.85)(0.85)	
4		
5		

2. Using symbols and words describe the functional relationship in this situation. Discuss the domain and range of the function rule and of the problem situation.

3. Describe how to determine the amount of medicine left in the body after 10 hours.

4. When will the amount reach 60 milligrams? Explain how you know.

5. Why would it be important for the patient to repeat the dosage after a prescribed number of hours?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

Scaffolding Questions:

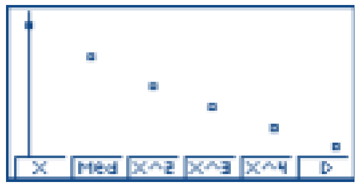
- How much is the initial dosage?
- What percentage of the medicine is left in the body after one hour?
- Express this percentage as a decimal.
- How is the amount in the body at two hours related to the amount at one hour?
- Identify the variables in this situation.
- Describe how the variables change in relation to each other.
- Create a scatterplot of the data in the table and describe the graph.
- Explain the difference between questions 4 and 5.

Sample Solution:

1. Repeated multiplication by 0.85 was used to complete the table as follows:

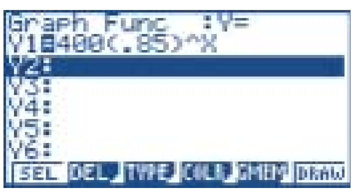
Number of Hours	Number of Milligrams Process	Number of Milligrams
0	400	400
1	400 (0.85)	340
2	400 (0.85)(0.85)	289
3	400 (0.85)(0.85)(0.85)	245.65
4	400 (0.85)(0.85)(0.85)(0.85) = 400 $(0.85)^4$	208.8
5	400 (0.85)(0.85)(0.85)(0.85)(0.85) = 400 $(0.85)^5$	177.48
x	400 $(0.85)^x$	

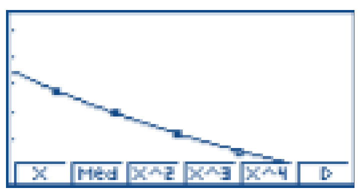
2. The amount of medicine remaining in a patient's system is 400 times the rate raised to the number of hours. The *x*-values represent the number of hours the medicine is in a patient's system. The *y*-values represent the amount of medicine (in milligrams) that is in a patient's system. A scatterplot may be used to analyze the data.



The graph is nonlinear. The ratio of successive terms is a constant ratio of 0.85.

A function that is used to model this situation is $y = 400(0.85)^x$.
This model will apply until another dose of medicine is administered.





The domain of the function is the set of all real numbers, but the domain of the problem situation is the set of nonnegative numbers because *x* represents time in this situation. The range for the problem situation is the set of all numbers less than or equal to 400 but greater than zero. Theoretically, the amount of medicine would never reach zero.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(A) uses patterns to generate the laws of exponents and applies them in problem-solving situations;

(C) analyzes data and represents situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundation for Functions
2 Using Patterns to Identify Relationships
  2.2 Identify More Patterns

III. Nonlinear Functions
3 Exponential Functions and Equations
  3.1 Exponential Relationships
  3.2 Exponential Growth and Decay
  3.3 Exponential Models

Connections to Algebra End-of-Course Exam:

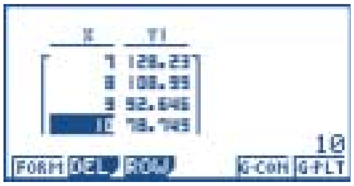
Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

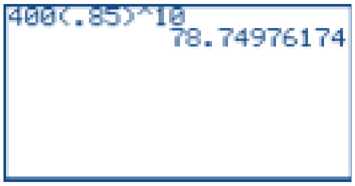
Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

3. The calculator can be used to explore the amount of medicine left in the patient's system after different numbers of hours. After 10 hours, there will be 78.75 milligrams left.





4. From the table it can be seen that a patient will have 60 milligrams left in his system between 11 and 12 hours after the initial dosage.

5. A patient would need to repeat the prescribed dosage to keep a constant amount of medicine in his system to fight the infection.

Extension Questions:

- If the rule had been $y = 500(0.85)^x$ instead of $y = 400(0.85)^x$, how would this situation be different from the given situation?

The initial amount had been changed from 400 to 500. The percent has not been changed.

- What would the equation be if the amount of medicine was reduced by 30 percent each hour?

If the amount was reduced by 30 percent each hour, then there would be 70% of the amount left. If the original amount was 400 milligrams, the equation would be $y = 400(0.70)^x$.

- If you took a new dosage of 400 milligrams at the 12th hour, how much would you expect to have in your system in the 15th hour?

From the table the amount in the system at the 12th hour is 56.897 milligrams. If 400 milligrams are added, the amount in the system is 456.897 milligrams. This amount would be multiplied by the factor .85 to obtain the amount at the 13th, 14th, and 15th hours. The amount in the system at the 15th hour is approximately 280.59 milligrams.

SUPPLEMENTAL

Chapter 6:

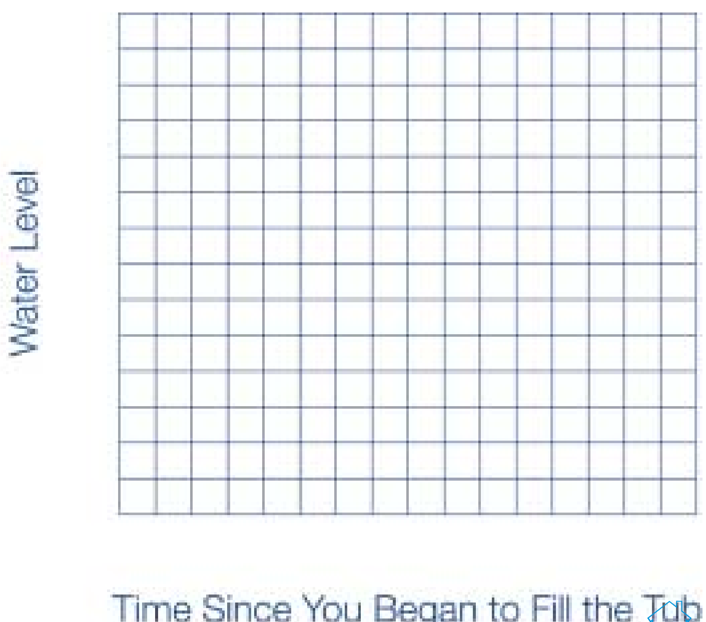
Function Fundamentals

Bathing the Dog

It's time for Shadow, your German Shepherd, to get a winter bath. Shadow does not enjoy getting a bath! You fill the bathtub halfway full, put Shadow in the tub, and begin to bathe her. Shadow tries to escape and gets halfway out of the tub. You pull her back into the tub and finish the bath. You get her out and then drain the tub. Assume a lengthwise cross section of the tub is trapezoidal with the tub sides nearly vertical.

1. Describe how the water level in the tub will vary before, during, and after Shadow's bath.

2. Sketch a graph of your description. Clearly label significant points on the graph.

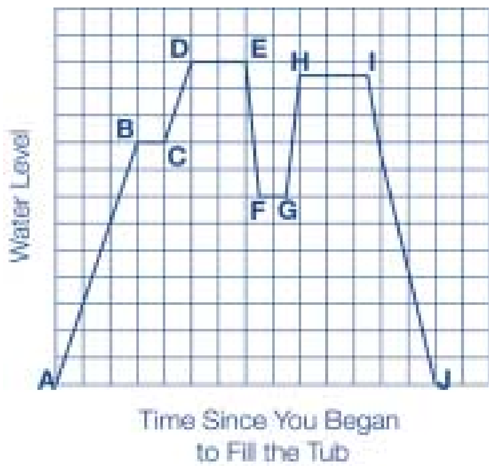


Teacher Notes

Scaffolding Questions:

- What is the water level when the tub starts to fill?
- How will the shape of the tub affect how the water level changes as the tub is filled?
- What happens to the water level when you put Shadow into the tub?
- What happens to the water level if Shadow climbs halfway out of the tub?
- What happens to the water level when you get Shadow completely back into the tub?
- What happens to the water level when you finish bathing Shadow and she gets out?
- How will the shape of the tub affect how the water level changes as the tub is drained?

Sample Solution:



First, the water level starts at (0,0) because the tub initially is empty. It rises at a nearly constant rate because the tub walls are nearly vertical (A to B on graph). In other words, the tub is basically a cylinder with an oval base. The water level stays constant for a moment while you go get Shadow (B to C on graph).

Second, the water level jumps up suddenly when you put Shadow into the tub (C to D on graph) because her body mass displaces water. Also, since she does not like her bath, she thrashes around, (D to E on graph).

Materials:
None required.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

Third, Shadow climbs halfway out of the tub for a moment, trying to escape her bath. This makes the water level drop (E to F on graph). Although she is halfway out of the tub, you continue to bathe her (F to G on graph).

Fourth, you get Shadow back into the tub so the water level jumps back up, but not as high, because by now some of the water has splashed out of the tub (G to H on graph). You finish bathing her (H to I on graph).

Finally, Shadow's bath is done. She gets out, and you pull the plug. As the tub drains, the water level drops at a nearly constant rate (I to J on graph).

Extension Questions:

- Is it possible to write a function rule to describe this situation? Why or why not?

It would be extremely difficult to write a function rule for this situation. The graph would need to be described by a different function for each phase of the graph. Also, the graph would change the next time Shadow gets a bath. The graph depends on a number of factors. How fast is the tub filling? Is the tub filling at a constant rate? How long will Shadow stay in the tub before she tries to climb out?

Data for one bath could be collected and a function fit to each of the various stages. However, the function for each stage would change with the next bath that Shadow gets!

- How would the graph change if the sides of the bathtub were graduated, that is if the sides of the bath tub widen more from tub bottom to top?

As the tub fills, the water level would rise more slowly. The graph would represent an increasing function whose graph is concave down. As the tub empties, the water level would drop more rapidly. The graph would represent a decreasing function that is concave down.

- When a dog enjoys getting a bath, the dog does not try to get out of the tub or thrash around. How would the graph change if Shadow enjoyed getting a bath?

The portion of the graph when Shadow is in the tub would be nearly horizontal because the water level would not change very much.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations of Functions
1 Developing Mathematical Models
1.1 Variables and Functions

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

- Explain the functional representations you used in describing this situation.

A verbal description was used to describe how the water level in the tub was related to the time (and events) for the bath of Shadow. A graph was used to describe how the water level in the tub varied as time during the bath elapsed. The water level is a function of the time needed for the bath. Thus, the water level is the dependent variable and time is the independent variable.

Distance and Time

The graph below represents distance as a function of time.

Create a situation this graph might represent. Choose appropriate units for time and distance. Describe your situation in detail.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- Describe some situations in which the variables could be distance and time.
- For the situation you choose, what are reasonable units for time and distance?
- Describe how to break the graph up into phases.
- What do the graphs in the phases show you in terms of the function increasing, decreasing, or being constant? What does this mean in your situation?
- Can you determine the slope of the graph in each phase? What will this mean in your situation?
- What are the *x*- and *y*-intercepts for the graph? What do they mean in the situation you chose?

Sample Solution:

A wind blows a leaf off a tree branch about 8 feet above the ground. The wind swirls the leaf upwards at a constant rate of 2 feet per second for one second. Now the leaf is 12 feet above the ground. The wind slows down. The leaf swirls upwards at a constant rate of 1 foot per second, reaching a height of 15 feet. From 5 seconds to 10 seconds the wind subsides. The leaf falls at a steady rate of 2 feet per second to 5 feet above the ground and lands on another tree branch. It stays on the branch for two seconds until a slight breeze catches the leaf and it falls to the ground at a steady rate of 5 feet per 2 seconds. The leaf's journey from tree branch to ground lasted 14 seconds.

Extension Questions:

- Consider the phases: ① 0 to 2, ② 2 to 5, ③ 5 to 10, ④ 10 to 12, and ⑤ 12 to 14. In which phases is the function increasing? Decreasing? Constant? What does this tell you about the slope of each phase?

The function is increasing in phases ① and ②, so the line slopes upward, and the rate of travel is positive.

The function is decreasing in phases ③ and ⑤, so the line slopes downward, and the rate is negative.

In phase ④ the distance remains constant; the line is horizontal; the rate of travel is 0 feet per second.

- What does the slope of each phase mean in the situation you created?

Since it equals change in distance between two points divided by change in time for the two points, it represents velocity or speed and direction at which the object travels.

- How would you interpret this graph if the dependent quantity was velocity instead of distance?

In Phase ①, an object is moving with constant positive speed. In phase ②, the velocity is still steady but slower than before. In phase ③, the distance is now decreasing at a steady rate; the velocity is a negative number. In phase ④, the velocity is zero because the object does not move. Finally, in phase ⑤, the distance decreases; the velocity is negative.

- If the first phase on the graph had been from the point (0,12) to the point (2,12), how would that change your description of the graph?

The object was still for the first two seconds before it started to move because the distance remained constant.

- Take the information from the graph and create a graph of the velocity of the object as a function of time. The velocity is the speed at which the object traveled.

The velocity can be found for each phase.

Time Interval	Velocity
0 to 2	2
2 to 5	1
5 to 10	-2
10 to 12	0
12 to 14	-2.5

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns
3 Interpreting Graphs
  3.1 Interpreting Distance versus Time Graphs

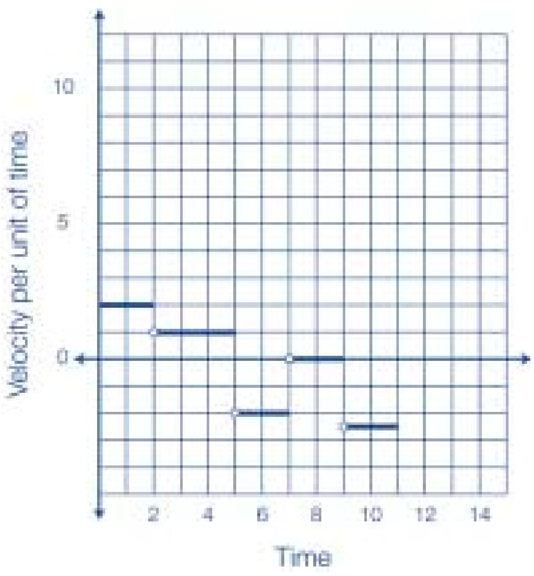
II. Linear Functions
1 Linear Functions
  1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

The graph is a series of horizontal line segments. There is an open circle at one end to indicate that a value of x may not have two function values.



Extracurricular Activities

Determine a function that represents each of the following situations. Describe the mathematical domain and range of the function and a reasonable domain and range for the situation. Use a table, graph, picture or other representation to explain your choice of domain.

A. You join a fitness club that charges an annual fee of $150 plus a facilities-use fee of $3.50 per visit. Your total annual cost is a function of the number of times you visit the club.

B. Over the past summer, you earned $500 by providing in-home pet care service to your neighbors when they went on vacation. You deposited this money in your savings account. Because you are an aspiring musician, you decide to join a CD music club. Their introductory offer allows you to order up to $325 on your first order with a minimum monthly payment of $25.78. No interest will be charged if you make your monthly payments on time and order no other CDs before paying off your first order. Your initial order totals $322.25. Your music club balance is a function of the number of months you pay on the balance.

C. A hiker walking across a desert mesa arrives at the edge just in time to see a hot-air balloon launched from the desert floor, 100 meters below the mesa. The balloon rises at a steady rate of 10 meters per second and can cruise at a maximum height of 300 meters above the mesa. The balloon's vertical distance from the mesa, as it rises to cruising altitude, is a function of the number of seconds that have passed since it was launched.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

For each situation,

- What are the constants?
- What is the dependent variable?
- What is the independent variable?
- In situation A, if you made 5 trips to the fitness club, how would you compute the cost?
- In situation B, as the number of payments increases, what happened to the balance?
- What type of function (linear, quadratic, exponential, inverse variation) relates the variables? How would you determine this?
- What restrictions does the function place on the independent variable?
- Should you use all real numbers for the domain? Why or why not?
- What representation would best help you see the domain and range?

Sample Solution:

A. The cost is equal to \$150 plus \$3.50 times the number of visits. The function rule is $f(n) = 150 + 3.5n$, where n = the number of visits during the year and $f(n)$ = the total annual cost.

The mathematical domain and range for this function are both the set of all real numbers because the function is a nonconstant, linear function.

A table of values helps show the domain and range that make sense for the situation.

Number of visits, n	Annual cost (dollars)
0	150
1	153.50
2	157
3	160.50
4	164
...	...
n	$150 + 3.50n$

The number of visits to the fitness center must be a whole number. Since the function value, $f(n)$, gives cost with initial value of $150 and $3.50 per visit, $f(n)$ must be 150 plus whole number multiples of $3.50. There must be a maximum number of visits, n, depending on how many times you can go in a year.

To summarize, the domain for the situation is {0, 1, 2, 3, 4,..., n} where n is the maximum number of visits per year, and the range is {150, 153.5, 157, 160.5,..., 150+3.5n}. Both the domain and the range are finite sets, limited by the number of visits you make in a year. The graph for the situation would show this as an increasing, dotted plot.

B. The music club balance is $322.25 minus 25.78 times the number of payments that you have made. The function rule is $c(n) = 322.25 – 25.78n$, where n = the number of months you pay on your music club balance and $c(n)$ = your music club balance. Since this is a nonconstant, linear function, the mathematical domain and range for this function are both all real numbers.

The domain values for the problem situation must be whole numbers. The number of payments must be a whole number. Suppose you make no additional charges on your music club account, and you make a payment of $25.78 each month.

The table below helps describe the domain and range that make sense for the situation.

Number of Payments	Music Club Balance
0	322.25
1	296.47
2	270.69
3	244.91
...	...
11	38.67
12	12.89

Your music club balance "zeroes out" at the thirteenth month, when you pay the final balance of $12.89.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

To summarize, the domain for the situation is the set {0, 1, 2, 3,..., 13}, and the range is the set {322.25, 296.47, 270.69,..., 12.89, 0}. Both domain and range are finite sets. The graph of the situation consists of only 14 points plotted in the coordinate plane. The graph of the mathematical function is a line with an infinite number of points.

C. The initial vertical distance from the balloon to the mesa at the time of the launch can be represented by -100. The vertical distance is increasing at a rate of 10 meters per second. The height of the balloon is the starting vertical distance plus 10 times the number of seconds that have elapsed since the launch.

$h(t) = -100 + 10t$, where t = time in seconds into launch and
$h(t)$ = height in meters of the balloon with respect to the mesa

Both the mathematical domain and range for the function are all real numbers since this is a linear function.

The domain for the situation, time t in seconds into launch, is the set of all t values where $0 \le t \le 40$. The range for the situation, height in meters into launch, is the set of all values $h(t)$ where $-100 \le h(t) \le 300$.

The domain is time and changes continuously from 0 seconds to 40 seconds because the balloon starts at time 0, and it takes 40 seconds to reach the altitude of 300 meters. The range is distance and changes continuously from -100 meters to 300 meters. The graph of the situation will be a line segment instead of a set of discrete points.

Extension Questions:

- What kind of function is needed to model each of the situations?

Each situation is modeled by a linear function since each situation involves a constant rate of change.

- What is the parent function for these functions? How does knowing the parent function for these functions help you determine the mathematical domain and range of the function for each of these situations?

The parent function is $y = x$. The domain and range of the parent function are the set of all real numbers. The only special case is a linear function that is *constant, which restricts the range to a single value. Multiplying x by m and adding b to get y = mx + b does not change the domain (set of x - values) or the range (set of y - values). It just changes the graph of the function in terms of where it crosses the y - axis and its slope.*

- In Situation B, describe how the domain and range would change if you changed the rate in the problem but not the initial value.

If you increase the rate at which you pay the balance, the domain and range will shorten, since you will pay off the credit card balance faster. If you decrease the rate, that is if you pay less each month, the domain and range will lengthen; it will take longer to pay off the credit card debt.

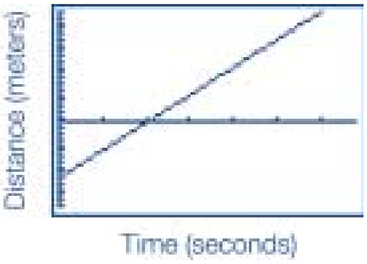
- In Situation C, describe how the domain and range will change if the launch altitude and cruising altitude of the balloon changed. Call this the "launch to cruise distance."

If the "launch to cruise distance" increases, the domain will be a longer time interval and the range will be a longer distance interval. If the "launch to cruse distance" decreases, the domain will be a shorter time interval and the range will be a shorter interval.

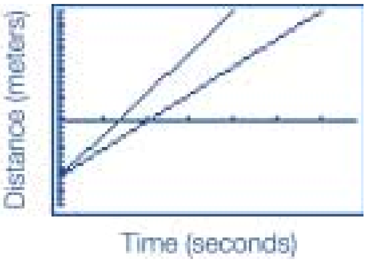
- In Situation C, describe how the domain and range will change if the launch to cruise distance remains the same but the balloon rises at a different rate.

If the balloon rises faster, it will take less time to cover the same distance. This means the domain will be a shorter time interval. The range interval will be the same. If the balloon rises more slowly, it will take more time to cover the same distance. This means the domain will be a longer interval of time, but the range interval will be the same. The graphs of these situations would look like this:

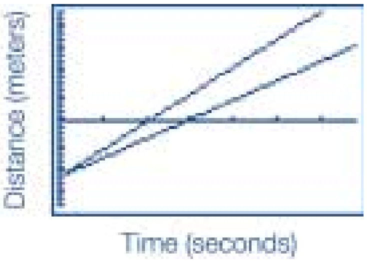
The original situation:



Faster rate:



Slower rate:



Making Stuffed Animals

Pocket sized stuffed animals cost $20 per box (100 animals) plus $2,300 in fixed costs to make them.

1. What does it cost to make 200 boxes of stuffed animals? 250 boxes? 320 boxes?

2. How many boxes of stuffed animals can be made with $5,000? $50,000? Explain how you found your solution.

3. How can you use a graph to predict the cost of 425 boxes?

Teacher Notes

Materials:
One graphing calculator per student.
One sheet of graph paper per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the variables?
- Identify the dependent variable.
- Identify the independent variable.
- What does the $2,300 fixed cost mean?
- What is the cost of one box of stuffed animals?
- What is the cost of three boxes of stuffed animals?

Sample Solution:

1. Compute the cost of the boxes and put the values in a table.

Number of Boxes	Cost of Boxes (dollars)
0	2,300
100	4,300
200	6,300
300	8,300
400	10,300
500	12,300

Every 100 boxes has an additional cost of $2,000 so 50 boxes would have an additional cost of $1,000. Use the table to find that 200 boxes cost $6,300 then add $1,000 to find the cost of 250 boxes. The cost of 250 boxes is $7,300. To find the cost of 320 boxes use the fact that 50 boxes cost an additional $1,000; so 10 boxes would have cost $200. The cost of 320 boxes would be 8300 + 200 + 200 = 8700.

2 The cost is 2300 plus 20 times the number of boxes. The function is

$c = 2300 + 20b$, where c is the cost in dollars and b is the number of boxes.

$$5000 = 2300 + 20b \qquad 50000 = 2300 + 20b$$
$$2700 = 20b \qquad 47700 = 20b$$
$$135 = b \qquad 2385 = b$$

3. Graph the line $y = 2300 + 20x$. Set an appropriate viewing window. Trace along the line to find the value of y when x is 425.

The cost of 425 boxes is $10,800.

Extension Questions:

- How are the graphs of the problem situation and the graph of the function rule different?

The graph of the problem situation is a set of points with x values that are counting numbers. The number of boxes may not be negative or a fraction. The graph of the function rule is a line.

- How does the $2,300 fixed cost affect the graph?

The $ 2,300 is the cost of zero boxes. It is the point (0,2300) on the graph line. However, it would not be a point on the graph of the situation. You would not purchase zero boxes.

- What limits the domain in this situation?

The domain values must be counting numbers, because the stuffed animals are sold by the whole box.

- What makes a relationship linear?

There is a constant increase or decrease in how one variable is related to the other variable. That is there is a constant rate of change.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 The Linear Parent Function
1.2 Y-Intercept

- Create an example of another linear relationship.

Linear situation:
If Jack makes $7.50 per hour, the amount of money Jack makes depends on the number of hours he works.

- Create an example of a nonlinear relationship.

Nonlinear situation:
If Jack is driving on a highway at varying speeds, the number of miles traveled depends on the time.

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Student Work

Making Stuffed Animals

B = Number of Boxes, T = Total Cost

The base cost is \$2,300 so you have to add that to other work. However many boxes you make they cost \$20 each for however many boxes you make.

Formula: $20B + 2300 = T$

1. $20B + 2300 = T$ Start with the formula
   $20(200) + 2300 = T$ Substitute B with 200 (the number of boxes)
   $4000 + 2300 = T$ Do the Multiplication and the Addition
   \$6300 of cost $= T$ You will get the Total cost

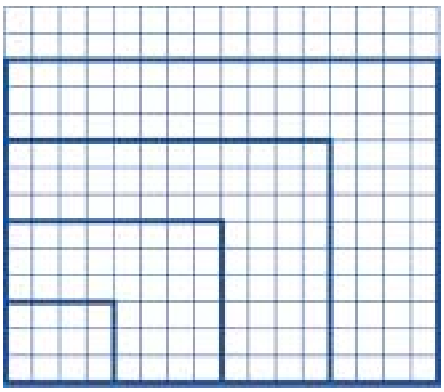
2. a. $20B + 2300 = T$ Start with the formula
   $20B + 2300 = 5000$ Substitute
   $20B = 2700$ Subtract 2300 from both sides then divide by 20
   $B = 135$ boxes Find the number of boxes

   b. $20B + 2300 = T$ Start with the Formula
   $20B + 2300 = 50000$ Substitute
   $20B = 47700$ Subtract 20 from both sides
   $B = 2385$ boxes Divide by 20

3. This problem involves a linear relationship
   You can tell that it does from a table because the relationship between the numbers is going up at a constant rate.
   You can tell from a graph because the line goes up at a constant rate and is using a formula to graph it.
   From a function rule you can tell it is a linear relationship due to the corresponing rule and it forms a line relationship

4. I could use a graph to find the cost of 425 boxes by graphing the function. Once you have the graph, find 425 on the x-axis and go up to the line and once you reach the line go across to the y-axis and you will find how much it costs.

Nested Rectangles



A set of similar rectangles has been placed on a grid.

1. Write a function rule that shows how the width of a rectangle depends on its length. Consider the length to be the measure of the horizontal side and the width to be the measure of the vertical side.

2. Is this a proportional relationship? Explain how you know.

3. Could a rectangle with dimensions 10 units by 8 units belong to this set? Justify your answer using two different methods.

4. Name four other rectangles that would belong to this set.

5. Describe verbally, symbolically, and graphically the relationship between the length of the rectangle and the perimeter of the rectangle.

6. Describe verbally, symbolically, and graphically the relationship between the length of the rectangle and the area of the rectangle. Compare this relationship to the relationship between length and perimeter. Is either of these relationships a direct variation?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- What are the length and width of the smallest rectangle?
- What is the relationship between these two numbers?
- What are the length and width of the second rectangle?
- What is the relationship between these two numbers?
- How could you organize the information you are collecting?
- What conditions must occur if there is a proportional relationship?
- How do you find the perimeter of the rectangle?
- In question 5 what is the dependency relationship?
- What are the variables to be considered in question 5?

Sample Solution:

1. Count the length and the width of each rectangle and record the information in a table.

Length	Width
4	3
8	6
12	9
16	12

The ratio of the width to the length is 3:4.

$$\frac{w}{l} = \frac{3}{4}$$

$$w = \frac{3}{4}l$$

The other rectangles also satisfy this relationship.

Length	Process	Width
4	$\frac{3}{4}(4)$	3
8	$\frac{3}{4}(8)$	6
12	$\frac{3}{4}(12)$	9
16	$\frac{3}{4}(16)$	12

The relationship between the length and the width is that $w = \frac{3}{4}l$.

2. The relationship is a proportional relationship (a direct variation) because the equation is of the form $y = kx$ where k is a constant. The ratio of the length to the width in any given rectangle is a constant.

3. Check the values of 10 for the length and 7.5 for the width in the equation.

$$\frac{3}{4}(10) = 7.5$$

The rectangle with measurements 10 units and 8 units would not belong to this set. If the length is 10 units, the width must be 7.5 units.

Another approach is to ask if the two ratios are equal.

$$\frac{8}{10} \neq \frac{3}{4}$$

Therefore, this rectangle would not be similar to the given rectangles.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(G) relates direct variation to linear functions and solves problems involving proportional change.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

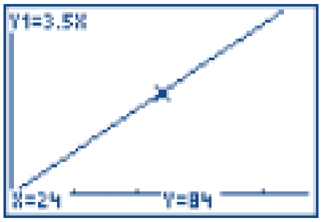
4. Possible answers are shown in the table.

Length	Process	Width
5	$\frac{3}{4}(5)$	3.75
9	$\frac{3}{4}(9)$	6.75
13	$\frac{3}{4}(13)$	9.75
14	$\frac{3}{4}(14)$	10.5

5. The perimeter is twice the length plus twice the width.

$$p = 2l + 2w$$
$$w = \frac{3}{4}l$$
$$p = 2l + 2\left(\frac{3}{4}l\right) = 2l + \frac{3}{2}l = \frac{7}{2}l$$
$$p = \frac{7}{2}l$$

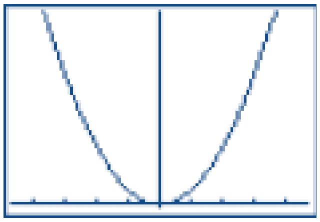
The perimeter is three and one-half times the length.
The graph of the function is a straight line.



6. The area of the rectangle is the product of the length and the width.

$$A = lw$$
$$w = \frac{3}{4}l$$
$$A = l\left(\frac{3}{4}l\right) = \frac{3}{4}l^2$$
$$A = \frac{3}{4}l^2$$

The area is three-fourths of the length squared.
The graph is a parabola with a vertex at the origin.



The portion of the parabola that makes sense in this situation is the portion in quadrant I.

The relationship for length and perimeter is linear, but this relationship between length and area is not linear. The length and perimeter relationship is a direct variation because it is of the form $y = kx$. Its graph is a line that passes through the origin.

Extension Questions:

- What restrictions must be placed on the domain of the function $w = \frac{3}{4}l$ for this problem situation?

The length may be any positive real number. The measurement of a side of a rectangle may not be negative or zero.

- If a rectangle, similar to the original rectangles, is created by doubling the length of one of the rectangles, how is the perimeter of the new rectangle related to the perimeter of the original rectangle.

The perimeter of the original rectangle is given by the formula

$$p = \frac{7}{2} l$$

The perimeter of the new rectangle would be

$$P = \frac{7}{2}(2l)$$

$$P = 2\left(\frac{7}{2} l\right) = 2p$$

The perimeter of the new rectangle would be twice the perimeter of the original rectangle.

- If a new similar rectangle is created by doubling the length of one of the rectangles, how is the area of the new rectangle related to the area of the original rectangle.

The area of the rectangle in this set is given by the formula

$$A = \frac{3}{4} l^2$$

If the length is doubled, the formula becomes

$$\text{New Area} = \frac{3}{4}(2l)^2$$

$$\text{New Area} = \frac{3}{4}(2^2 l^2)$$

$$\text{New Area} = \frac{3}{4} \bullet 4l^2$$

$$\text{New Area} = 4\left(\frac{3}{4} l^2\right)$$

The new area is 4 times the area of the original rectangle.

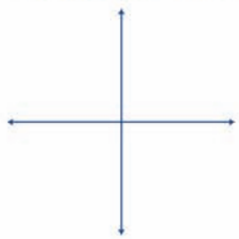
Chapter 7:

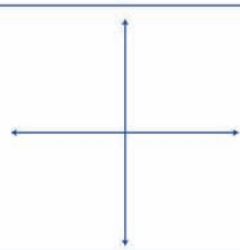
Linear Functions, Equations, and Inequalities

Analysis of a Function

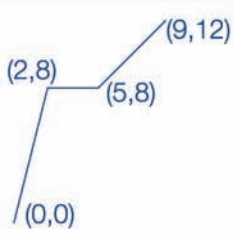
I. If given a function, sketch a complete graph. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.

II. Describe the domain and range for each mathematical situation. Explain your thinking.

<table>
<tr><th>Function</th><th>Graph or Table</th><th>Domain and Range</th></tr>
<tr><td>1. $f(x) = 5 - 2x$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>2. $y = -2$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>3.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>6</td><td>6</td></tr><tr><td>12</td><td>9</td></tr></table></td><td>Domain:

Range:</td></tr>
<tr><td>4.</td><td></td><td>Domain:

Range:</td></tr>
</table>

III. Write a summary of the functions, comparing their domains and ranges and their graphs.

IV. Describe a practical situation that each of these functions might represent. What restrictions will the situation make on the mathematical domain and range of the function? How will the situation affect the graph of the mathematical function?

Teacher Notes

Scaffolding Questions:

- What type of function relates the variables?
- What is the dependent variable? What is the independent variable? How do you know?
- What are the constants in the function? What do they mean?
- What restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?

Materials:
One graphing calculator per student.

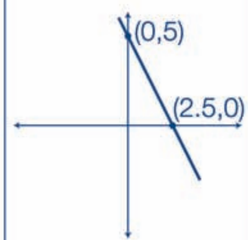
Connections to Algebra I TEKS and Performance Descriptions:

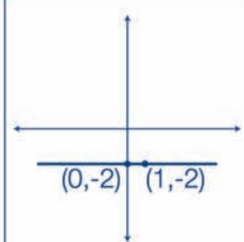
(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

Sample Solution:

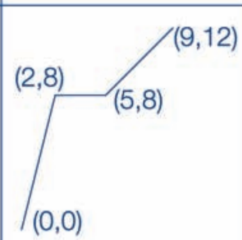
I & II:

<table>
<tr><th>Function</th><th>Graph or Table</th><th>Domain and Range</th></tr>
<tr><td>1. $f(x) = 5 - 2x$</td><td></td><td>Domain: The domain is the set of all real numbers because $5 - 2x$ is defined for any value of x.

Range: The range is the set of all real numbers since any number can be generated by $5 - 2x$.</td></tr>
<tr><td>2. $y = -2$</td><td></td><td>Domain: The domain is the set of all real numbers because the function $y = -2$ means "y is equal to -2 no matter what x is." This is a constant function.

Range: y is always -2, thus the range is just the number -2.</td></tr>
<tr><td>3. The table shows a constant rate of change of $\frac{1}{2}$ so this is a linear function. The y-intercept is (0,3). The function that models this set of points is $y = \frac{1}{2}x + 3$.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>6</td><td>6</td></tr><tr><td>12</td><td>9</td></tr></table></td><td>Domain: The domain is the set of given x values {-2,2,6,12}.

Range: The range is the set of y values {2,4,6,9}.

The domain and range of the function that models this data are both the set of all real numbers.</td></tr>
<tr><td>4.
• If $0 \leq x \leq 2$, then the function is $y = 4x$.
• If $2 < x \leq 5$, then the function is $y = 8$.
• If $5 < x \leq 9$, then the function is $y = x + 3$.</td><td></td><td>Domain: The domain is the set of all real numbers x, $0 \leq x \leq 9$, since this is what the graph shows.

Range: The range is the set of all real numbers, y, $0 \leq y \leq 12$, by the same reasoning.</td></tr>
</table>

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

III. Summary of the functions:

The functions in problems 1, 2, and 3 are linear functions, having the form $y = mx + b$. They all have as their domains the set of all real numbers because the expression for each function is never undefined. The functions in 1 and 3 have as their ranges the set of all real numbers because every real number can be generated by the expressions for those functions. The function in problem 2 has as its range the single number -2 because it is a constant function.

The graphs of the functions in problems 1, 2, and 3 are lines. The graph of the function in problem 1 has a *y*-intercept of (0,5) and an *x*-intercept of (2.5,0). The line "falls" from left to right because the slope is negative. This is a decreasing function. The graph of the function in problem 2 has a *y*-intercept of (0,-2) and no *x*-intercept. It is a horizontal line with slope zero. This is a constant function.

The graph of the function that models the data given in problem 3 has *y*-intercept (0,3) and an *x*-intercept of (-3,0). The line "rises" from left to right because the slope is positive. This is an increasing function.

The graph of the function in problem 4 consists of three linear segments, and so it requires three different functions to describe it.

For the first piece, $0 \le x \le 2$, and the graph is the corresponding part of the line with *y*-intercept (0,0) and slope $\frac{8-0}{2-0} = 4$. $y = 4x$. This graph "rises," terminating at the point (2,8).

For the second piece, $2 < x \le 5$, and the graph is a horizontal segment (slope = 0) terminating at (5,8). $y = 2$.

For the third piece, $5 < x \le 9$, and the graph is the corresponding part of the line with slope $\frac{12-8}{9-5}$. The equation is of the form $y = 1x + b$.

Use the point (9,12). Substitute 9 for x and 12 for y.

$12 = 1(9) + b$. $b = 3$
$y = 1x + 3$.

This graph starts at (5,8) and "rises." The graph of the first piece is steeper than the graph of the third piece, because the slope for the first piece is greater than the slope for the third piece.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
 2.1 Identifying Patterns
3 Interpreting Graphs
 3.1 Interpreting Distance versus Time Graphs
 3.2 Interpreting Velocity versus Time Graphs

II. Linear Functions
1 Linear Functions
 1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

IV. Description of a practical situation for each function:

The function $y = 5 - 2x$ could represent a small toy race car starting 5 feet away from the finish line and moving forward at 2 feet per second, where y = the distance in feet between the toy car and the finish line and x = the time in seconds the car has been moving. For this situation, the domain is the set of all numbers x, $0 \le x \le 2.5$ representing the time to start and complete the race. The range would be the set of all numbers y, $0 \le y \le 5$ representing the range of distance traveled by the car. The graph would simply be the segment from (0,5) to (2.5,0).

The function $y = -2$ could represent an ocean diver in the waters near a beach in Hawaii. The diver is swimming at a constant rate and is 2 meters below the water surface. For this situation, the domain is time, x in minutes, that the swimmer is at this depth. For example, the domain could be the set of all numbers x, $0 \le x \le 15$ and the range is $y = 2$. The graph would be a horizontal segment from (0,-2) to (15,-2).

The function $y = \frac{1}{2}x + 3$ could represent the allowance that a very young child gets each week. The parent puts $3 in the child's piggy bank to start the child saving. Each week, the child gets a 50 cent allowance and adds it to the piggy bank. The child has been told that if he saves his allowance each week for 6 months, then he will get an increase. For this situation, the domain is the set of all values x, $x = 0,1,2,3,\ldots,24$, because there will be roughly 24 weeks in the six-month period. The range will be the set of all values y, $y = 3, 3.5, 4, 4.5, \ldots., 15$. The graph will be a discrete graph because it will simply be a plot of a set of 25 points.

The fourth function is the function consisting of three linear pieces. These could be defined as:

1. If $0 \le x \le 2$, $y = 4x$.
2. If $2 < x \le 5$, $y = 8$.
3. If $5 < x \le 9$, $y = x + 3$.

This function could represent a student's pace on a reading assignment. During the first two minutes, the student rapidly reads 8 short paragraphs of the 12 he has to read. He stops reading for 3 minutes so that he can reflect on what he just read. He realizes that he needs to read the remaining 4 paragraphs more carefully, so he finishes the remaining paragraphs at a slower rate. The domain for this situation is the time, x in minutes, that it takes the student to complete the reading assignment.

This will be the time interval from 0 minutes to 9 minutes. The range will be the amount of text the student has read during this time interval, 0 paragraphs to 12 paragraphs. The graph will be connected line segments: from (0,0) to (2,8), from (2,8) to (5,8), and from (5,8) to (9,12).

Extension Questions:

- For problems 1, 2, and 3, determine the equation of a line perpendicular to each of the given lines and having the same *y*-intercept.

If a function is not a horizontal line, find the slope of the line and determine the opposite reciprocal of this slope. If the line is a horizontal line, the perpendicular line will have undefined slope.

In problem one, the slope is -2; the slope of a perpendicular line would be $\frac{1}{2}$*.*

The equation of the line is $y = 5 + \frac{1}{2}x$*.*

In problem 2, the line is horizontal with y-intercept -2. The perpendicular line will be a vertical line. The slope of a vertical line is undefined. The line is of the form x equals a constant. Any vertical line would be perpendicular to y = -2. The equation would be x = k, where k is any number. If a line must contain the y-intercept point (0,-2), the equation would have to be x = 0.

For problem 3, the slope of the line is $\frac{1}{2}$*. The perpendicular line would have slope -2. The equation of the line would be y = -2x + 3.*

- Describe the domain of these three perpendicular lines.

The domain and range of the perpendicular lines in problems 1 and 3 would be all real numbers. The domain of the line x = k is the number k. The range is all real numbers.

- Do these perpendicular lines represent functions?

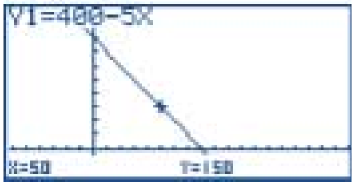
The perpendicular lines in problems 1 and 3 represent functions because for each x there is only one y value. However, x = k does not represent a function because the x value 0 is paired with an infinite number of y values.

Create a Situation

Create and describe in detail a situation that each of the following graphs could represent:

A

B



Teacher Notes

Scaffolding Questions:

- What type of function are these graphs presenting?
- What are the constants in these functions?
- Are the functions increasing or decreasing?
- How can you use this information to help you describe a situation each function might represent?

Sample Solution:

A. The function for the graph is given to be $y = 10.25x - 150$. The function rule implies that the rate of change is 10.25 in the y values for every unit change in the x value and the starting amount is -150. The following money situation could be modeled by this graph and function. You decide to start up a lawn mowing business. You borrow $150 from your dad to buy a new mower. You charge $10.25 for each lawn you mow.

The graph represents your cash assets when you have mowed x lawns. You will make a profit once the y values are positive. You are "in the red" until you mow the 15th lawn, since your "break-even point" (x-intercept) is between 14 and 15. Now you show a profit since your y-values are positive when you mow 15 or more lawns.

B. The function for the graph is $y = 400 - 5x$. The starting value is 400, and the y value is decreased by 5 units for every increase of one in the x unit. This could represent the altitude of a skydiver whose parachute opens at 400 meters. The skydiver is gently drifting to a landing at a rate of 5 meters per second. The graph for this situation would just be the first quadrant region. The y-intercept, (0,400) represents when the parachute opens. The x-intercept, (80,0) represents the number of seconds it takes him to land. This could be extended to include the second quadrant region by assuming that $x = 0$ is when the skydiver is first sighted by someone on the ground and that he opened his parachute before that.

Extension Questions:

- What would happen to the graph of the function in Part A if the function were $y = 10.25x - 129.5$? How would this change the situation you described?

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.

The graph will be a line with the same slope but with a different y-intercept, (0,-129.50).

In the situation it could mean that you need to mow fewer yards because you found a mower that cost $129.50.

- What would happen to the graph of the function in Part B if the function were $y = 400 – 4x$? How would this change the situation you described?

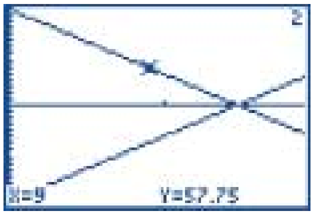
The graph will be a line with the same y-intercept. It will not be as steep since the slope is -4 instead of -5. The x-intercept will change from (80,0) to (100,0).

In the situation, it would mean that the skydiver is drifting to his landing at 4 meters per second, and lands in 100 seconds.

- For problems A and B, what would the graphs look like if you reflected the original graphs over the *x*-axis? How would this change the function describing the graph? How would it change the situation you chose to represent the graph?

The function for A would become y = -10.25x + 150 since reflecting over the x-axis is the same as multiplying the expression 10.25x – 150 by -1. This is a decreasing linear function. It could no longer represent a “money-earned” situation. It could represent a “money spent out of $150” situation. For example, Jack has 150 dollars in his savings account. He withdraws $10.25 each week. If he does not add any money to the account, y represents the amount of money in the savings account at x weeks.

y = 10.25x – 150
y = -10.25x + 150



Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

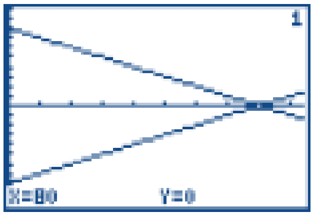
I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Similarly, the function for B would become y = 5x – 400. The graph would have a negative y-intercept, (0, -400). This is an increasing function that starts at a negative value and could not represent the skydiver's altitude as he drifts to his landing. The altitude at time zero may not be negative. One must think of a situation that begins with a negative value. For example, Lance borrows $400 from his sister, and pays her back at the rate of $5 per week. If he continues to pay her at the constant rate, y represents the amount of money he owes her, and x represents the number of weeks. The x-intercept is 80; this means that after 80 weeks the amount he owes her is 0 dollars.

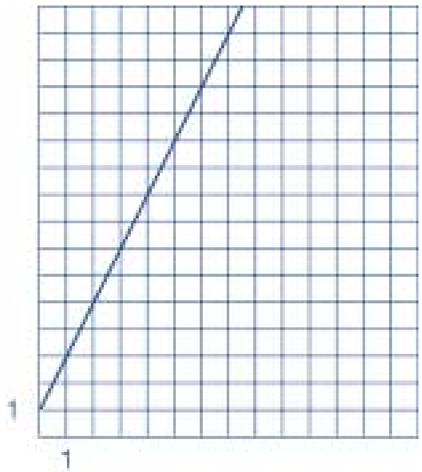


Finding Pairs

Functions are represented in different ways in letters A through L. Compare and contrast the function rules, tables, graphs, and the situations. Separate the letters into the six pairs of letters that then show representations of the same functional relationship.

A. $y = 2x - 1$

B.



C. The huge beautiful plant was growing at a rate of 1 and a 1/2 inches per week.

D. The Math Club found a place that will sell them t-shirts for $5.00 each, but there is a set-up fee of $50.

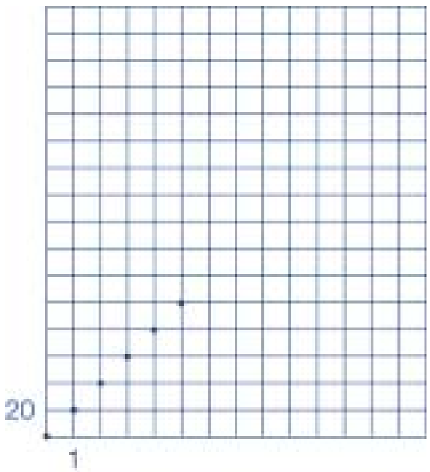
E.

x	y
0	50
1	55
2	60
3	65
4	70
5	75

F. $y = 2x + 1$

G. $y = 1.5x$

H.



I.

x	y
-2	-5
-1	-3
0	-1
1	1
2	3
3	5

J.

x	y
-2	1.5
-1	1.5
0	1.5
1	1.5
2	1.5
3	1.5

K.

x	y
0	0
1	20
2	40
3	60
4	80
5	100

L. $y = 1.5$

Teacher Notes

Scaffolding Questions:

- What type of relationship is described by the graphs, tables, and functions?
- What do the coefficients of *x* represent?
- What patterns do you notice in the tables? What is causing these patterns?
- What does it mean when you are subtracting a term?

Sample Solution:

Table D and situation E are a pair; the $50 set-up fee is the entry $x = 0$, $y = 50$. The $5.00 per shirt is evident in the table as increments of 5 under the *y* column.

Table I and function A are a pair. The point (0,-1) from the table indicates a *y*-intercept of -1. The increments of 2 under the y column, for every corresponding increment of 1 in the x column, mean that the slope is 2. The equation is $y = 2x – 1$.

Table K and graph H are pairs because the point (0,0) indicates that the graph passes through the origin. In the table y is increasing by a constant rate of 20, and the slope of the line is 20 because for every unit to the right there are 20 units up to get to a point on the line.

Table J and function L represent the same information. The table shows 1.5 under the entire *y* column, which means *y* is always equal to 1.5.

Function F and graph B match because the graph passes through point (0,1), which means the *y*-intercept is 1. The coefficient of *x* is 2, and 2 is also the slope of the line in graph B.

Function G and situation C are pairs. The coefficient of *x* is 1.5, and that is the same as the rate of change for the plant.

Extension Questions:

- How do the numbers in the functions affect the table?

The coefficient, m, of x in the function $y = mx + b$ is the slope or rate of change that can be determined from the table. The constant b in the function corresponds to the data point (0,b).

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes functions or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve functions and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves functions, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

- Make a list of patterns you notice in the tables, and explain what causes the patterns.

In table E as x increases by 1, the values of y increase by 5. This is because the constant rate of change is 5. In a function rule it will show up as the coefficient of x.

The constant rate of change for table I is 2.

The constant rate of change for table K is 20.

The constant rate of change for table J is 0. The y values do not change as x changes. In the equation $y = mx + b$, the slope is zero. Therefore, $y = 0x + b$ or $y = b$ for every x.

The constant rate of change tells me the coefficient of x is 0. This indicates that the matching function does not have a term with an x.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
- 1.1 The Linear Parent Function
- 1.2 The Y-Intercept
- 1.3 Exploring Rates of Change
- 1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (functions of lines) to model problems involving real-world and mathematical situations.

First-Aid Supplies

Mark is the trainer for the Little Kids soccer team. He is at the store to pick up 50 bandages and 3 ice packs for the first-aid kit. Mark may spend at most $12. Prices vary for the different ice packs, but every brand of bandage costs the same: $4.50 for 50 bandages. The sales tax is 9%.

1. Write an inequality to identify the number of ice packs Mark can purchase. Identify your variable.

2. How much can Mark spend for each ice pack and keep within the $12 budget?

3. Suppose the booster club gives Mark another $10 to spend for ice packs. Describe your solution verbally and algebraically.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- Can Mark spend less than the amount of money he has? Can he spend more?
- How does the tax rate affect your inequality?

Sample Solution:

1. The cost of the bandages is \$4.50. The cost of the ice packs depends on the price of the ice pack.

   Let x = the price of one ice pack.
   Since Mark needs 3 ice packs, the cost of the ice packs is 3 times the price of one ice pack or $3x$. The expression for the cost of the ice packs plus the cost of the bandages is $3x + 4.50$.
   The sales tax is 9% of the cost or $0.09(3x + 4.50)$.
   The total cost including the tax must be less than or equal to \$12.

   $(3x + 4.50) + 0.09(3x + 4.50) \le 12.00$

2.

$$\begin{aligned} 3x + 4.50 + 0.27x + 0.41 &\le 12.00 \\ 3.27x + 4.91 &\le 12.00 \\ 3.27x &\le 7.09 \\ x &\le 2.168195719 \end{aligned}$$

   x represents a dollar amount and must be expressed to the nearest hundredth. If you round up to \$2.17, the cost would be

   $3(2.17) + 4.50 + 0.09(3(2.17) + 4.50) =$
   $11.01 + 0.09(11.01)\ 11.01 + 0.99 = 12.00$

   Mark can spend up to \$2.17 per ice pack.

3. If the booster club gives Mark an additional \$10 to spend for the ice packs, the only difference in the solution will be the total amount budgeted for purchase. Rather than being \$12, the new amount will be \$22. The additional money will allow Mark to purchase more ice packs at a cheaper price.

Let x = the price of an ice pack.
The total cost may now be less than or equal to \$12.00 plus \$10.00.

$(3x + 4.50) + 0.09(3x + 4.50) < 22.00$

Values are rounded to the nearest hundredth.

$$\begin{aligned} 3x + 4.50 + 0.27x + 0.41 &\leq 22.00 \\ 3.27x + 4.91 &\leq 22.00 \\ 3.27x &\leq 17.09 \\ x &\leq 5.226 \end{aligned}$$

If this answer is rounded to \$5.23 and Mark spent \$5.23 per ice pack, the cost would be \$22.01, so he can spend at most \$5.22.

Extension Questions:

- Suppose Mark found the bandages on sale for \$3, and he could spend no more than \$15. How much could he spend per ice pack?

If the bandages are on sale for \$3, and Mark can spend up to \$15, he could spend up to \$3.59 per ice pack. Values are rounded to the nearest hundredth.

Let x = the price of an ice pack.

$$\begin{aligned} (3x + 3.00) + 0.09(3x + 3.00) &\leq 15.00 \\ 3x + 3.00 + 0.27x + 0.27 &\leq 15.00 \\ 3.27x + 3.27 &\leq 15.00 \\ 3.27x &\leq 11.73 \\ x &\leq 3.59 \end{aligned}$$

- Would a 7% tax rate affect the number of ice packs Mark could purchase if he has a maximum of \$15 for the purchase?

Let x = the price of an ice pack.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

$$
\begin{aligned}
(3x + 3.00) + 0.07(3x + 3.00) &\leq 15.00 \\
3x + 3.00 + 0.21x + 0.21 &\leq 15.00 \\
3.21x + 3.21 &\leq 15.00 \\
3.21x &\leq 11.79 \\
x &\leq 3.67
\end{aligned}
$$

A 7% tax rate does not have a great impact on the price of the ice packs. Therefore, the number of ice packs Mark can purchase is not affected by the lower tax rate.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Linear Functions
3 Linear Equations and Inequalities
3.3 Solving Linear Inequalities

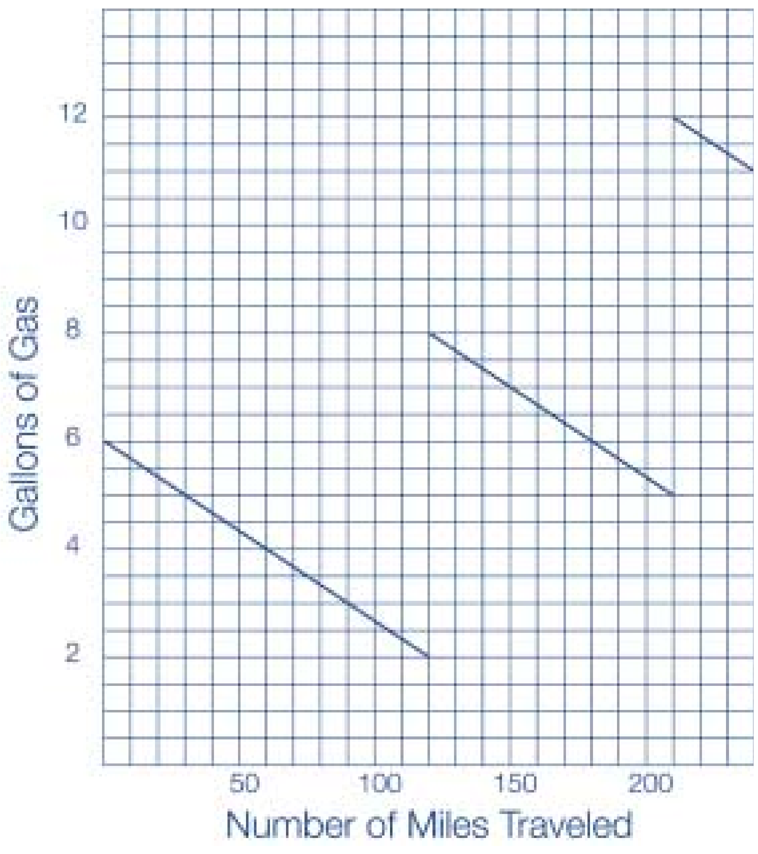
Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

Gas Tank

The following graph shows how the amount of gasoline in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the shape of the graph for this situation. Include in your description an interpretation of the slopes of the segments.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Scaffolding Questions:

- How many phases do you see in the graph?
- How does the graph behave in each phase? What does this mean in the situation? How does the graph behave between phases? What does this mean in the situation?
- How does the amount of gas in the tank vary during the first 100 miles of the trip? During the next 120 miles? During the last 40 miles?

Sample Solution:

The gas tank starts out with 6 gallons of gas, and for the first 120 miles traveled drops at a steady rate to 2 gallons. At 120 miles, the number of gallons jumps to 8 gallons, which suggests stopping to get gas. Over the next 90 miles traveled (from mile 120 to mile 210), the gas amount drops steadily to 5 gallons. Again, at 210 miles the number of gallons jumps suddenly to 12 gallons and then drops steadily over the next 30 miles.

The capacity of the tank is at least 12 gallons since that is the maximum *y*-value we see. Thus, at the beginning of the trip the tank was not full, and on the first refill was not filled to capacity.

The rate of change in gas in all three phases is 1 gallon used per 30 miles (slope of $-\frac{1}{30}$), so gas consumption (gallon/mile) is occurring at a steady rate.

Extension Questions:

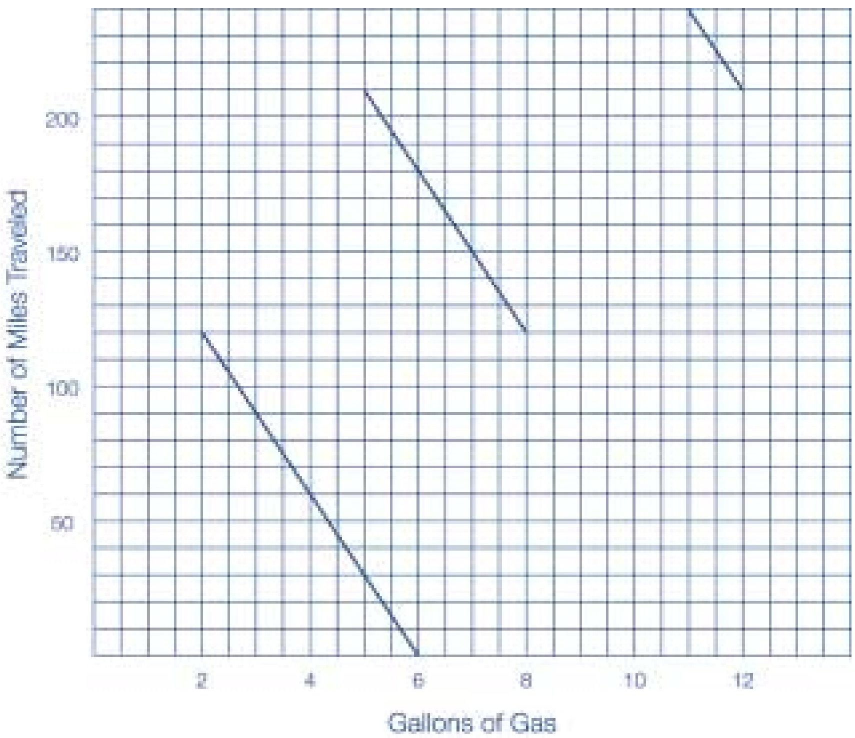
- What does the graph tell you about the capacity of the tank?

The capacity of the tank has to be at least 12 gallons because that is the greatest amount of gas in the tank that the graph shows. However, the capacity could be more than that and the tank is not being filled to capacity each time.

- How would the graph be different if you know that the capacity of the gas tank is 15 gallons and the tank was filled to capacity at each of the stops?

Each of the segments would begin with a y-coordinate of 15.

- Create a new graph that would be the result of switching the independent and dependent variables of the original graph.



- What does the resulting rate of change (slope) in each phase now represent?

The rate of change would be miles traveled per gallon and is a decrease of 30 miles per gallon in each phase.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
3 Interpreting Graphs
 3.1 Interpreting Distance versus Time Graphs

II. Linear Functions
1 Linear Functions
 1.1 The Linear Parent Function
 1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Greetings

The school choir purchased customized cards from a company that charges $100 for a set-up fee and $2 per box of cards. The choir members will sell the cards at $3 per box.

The function describing their profit, p dollars, for selling x boxes of cards is $p = 3x - (100 + 2x)$.

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?

2. How much money will the choir make if they sell 200 boxes? Show your strategy.

3. How many boxes must the choir sell to make a $200 profit? Explain how you found your answer.

4. How many boxes must the choir sell to make a $500 profit? Use a different strategy than the one you used in number 3.

5. How many boxes will the choir have to sell to break even?

6. The choir will not consider this project unless they can raise at least $1,000. Write and solve an inequality that will help them determine if they should do this project.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Scaffolding Questions:

- What does the word "profit" mean?
- What does the 3 in the expression $3x$ represent?
- What does x represent?
- What does p represent?
- Why are there parentheses used in the function rule?
- How can you use the distributive property to simplify the expression?
- Which of the variables are you given in question 2?
- Which of the variables represents $200 in question 3?
- What does it mean to break even?
- Describe how you might use a table to answer question 3.
- Describe how you might use a graph to answer question 3.

Sample Solution:

1. The expression $3x$ represents the amount in dollars collected from the sale of x boxes. The $(100 + 2x)$ means that you have to pay $100 plus $2 per box that you sell.

2. If they are going to sell 200 boxes, you must evaluate the function for $x = 200$.

   $p = 3x - (100 + 2x)$
   $p = 3(200) - (100 + 2(200))$
   $p = 600 - (100 + 400)$
   $p = 600 - 500$
   $p = 100$

   They would make a profit of $100.

3. Generate a table that shows the number of boxes and the amount of profit made. Use the table to determine the number of boxes that will make a $200 profit.

Number of Boxes	Profit (dollars)
0	-100
100	0
200	100
300	200
400	300
500	400
600	500

You must sell 300 boxes to make a $200 profit.

4. The symbolic method may be used to determine how many boxes the choir should sell to make a $500 profit.

   Simplify the rule.
   $p = 3x - (100 + 2x)$
   $p = 3x - 100 - 2x$

   $p = x - 100$,
   Substitute 500 for p.

   $500 = x - 100$
   $x = 600$

   They must sell 600 boxes to make $500.

5. To break even means that the cost equals the revenue or that the profit is 0.

   $p = x - 100$
   $0 = x - 100$
   $x = 100$

   The choir must sell 100 boxes to break even.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

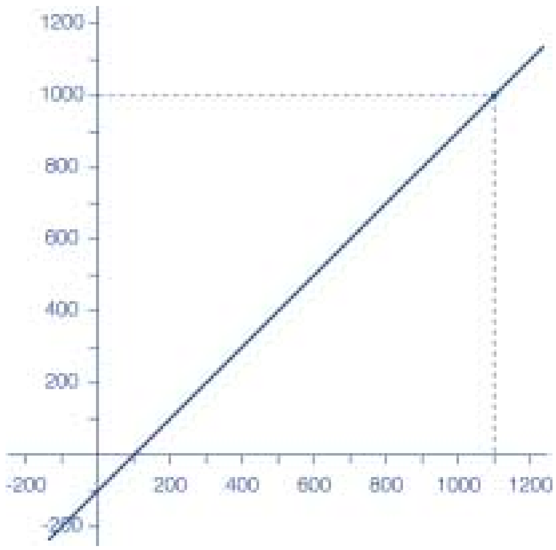
Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

6. If they want to make at least $1,000, it means that the profit must be greater than or equal to $1,000.

$$\begin{aligned} p &\geq 1000 \\ x - 100 &\geq 1000 \\ x &\geq 1100 \end{aligned}$$

The choir must sell at least 1100 boxes of cards. If they feel they cannot sell at least 1100 boxes they should not do this project.

Another approach is to examine the graph.
The graphs of $y = 3x - (100 + 2x)$ and $y = 1000$ intersect in the point (1100,1000). That means that when they sell 1100 boxes, the profit is $1000. The graph of the profit is above the graph of $y = 1000$ for values of x greater than 1100. They must sell at least 1100 boxes to make a profit of at least $1000.



Extension Questions:

- What will happen in this situation if the $100 set-up fee is omitted?

You would have to pay less. The profit would be represented by is p = 3x – 2x or p = x. You will make $1.00 per box. Now the y-intercept is zero. The rate of change is still one dollar per box.

- For another situation the profit is represented by $p = 3x - (30 + 2.50x)$. Describe the cost and selling process for this situation.

Profit is revenue minus cost. The expression 3x means that they are charging $3 per box. The cost is represented by 30 + 2.50x. They must be charged a set-up fee of $30 plus $2.50 per box.

- Under what conditions is the second situation better than the first?

Determine when the two are equal in value.

$$\begin{aligned} 3x - (30 + 2.50x) &= 3x - (100 + 2x) \\ 0.5x - 30 &= x - 100 \\ -0.5x &= -70 \\ x &= 140 \end{aligned}$$

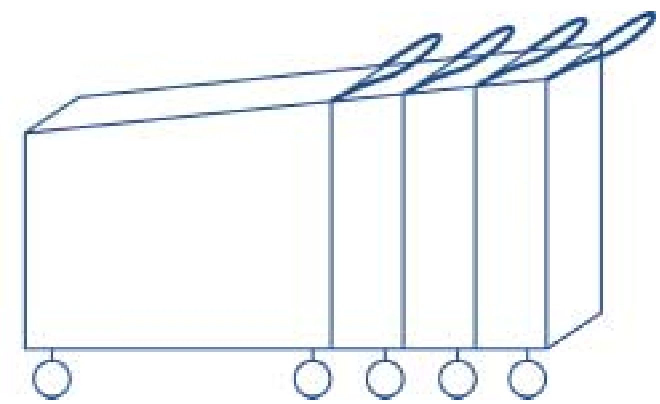
Examine the graph to determine which function has the greater value after x = 140.

When x is greater than 140 the function y = x – 100 has the greater value. The profit is greater for the first situation for 140 or more boxes.

Grocery Carts

Randy must try to fit grocery shopping carts in an area that is 80 feet long and wide enough for the width of a grocery cart. He did some measurements to use in his computations. The table shows the length of a set of grocery carts as they are nested together.

Number of Grocery Carts	Length in Inches
1	37.5
8	116.25



Randy recently finished his algebra class and decided he could determine an expression for the length of the nested grocery carts.

1. What is a function for the length in inches in terms of the number of nested grocery carts?

2. What is the length of a nested set of 50 grocery carts?

3. How many carts would fit in a space 80 feet long if the space is wide enough for one grocery cart?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- If one shoping cart is 37.5 feet long, how could you find the additional length for each nested grocery cart?
- Compute the finite differences in the table.
- What is the rate of change for the situation?
- Complete this new table with the missing values.

Number of Grocery Carts	Length in Inches Process	Length in Inches
1	37.5	
2	37.5 +	
3	37.5 +	
4	37.5 +	
5	37.5 +	
n		

Sample Solution:

1. The length for one grocery cart is 37.5 inches. The rate of change is 78.75 inches for 7 grocery carts or 11.25 inches for 1 grocery cart. The total length is 37.5 plus 11.25 for every additional grocery cart.

   $L = 37.5 + 11.25(n - 1)$ or
   $L = 26.25 + 11.25n$

   where n is the number of grocery carts and L is the length of the set of grocery carts.

2. When the number of grocery carts is 50, the length is

   $L = 26.25 + 11.25(50) = 588.75$ inches

3. 80 feet is 960 inches. When is the length 960 inches?

   A table or graph may be used to determine when y is 960. When a table is set with increments of 10, it shows that the value of x is between 80 and

90. When the table is set with increments of 1, the value of x that gives a y value of 960 is x = 83.

83 carts will fit in a length of 80 feet.

Extension Questions:

- What is a reasonable domain for the function you have created?

The function L = 32 + 11.25n is a linear function. The domain of the function is all real numbers.

- What is a reasonable domain for the problem situation?

The domain for the problem situation represents the number of grocery carts and must be the set of positive integers. However, the domain is determined by the physical, logistical constraints of the situation, such as the available space for storage and customer capacity.

- How do the numbers in the equation relate to the physical grocery carts?

The 26.25 inches is the length of the cart that slides into the remaining carts each time; the 11.25 inches is the amount that hangs out for each new cart.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

- If the equation had been determined to be $L = 32 + 11.25n$ for a different set of grocery carts, what is the same about the two sets of grocery carts?

The portion that is added for each new grocery cart is the same, because the rate, 11.25, has not changed. However, the y-intercept has changed, so the part that is nested into the rest of the carts is not the same.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Hull Pressure

When a submarine descends into the ocean, the pressure on its hull increases in increments as given in the following table. (Pressure is measured in kilograms per square centimeter, and depth is measured in meters.)

Depth	0	300	600	900	1200	1500
Pressure	0	32	64	96	128	160

1. Describe verbally and symbolically a function that relates the depth of the submarine and the pressure on its hull.

2. How will the situation restrict the domain and range of the function?

3. What will be the pressure on the submarine's hull when it is at a depth of 1575 meters?

4. If the pressure on the submarine's hull is 240 kg/cm^2, what is the depth of the submarine?

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations; and

(G) relates direct variation to linear functions and solves problems involving proportional change.

Teacher Notes

Scaffolding Questions:

- How does the pressure change as the depth of the submarine increases?
- What is the initial pressure on the submarine's hull?
- What should the dependent variable represent?
- What should the independent variable represent?
- What is the rate of change in the pressure?
- How will you find the pressure for a given depth?
- How will you find the depth for a given pressure?

Sample Solution:

1. When the submarine is at the ocean's surface, the pressure on its hull is 0 kg/cm^2.

   For every 300 meters the submarine dives, the pressure on its hull increases by 32 kg/cm^2.

   A linear function with intercept (0,0) and slope $m = \frac{32}{300} = \frac{8}{75}$

   represents the situation, that is, $p = \frac{8}{75}d$, where p is the pressure in kg/cm^2 and d represents the depth in meters.

2. While the mathematical domain and range for this function are both the set of all real numbers, the situation restricts the domain to the real numbers from zero to the maximum depth the submarine can dive. The situation restricts the range to the real numbers from zero to the maximum pressure the submarine's hull can withstand. This would depend on the construction and size of the submarine.

3. If the depth of the submarine is 1575 meters, then d = 1575 and

   $$p = \frac{8}{75}d = \frac{8}{75}(1575) = 168.$$

   The pressure on the submarine's hull is 168 kg/cm^2.

4. If the pressure on the submarine's hull is 240 kg/cm^2, then $p = 240$ and the following equation can be solved for d.

$$240 = \frac{8}{75}d$$

$$d = 240 \cdot \frac{75}{8} = 2250$$

The submarine's depth is 2250 meters.

The problem could also be solved by finding the intersection of the graphs of $y = 240$ and $y = \frac{8}{75}x$.

The value of x when $y = 240$ is 2250.

Extension Question:

- Is there a proportional relationship between the hull pressure and the depth? Explain how you know whether or not the relationship is proportional.

The graph of the function is a straight line that contains the point (0,0). Therefore, there is a proportional relationship between the hull pressure and the depth.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
  1.4 Finite Differences

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

Math-a-thon

Catrina is participating in the school math-a-thon to raise money for the end-of-year field trip. Her mother is donating $25.00 to get her started. She will also receive 75 cents for every problem she answers correctly.

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.

2. Catrina's grandmother gives her an extra $20.00 to add to her field-trip money. How would this change the previous situation's rule, graph, and table?

3. What part of the situation would you change in order to produce a lesser or greater slope? Explain how you know.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- How much money will Catrina raise if she works 20 problems correctly?
- What are the constants in this situation?
- Describe the variables in this problem.
- What type of graph do you think this situation will produce?
- What role does the $25.00 play in the graph of this situation?
- What does adding $20.00 due to the graph of the situation?
- What is the rate of change for the original situation?
- What is the rate of change for the second situation?

Sample Solution:

1. The amount of money she will have is $25.00 plus $0.75 times the number of problems she gets correct. The function rule is

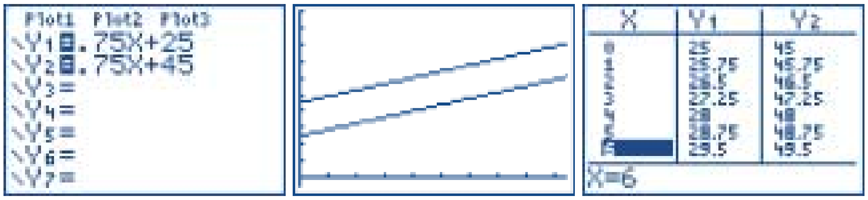
   $d = 0.75p + 25.00$.

   The $0.75 is the amount of money Catrina will get for every problem she works correctly. The $25.00 is the amount of money Catrina is going to get from her mother regardless of the number of problems she works. The variable *p* represents the number of problems worked correctly and the variable *d* is the total donation.

2. The $25.00 in the first function rule will change to $45.00 because now Catrina will start with $25.00 plus $20.00. The new rule is

   $d = 0.75p + 45.00$.

   The table now shows when *x* is 0, then *y* is 45 instead of 25. It still increases by 0.75 for every problem. The graphs will show parallel lines, one starting at (0,25) and the other starting at (0,45).



3. The amount of money Catrina receives per correct problem affects the rate of change. This rate of change determines the steepness of the line.

   To get a greater slope in this function rule the amount of money Catrina receives per correct problem needs to increase. Anything more than $0.75 will result in a greater slope, and anything less than $0.75 will produce a line with a lesser slope.

Extension Questions:

- How do the domain of the function rule and the domain of the problem situation compare?

The domain for the function is all real numbers. However, in the problem situation the number of problems must be a whole number. The number of problems in the competition would be the maximum number she could get correct, so the domain of the first problem situation is a subset of the set of whole numbers.

- How do the graphs of the function rule and the situation compare?

The graph of the function would be a straight line, but the graph of the problem situation would be a set of points on a straight line in the first quadrant.

- Write another scenario that will produce a similar function rule, graph, and table.

Johnny has a basket with 20 apples and starts picking apples at a rate of 5 apples for every minute. How many apples will he have in 10 minutes?

- Jackie did not receive a starting donation; can she still collect as much money as Catrina? Explain your answer.

Yes, she can do more problems than Catrina, or she can collect more per problem she works or a combination of these two things.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
  1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Recycling

At Dobie Middle School, the number of cans, $n(p)$, collected for recycling after a baseball game depends on the number of people, p, who attend the game. At least 80 people attend each game. The approximate relationship is given by the function $n(p) = 2.5(p - 40) - 100$.

1. If 400 people attended the game for the semifinals of the district championship, how many cans would you expect to be collected? Show at least two different strategies to determine the number of cans.

2. If 300 cans were collected at a game, how many people would you expect to have attended the game? Solve this two different ways.

3. If 673 cans were collected at another game, how many people would you expect to have attended that game?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Scaffolding Questions:

- What does the 400 given in problem 1 represent?
- What does the 300 represent?
- How is problem 2 different from problem 1?
- Will it help you to construct a table to solve the problem?
- What will the graph look like? How can the graph help you answer the questions?

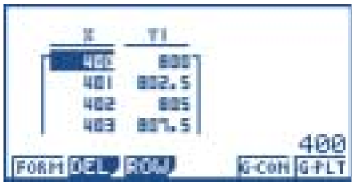
Sample Solution:

1. To determine the number of cans for 400 people, evaluate the function for $p = 400$.

$$
\begin{aligned}
n(p) &= 2.5(p - 40) - 100 \\
n(400) &= 2.5(400 - 40) - 100 \\
&= 2.5(360) - 100 \\
&= 900 - 100 \\
&= 800
\end{aligned}
$$

If 400 people attended the game, the band can expect to collect 800 cans.

A different approach to solve this would be to simplify the equation by using the distributive property. The equation will then be $n(p) = 2.5p - 100 - 100$ or $n(p) = 2.5p - 200$. Next make a table and find the value of c when $p = 400$.

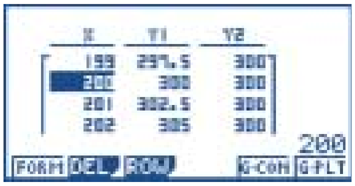


Looking at the table if 400 people attended the game, the band can expect to collect 800 cans.

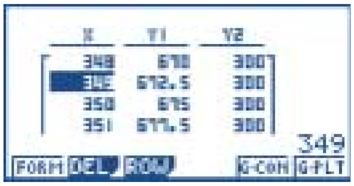
2. If 300 cans were collected using the table, it can be determined that 200 people attended the game that day. The equation can be used by substituting 300 for $n(p)$ and solve for p.

$$\begin{aligned} 300 &= 2.5p - 200 \\ 300 + 200 &= 2.5p \\ 500 &= 2.5p \\ 200 &= p \end{aligned}$$

Two hundred people attended the game the day the band collected 300 cans. Another approach is to use a graphing calculator: enter the functions $y = 2.5(x - 40) - 100$ and $y = 300$. The point of intersection on the graph may be found or the x-value may be located when the y-value is 300.



3. To determine the number of people if 673 cans were collected, look for 673 in the table for the x-value.



There are 349 people for 672.5 cans. This number does not have meaning because the number of cans and the number of people must be whole numbers.

The best estimate is for 350 people collecting 675 cans.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Another approach would be the symbolic solution.

$$\begin{aligned} n(p) &= 2.5p - 200 \\ 673 &= 2.5p - 200 \\ 873 &= 2.5p \\ p &= 349.2 \end{aligned}$$

Since p represents the number of people it may not be a fraction. There would be more than 349.2 people to collect at least 673 cans.

Extension Questions:

- Explain the meaning of the slope of the function in this problem situation.

The slope is 2.5 cans per person; it represents the estimated number of cans each person would throw away during the game.

- Explain the meaning of the *y*-intercept of the function.

The y-intercept of the function is -200; that is y = -200 when x = 0. However, this value of y does not have a meaning in the context of the problem because the number of people is greater than or equal to 80.

- What is the *x*-intercept, and what is its meaning in the context of the problem?

The x-intercept is 80, the minimum number of people that attend the game.

Shopping

Celeste is going shopping to buy 2 pairs of shoes and some earrings. She can spend $100 at the most. The shoes Celeste wants to buy cost $24.99 per pair. Earrings cost $12.99 a pair. What is the greatest number of earrings she can buy? The sales tax on the total sale is 8% of the amount.

1. Write an inequality to identify the number of earrings she can purchase.
2. Would it be possible for Celeste to purchase 4 pairs of earrings? Explain your answer.
3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for $19.99?

Teacher Notes

Scaffolding Questions:

- Identify the variable and describe the situation verbally and symbolically.
- Can Celeste spend less than the amount of money she has? Can she spend more?

Sample Solution:

1. Let x = the number of pairs of earrings Celeste can buy
   $24.99 = the cost of one pair of shoes
   $12.99 = the cost of one pair of earrings

   The cost of two pairs of shoes at $24.99 each + x pair of earrings at $12.99 may be represented by 2(24.99) + 12.99 x.

   The tax of 8% on the sale is represented by
   0.08 [2(24.99) + 12.99 x].

   The cost plus the tax may total to no more than $100. This inequality describes the restriction:

   $$2(24.99) + 12.99x + 0.08[2(24.99) + 12.99x] \le 100.00$$

   Use the distributive property to simplify before solving and round to the nearest hundredth:

   $$\begin{aligned} 49.98 + 12.99x + 4.00 + 1.04x &\le 100.00 \\ 53.98 + 14.03x &\le 100.00 \\ -53.98 \qquad &\quad -53.98 \\ \hline 14.03x &\le 46.02 \\ x &\le 3.28 \end{aligned}$$

   Celeste can buy no more than 3 pairs of earrings. She doesn't have enough money for 4 pairs of earrings, but she will have some money left.

   If Celeste finds the shoes on sale for $19.99 a pair, the inequality would change as follows:

   $$2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] \le 100.00$$

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{aligned} 39.98 + 12.99x + 3.20 + 1.04x &\leq 100.00 \\ 43.18 + 14.03x &\leq 100.00 \\ -43.18 \qquad\qquad &\quad -43.18 \\ \hline 14.03x &\leq 56.82 \\ x &\leq 4.05 \end{aligned}$$

Celeste would be able to buy 4 pairs of earrings and still have money left over.

Extension Questions:

- Suppose Celeste wants to have $20 left. Describe and write your solution algebraically.

In order for Celeste to have $20 left, the total amount she can spend has to be reduced by $20. Rather than having $100 to spend, she only has $80. The inequality is:

$$\begin{aligned} 2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] &\leq 80.00 \\ 49.98 + 12.99x + 4.00 + 1.04x &\leq 80.00 \\ 53.98 + 14.03x &\leq 80.00 \\ -53.98 \qquad\qquad &\quad -53.98 \\ \hline 14.03x &\leq 26.02 \\ x &\leq 1.85 \end{aligned}$$

Celeste would only be able to purchase 1 pair of earrings if she sets aside $20.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
1.2 Valentine's Day Idea

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

- How would your solution change if Celeste finds the shoes on sale for 25% off of the original price. Show your solution algebraically.

If Celeste finds the shoes on sale for 25% off the original price of $24.99, the new shoe price could be found by realizing the shoes will cost 75% of the original price. Multiplying $24.99 by 0.75 will give you the new shoe price. The solution is similar to the sample solution:

$24.99 * 0.75 = $18.74

The new inequality will be:

$$
\begin{aligned}
2(18.74) + 12.99x + 0.08[2(18.74) + 12.99x] &\le 100.00 \\
37.48 + 12.99x + 3.00 + 1.04x &\le 100.00 \\
40.48 + 14.03x &\le 100.00 \\
-40.48 \qquad\qquad &\quad -40.48 \\
\hline
14.03x &\le 59.52 \\
x &\le 4.24
\end{aligned}
$$

Celeste would be able to purchase 4 pairs of earrings and still have some money left over.

- How would a 9% tax rate have an effect on the number of earrings Celeste could purchase at the original show price?

$$2(24.99) + 12.99x + 0.09[2(24.99) + 12.99x] \le 100.00$$

Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$
\begin{aligned}
49.98 + 12.99x + 4.50 + 1.17x &\le 100.00 \\
54.48 + 14.16x &\le 100.00 \\
-54.48 \qquad\qquad &\quad -54.48 \\
\hline
14.16x &\le 45.52 \\
x &\le 3.21
\end{aligned}
$$

A 9% sales tax rate will cost a little more, but Celeste will still be able to purchase 3 pairs of earrings and have a little money left.

Sound Travel

Many fishing boats and salvage ships are equipped with sonar to help them find shipwrecks and large schools of fish. Sound travels through water at about 1463 meters per second. By measuring the time taken for the sound waves to travel through the water from the boat to the fish it is possible to calculate the distance from the boat to the fish.

1. Write a function that is a model for the relationship between the number of seconds it takes the sound signal to return to the boat and the distance from the boat to the school of fish. Identify your variables.

2. Describe the graph of this function including the domain and the range. Explain how you know whether or not there is a direct variation between the number of seconds and the distance in meters.

3. Suppose the sound signal returned to the boat in 0.05 seconds. Estimate the distance to the school of fish.

4. If the distance from the boat to the school of fish is 24,000 meters, how long will it take the signal to return to the boat?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(G) relates direct variation to linear functions and solves problems involving proportional change.

Scaffolding Questions:

- If the sound returns to the boat in 1 second, what is the distance to the school of fish? 2 seconds?
- What is the relationship between the distance to the school of fish at 1 second and at 2 seconds? At 2 seconds and at 3 seconds?
- What does the 1463 mean in the function?

Sample Solution:

1. Let d = the distance to the school of fish in meters.
   Let t = the time in seconds for the sound to return to the boat.

   The time for the signal to return to the boat depends on the distance in meters. The time is the dependent variable, and the distance is the independent variable.

   The table shows the number of seconds and the distance in meters.

Time (seconds)	Distance (m)
0	0
1	1463
2	2926
3	4389

   The function rule is linear because the rate of change is constant. The difference in the time in the table is 1 second. The difference in the distance is an increase of 1463 kilometers for every increase of one second. The function rule is $d = 1463t$.

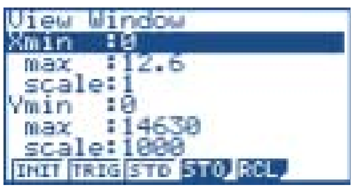
2. There is a direct variation (proportional relationship) between the distance and the time in seconds because the graph of the function is a straight line that passes through the origin.

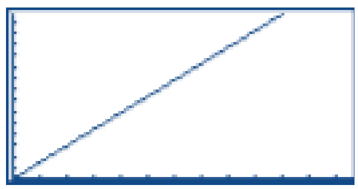
   The domain of the function represents the time it takes for the sound to return to the boat. The time in seconds will have to be greater than

0 or $x > 0$. The range represents the distance to the school of fish. The distance must also be greater than 0, or $y > 0$.

The rate of change in the distance is an increase of 1463 meters for every second. The slope of the equation is 1463, representing an increase of 1463 meters per second. The y-intercept is 0 because at 0 seconds, there is no distance to be recorded.

The graph of the function is:





If the sound signal returned to the boat in 0.05 seconds, the distance could be found by substitution into the function as follows:

$d = 1463t$
$d = 1463(0.05)$
$d = 73.15$ meters

If the distance to the fish is 24,000 meters, it will take the sonar signal approximately 16.4 seconds to return to the boat. Because

$$\begin{aligned} d &= 1463t \\ 24000 &= 1463t \\ 16.4 &= t \end{aligned}$$

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Linear Functions
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

Extension Questions:

- If the time is doubled, will the distance be doubled? Justify your answer.

Because there is a proportional relationship, the distance will be doubled if the time is doubled.

$d = 1463t$
$1463(2t) = 2(1463t) = 2d$

- Describe the difference in the two questions asked in problems 3 and 4.

In problem 3 you are given the domain value and asked to evaluate the function. In problem 4 you are given the function value (range value) and asked to find the domain value.

- Determine a function rule that expresses the time as a function of the distance. What type of relationship is this?

Solve the rule for t.

$$d = 1463t$$
$$t = \frac{d}{1463}$$

This function is also linear.

Taxi Ride

The cab fees in Chicago are $1.40 for the first one-fifth mile and 20¢ for each additional one-tenth of a mile.

1. What is the longest distance you can travel for $10.00?

2. How much will you have to pay if you need to get to a restaurant that is 20 miles away from your hotel? Solve using two different techniques.

3. If you want to include a 15% tip, what is the longest distance you can travel for $10.00?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the variables in this situation?
- Which variable is independent?
- What kind of relationship is there between the two variables?
- How much money would you have to pay for the first mile?
- How much money would you have to pay for the second mile?
- What is the rate of increase per mile?
- What is the *y*-intercept?
- What is multiplied by 20¢? Could you use 40¢ instead? How?
- How many tenths are there in 1/5?

Sample Solution:

1. Make a table to look for patterns using the information given, and from the pattern generate the rule for the situation. The pattern shows that the cost will increase by 40¢ for every additional fifth of a mile.

Miles	Cost (dollars)
$\frac{1}{5}$	1.40
$\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$	1.60
$\frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

Determine how many tenths are left after paying $1.40 for the first fifth of a mile. Let the number of miles be represented by *m*.

The number of miles left after the first one-fifth of a mile would be $m - \frac{1}{5}$.

In each mile there would be 10 tenths of a mile.

The number of tenths of a mile left after the first one-fifth of a mile would be represented by $10(m - \frac{1}{5})$ or $10m - 2$.

The cost is 20 cents for every tenth of a mile or $10(m - \frac{1}{5})(0.20)$ or $2m - 0.40$.

The total charge would be

$\$1.40 + 10(m - \frac{1}{5})(0.20) = \$1.40 + 2m - 0.40 = 2m + 1$.

To determine how many miles can be traveled with $10.00, substitute the $10.00 for the cost and solve the inequality.

$$10 \geq 1.40 + 10(m - \frac{1}{5})(0.20)$$
$$10 \geq 1.40 + (10m - 2)(0.20)$$
$$10 \geq 1.40 + 2m - 0.40$$
$$10 \geq 2m + 1$$
$$9 \geq 2m$$
$$m \leq 4.5$$

The greatest number of miles that could be traveled with $10.00 is 4.5 miles.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations;

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

2. Generate a table for the amount for traveling whole miles distances.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Miles	Cost (dollars)
1	3.00
2	5.00
3	7.00
4	9.00
5	11.00
6	13.00

The first mile will cost $3.00, but after the first mile the other miles will cost $2.00 each. Continue the table until 20 miles, or use the patterns to figure the rule. The function for this situation is the cost per mile is $2 for every mile but not the first mile that costs $3.

In symbols: $c = 2(m - 1) + 3$
Substitute the 20 miles for the *m* and determine the cost.

$c = 2(20 - 1) + 3$
$c = 2(19) + 3$
$c = 38 + 3$
$c = 41$

It would cost $41.00 to travel 20 miles.

Another way to solve this problem would be to use the rule for the tenth of a mile. Substitute the 20 for the *m*.

$$c = 1.40 + 10(m - \frac{1}{5})(0.20)$$
$$c = 1.40 + 10(20 - \frac{1}{5})(0.20)$$
$$c = 1.40 + 10(19.5)(0.20)$$
$$c = 41.00$$

3. If the cost is to include a tip of 15%, the computed cost must be multiplied by 1.15.

$$\begin{aligned} 10 &\geq [1.40 + 10(m - \tfrac{1}{5})(0.20)]1.15 \\ 10 &\geq 1.61 + (10m - 2)(0.23) \\ 10 &\geq 1.61 + 2.3m - 0.46 \\ 10 &\geq 2.3m + 1.15 \\ 8.85 &\geq 2.3m \\ m &\leq 3.847826 \end{aligned}$$

If you want to leave a 15% tip and travel under $10.00, the longest distance you can travel is about 3.8 miles.

Extension Questions:

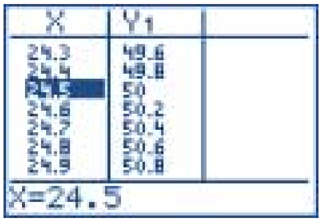
- What are reasonable domain and range values for this situation?

The domain values are every tenth of a mile after the first mile. The range values are $1.40 and every increment of 20¢ after $1.40.

- If a shuttle service charges a fee of $50.00 to any location from the airport, under what circumstances would it be more cost- effective to take a taxi?

The question is when is the taxi going to cost less than $50.00.

Examine the table for the function.



X	Y1	
24.3	49.6	
24.4	49.8	
24.5	50	
24.6	50.2	
24.7	50.4	
24.8	50.6	
24.9	50.8	

X=24.5

It would be more cost-effective to take the taxi for anywhere less than 24.5 miles.

- If the rates increase, how would it affect the representation of your data?

If the rate per tenth of a mile increases, the table entries would be greater and the slope of the graph would be steeper. If the increase happened in the first fifth mile, the y-intercept would also change.

The Contractor

Lupe is a flooring contractor. He sets floor tile for a living. He submits a bid for each new job. Every time he bids for a job, he measures the area of the floor that he will tile and then figures out how much material he will need. He charges the following prices:

Subflooring:	\$1.27 per square foot
Tile:	\$6.59 per square foot
Adhesive:	\$31.95 per job
Grout:	\$55.95 per job
Labor:	\$125 base price plus \$0.79 per square foot

1. Write a rule to determine the total cost of the materials and labor for a typical job. Explain what the numbers and symbols in the rule mean.

2. Make a table and a graph that will help Lupe see the amount of money he should charge for jobs with various amounts of square footage.

3. If Lupe was awarded a job with an area of 550 square feet, what was the amount of the bid, based on the materials listed above?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

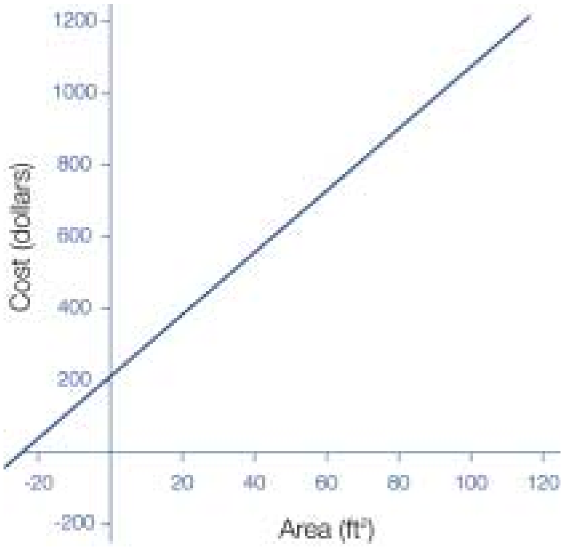
- What are the constants in this problem situation?
- What are the variables?
- Make a list of all the charges he must consider.
- How would he compute the cost for the tile?
- How would he compute the cost of the subflooring?
- How would he compute the labor charges?

Sample Solution:

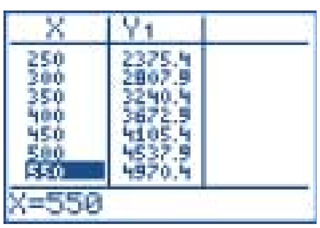
1. The rule would be cost is equal to the cost of the subflooring plus the cost of the tile plus the cost of the adhesive plus the cost of the grout plus the cost of the labor. If the area is the variable x and the total cost of the job is c, then, $c = 1.27x + 6.59x + 31.95 + 55.95 + 125 + 0.79x$. This can be rewritten as $c = 8.65x + 212.90$; c is the total cost of the job, x is the area of the space to be tiled, and \$8.65 is the combined cost of the items per square foot of area. \$212.90 is the total of the fixed costs that do not depend on area.

2. Use the rule and put the values in a table:

Area (ft^2)	Cost (dollars)
0	212.90
50	645.40
100	1077.90
150	1510.40
200	1942.90
250	2375.40
300	2807.90
350	3240.40
400	3672.90
500	4537.90
600	5402.90
700	6267.90
800	7132.90
900	7997.90
1000	8862.90



3. Use the table and look for the *x* value of 550.



X	Y_1
250	2375.4
300	2807.9
350	3240.4
400	3672.9
450	4105.4
500	4537.9
550	4970.4

X=550

If the area was 550 square feet, the cost will be $4970.40.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities; and

(C) for given contexts, interprets and determines the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.1 Solving Linear Equations

Connections to Algebra End-of-Course Exam:

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Extension Questions:

- Suppose Lupe wants to make a 20% profit on each job. Write a new rule that he can use to compute how much he should charge his customers including his profit.

The new price can be determined adding an additional 20% of the original cost to the original cost. Using symbols it could be .20(8.65x + 212.90) + 8.65x + 212.90 or 1.20(8.65x + 212.90).

- The price of the adhesive has increased to $45.95 per job. How is this going to affect the cost of the jobs? How is it going to show up in the rule, the graph, and the table?

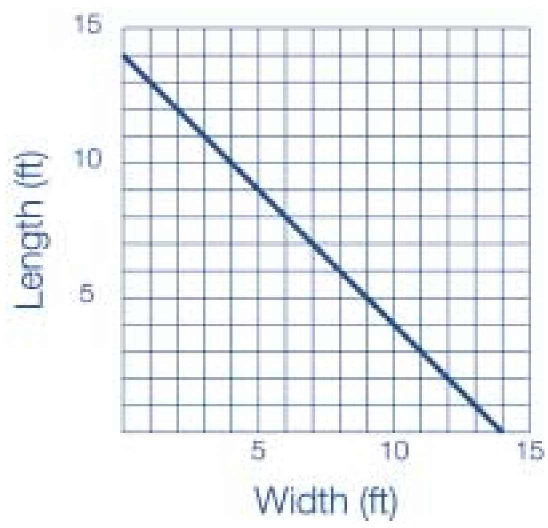
The adhesive cost is part of the fixed amount per job, $212.90. The fixed costs will increase by the difference in the new cost and the original cost, $45.95 – 31.95 or $14. The new fixed costs will be 212.90 + 14 or $226.90. The table values will all increase by this same amount. In the graph, the y-intercept will be 226.90.

- If the charge for labor has changed to $130 plus $0.85 per square foot, how is this going to affect the function that represents the cost of the jobs?

The price for labor has a fixed amount, and it is also part of the charge per square foot. An increase in the fixed amount of the labor will affect the y-intercept. If the fixed amount was increased from $125 to $130, the y-intercept will be increased by 5. An increase in the charge per square foot charge will increase the slope of the graph. The increase from $0.79 to $0.85 will increase the slope of the graph by 0.06.

The function rule will be changed from c = 8.65x + 212.90 to c = 8.71x + 217.90.

The Garden



1. Lance had a certain amount of fencing to enclose his garden. He created a graph to represent the relationship between the length of the garden and the width of the garden. Describe verbally and symbolically the relationship between the length and the width.

2. What are reasonable domain and range values for this function?

3. Explain what the graph tells you about the perimeter of the garden.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions

- What type of relationship is described by the graph?
- Name some points on the graph. What does that tell you about the possible dimensions of the garden?
- Define the independent variable and the dependent variable for this problem situation.
- What would the length be if the width were 12 feet?
- What are the restrictions on the length and the width?

Sample Solution:

1. The graph is a line.

   The starting value (y-intercept) is 14. The x-intercept is 14.

   Other points are (1,13) and (2,12). The rate of change is a decrease of 1 in the length for every increase of 1in the width. Thus, the slope of the line is -1.

   $l = -1w + 14$
   $0 \le w \le 14$

   However, there would be no garden if the width were 0 or 14 feet.

2. From the graph you can see that both the length and the width must be positive numbers less than 14.

Width (ft)	Length (ft)
0	14
2	12
4	10
8	6
10	4
14	0

The points (0,14) and (14,0) do not have meaning in this problem situation. There would be no garden if either the width or the length was 0.

3. Notice from the table that the length plus the width must be 14.

   $l + w = 14$
   or
   $l = -1w + 14$

   The perimeter is twice the sum of the width and the length.

   $2(l + w) = 28$

   The perimeter or amount of fencing is 28 feet.

Extension Questions:

- How is the domain for the function you have written different from the domain for the problem situation?

The domain for the function rule is the set of all real numbers. The domain for the problem situation is the set of all real numbers from 0 to 14.

- How would the graph have been different if the total amount of fencing had been 24 feet?

Twice the sum of the length and width would have been 24 feet. The sum of the length and the width would have been 12 feet. The x and y intercepts would have both been 12.

- How would the graphs have been the same if the total amount of fencing had been 24 feet?

The slope for both graphs would have been –1.

- Describe the relationship between the area of the garden and the width of the garden.

The area is the length times the width. The length is represented by 14 – w.
A = (14 – w)w

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions;

(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
3 Interpreting Graphs
3.1 Interpreting Distance Versus Time Graphs

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

The Submarine

A submarine is cruising at 195 meters beneath the ocean's surface and begins rising toward the surface at 12 meters per minute.

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.

2. How long does it take the submarine to reach the ocean's surface? Explain.

3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this will change the function and the graph of the original situation. What is the new function, and how did you determine it?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions:

- What are the constants in the problem? What quantities vary?
- What quantity will be the dependent variable? The independent variable?
- Create a table and/or a graph to help verify your function rule for Problem # 1.
- What kind of function models the situation?
- What is the submarine's depth when it is at surface level?
- What equation will you write and solve?
- If the submarine must surface 5 minutes sooner, how long will it take to surface?
- What quantity in the original function rule must change?
- Will the submarine rise at the same rate? Slower? Faster?
- Try different rates in your original function rule. Use tables and/or graphs to estimate the rate at which the submarine needs to rise to get to the surface 5 minutes sooner.
- What equation can you write and solve to determine this new rate?

Sample Solution:

1. The submarine's position, *D* meters below the surface, depends on the time, *t* minutes, it has been rising. The submarine starts rising from 195 meters below the surface, so its initial position is -195. It is rising toward the surface at 12 meters per minute, so it is rising at a constant rate of 12 m/min. The function will be a linear function because the rate at which it is rising is a constant.

   The distance will be the starting value plus the rate of change times the number of minutes.

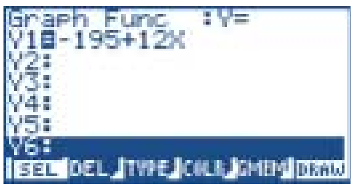
   This gives the function: $D = -195 + 12t$

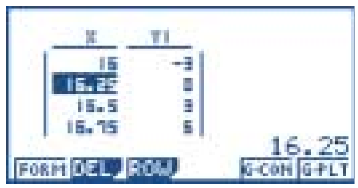
2. When the submarine surfaces, its depth below the surface is 0 meters, so

$$\begin{aligned} D &= -195 + 12t \\ 0 &= -195 + 12t \\ -12t &= -195 \\ t &= 16.25 \end{aligned}$$

   So, it takes the submarine 16.25 minutes to surface if it starts at 195 meters below the surface and rises at a rate of 12 meters per minute.

Another approach to answering the questions is to use a graphing calculator:



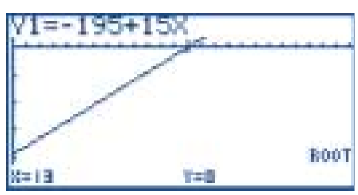


3. If the submarine needs to surface 5 minutes sooner, it needs to rise at a faster rate than 12 meters per minute. Before, it took the submarine 16.25 minutes to surface, and now it is to surface in 11.25 minutes.

   Experiment with different rates (more than 12 meters per minute) by using tables and graphs.

   If the submarine rises at 15 meters per minute, the equation is $D = -195 + 15t$.

   From the graph and table it can be seen that it takes 13 minutes to reach the surface.



(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of y = mx + b;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

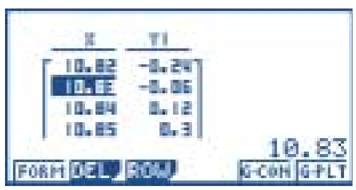
(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

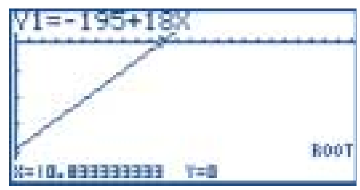
The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

If the submarine rises at 17 meters per minute, the equation is $D = -195 + 17t$. It takes about 11.5 minutes to reach the surface.

If the submarine rises at 18 meters per minute, the equation is D = -195 + 18t. It takes about 10.8 minutes to reach the surface.





The submarine needs to surface at a rate between 17 and 18 meters per minute.

Another approach would be to let the rate be a variable.

$D = -195 + rt$
D must be equal to 0 when $t = 11.25$.

$$\begin{aligned} 0 &= -195 + r(11.25) \\ 195 &= 11.25r \\ r &= 17.3 \end{aligned}$$

The rate should be 17.3 meters per minute.

Extension Questions:

- How do the domain and range of the situation compare with the mathematical domain and range of the function representing this situation? What effect does this have on how you graph the situation?

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
  1.1 Variables and Functions
  1.2 Valentine's Day Idea
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
  1.2 Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

The mathematical domain and range of the function are both all real numbers because it is a (nonconstant) linear function. The situation restricts the domain to 0 to 16.25 minutes and the range to -195 to 0 meters. Knowing the domain and range of the situation helps determine an appropriate window.

- What are the intercepts of the graph of the function? What information do they give about the situation?

The y-intercept is (0,-195). The initial depth of the submarine is 195 meters below surface. The x-intercept is (16.25,0). It takes the submarine 16.25 minutes to reach the ocean surface.

- Suppose the submarine must rise from 195 meters below the surface to the ocean's surface within 10 to 20 minutes. How will this affect the rate at which the submarine rises toward the ocean surface?

In this case, the constant is a range in time to rise to the surface instead of the rate at which the submarine rises. So, let r be the rate, in meters per minute, that the submarine rises and D the depth, in meters, of the submarine.

$D = rt - 195$

If $d = 0$ *at 10 minutes,*

$$\begin{aligned} 0 &= r(10) - 195 \\ 10r &= 195 \\ r &= 19.5 \end{aligned}$$

If $d = 0$ *at 20 minutes,*

$$\begin{aligned} 0 &= r(20) - 195 \\ 20r &= 195 \\ r &= 9.75 \end{aligned}$$

The rate, r meters per minute, must range from 9.75 to 19.5.

- Suppose a student modeled this situation with the function rule $y = 195 - 12t$. What do the variables represent for this rule?

y would represent distance from the surface to the submarine. The rate of travel would be – 12 feet per minute, because the distance from the surface to the submarine would be decreasing at a rate of 12 feet per minute.

Student Work

1. The equation y=12x−195 relates to the submarines position and the time its been rising -195 meters is where it starts, which is the "y" intercept and it rises at 12 feet per minute which is "mx". you could either graph the function or use the table to solve for the function to find when the submarine reaches the surface

2. It takes 16.25 minutes for the submarine to reach the surface because "x" is 16.25 when "y" is 0, which is the measure of the surface. 0=12x−195, 195=12x, 16.25=x

3. It will change it to 17.$\bar{3}$ per minute. the graph will be closer to the y-axis. my new function is

$$y = 195 - (52/3x)$$

I subtracted 5 from 16.25 and got 11.25 and then I divided 195 by 11.25 and got 52/3.

Chapter 8:

Interacting Linear Functions, Linear Systems

Bonnie's Dilemma

Bonnie and Carmen are lab partners in a chemistry class. Their chemistry experiment calls for a 5-ounce mixture that is 65% acid and 35% distilled water. There is no pure acid in the chemistry lab, but they did find two mixtures that are labeled as containing some acid. Mixture A contains 70% acid and 30% distilled water. Mixture B contains 20% acid and 80% distilled water. How many ounces of each mixture should they use to make a 5-ounce mixture that is 65% acid and 35% distilled water? Justify your solution using symbols, tables, and graphs.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Scaffolding Questions:

- If you take 1 ounce Mixture A, how much of this is distilled water? Describe how you determine that amount.
- How many ounces of acid are there in 4 ounces of Mixture A?
- How many ounces of distilled water are there in 2 ounces of the Mixture B?
- What are the variables in this situation?

Sample Solution:

Create a table to compute possible combinations of the two mixtures and the percent of acid and of water in the new mixture.

Amount of Mix A	Amount of Mix B	Amount of acid in New Mix	Amount of distilled water in New Mix	% of New Mix that is acid	% of New Mix that is distilled water	Is it 65% acid and 35% distilled water?
1	5 – 1 = 4	0.7(1) + 0.2(4) = 1.5	0.3(1) + 0.8(4) = 3.5	1.5 out of 5 = 30%	3.5 out of 5 = 70%	Too much water
2	5 – 2 = 3	0.7(2) + 0.2(3) = 2	0.3(2) + 0.8(3) = 3	2 out of 5 = 40%	3 out of 5 = 60%	Too much water
3	5 – 3 = 2	0.7(3) + 0.2(2) = 2.5	0.3(3) + 0.8(2) = 2.5	2.5 out of 5 = 50%	2.5 out of 5 = 50%	Too much water
4	5 – 4 = 1	0.7(4) + 0.2(1) = 3	0.3(4) + 0.8(1) = 2	3 out of 5 = 60%	2 out of 5 = 40%	Too much water
5	0	0.7(5) + 0 = 3.5	0.3(5) + 0 = 1.5	3.5 out of 5 = 70%	1.5 out of 5 = 30%	Not enough water

The value must be between 4 and 5.

4.5	0.5	0.7(4.5) + 0.2(0.5) = 3.25	0.3(4.5) + 0.8(0.5) = 1.75	3.25 out of 5 = 65%	1.5 out of 5 = 35%	The correct amts are 4.5 Mix A and 0.5 Mix B

Another approach using symbols requires that the variables be defined. The variables are the amount of coffee to be used from each mix.

x = the number of ounces of Mixture A
y = the number of ounces of Mixture B

The total amount must be 5 ounces.

Equation 1: $x + y = 5$

The amount of acid in Mixture A plus the amount of acid in the Mixture B must be 65 percent of 5 ounces. The amount of acid in the Mixture A may be expressed as $0.7x$. The amount of acid in the Mixture B may be expressed as $0.2x$.

Equation 2: $0.7x + 0.2y = 0.65(5)$

Similarly, the amount of Distilled water in the Mixture A plus the amount of Distilled water in the second blend must be 35 percent of 5 ounces.

Equation 3: $0.3x + 0.8x = 0.35(5)$

To solve symbolically, use two of the equations and the substitution method:

$x + y = 5$
$0.3x + 0.8y = 0.35(5)$

$$\begin{aligned} y &= 5 - x \\ 0.3x + 0.8(5 - x) &= 0.35(5) \\ 0.3x + 4 - 0.8x &= 1.75 \\ -0.5x &= -2.25 \\ x &= 4.5 \\ y &= 5 - 4.5 = 0.5 \end{aligned}$$

She should take 4.5 ounces of Mixture A and 0.5 ounces of the Mixture B.

Graphs may also be used to solve the problem. To use the graphing calculator to graph or make a table, solve the equations for y:

$y = 5 - x$

$$y = \frac{0.65(5) - 0.7x}{0.2}$$

$$y = \frac{0.35(5) - 0.3x}{0.8}$$

Graph the equations and make a table of values to find the common point. The common point is (4.5,0.5). This means that she should take 4.5 ounces of Mixture A and 0.5 ounces of the Mixture B.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Objective 4:
The student will formulate and use linear equations and inequalities

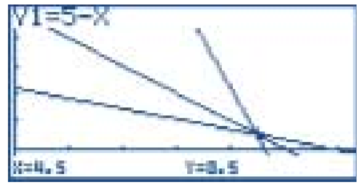
Connections to Algebra I: 2000 and Beyond Institute:

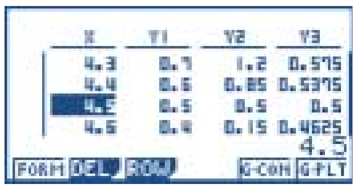
II. Linear Functions
3 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.





Extension Questions:

- Was it necessary to have three equations to solve the problem?

The second and third equations were complements of each other because the result formed the amounts that remained when the given portions were removed. For instance, 70% of the water in one mixture meant 30% of the acid was in the mixture. Three equations are not necessary.

- Does it matter which pair of equations is used?

From the graph it is evident that it does not matter which pair of equations are used. If you use any pair of equations, their graphs intersect at the point (4.5,0.5).

- What could have been determined if the total amount or 5 ounces was not given?

The total amount could be expressed as $x + y$.

The number 5 would be replaced by $x + y$ in the second and third equations.

The amount of Acid would be expressed as $0.7x + 0.2y = 0.65(x + y)$.

Similarly, the amount of distilled water in the Mixture A plus the amount of distilled water in the Mixture B must be 65 percent of x + y ounces.

$0.3x + 0.8y = 0.35(x + y)$

Simplify the equations:

$$\begin{aligned} 0.7x + 0.2y &= 0.65x + 0.65y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

$$\begin{aligned} 0.3x + 0.8y &= 0.35x + 0.35y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

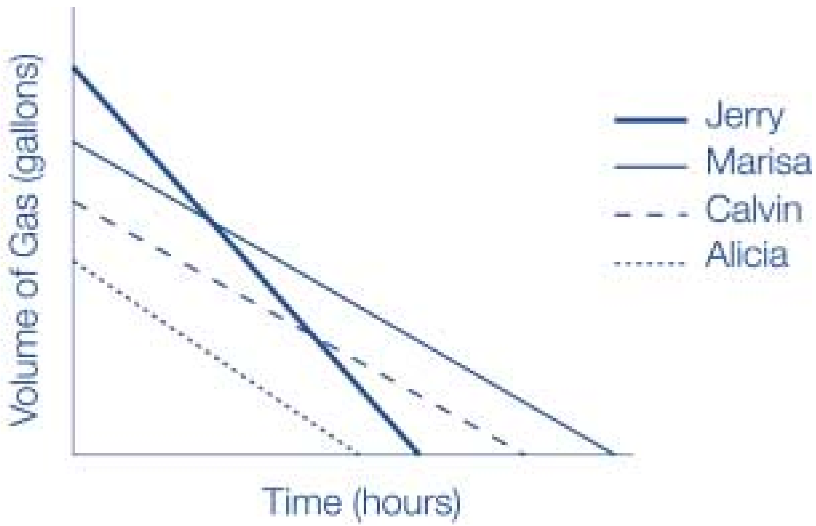
The system could not be solved for specific values of x and y, but one would know what the ratio of x to y must be for any solution.

$$\begin{aligned} 0.05x - 0.45y &= 0 \\ 0.05x &= 0.45y \\ \frac{x}{y} &= \frac{0.45}{0.05} = \frac{9}{1} \end{aligned}$$

To meet the conditions of the problem, the amount of Mixture A must be 9 times the amount of Mixture B.

Four Cars

Jerry, Alicia, Calvin, and Marisa wanted to test their cars' gas mileage. Each person filled their car to the maximum capacity and drove on a test track at 65 miles per hour until they each ran out of gas. The graphs given below show how the amount of gas in their cars changed over time.



Justify your answers to each of the following questions:

1. Whose car had the largest gas tank?
2. Whose car ran out of gas first?
3. Whose car went the greatest distance?
4. Whose car gets the worst gas mileage? What about the graph helped you decide?

5. How would parameters in the function for Calvin's graph compare to the function for Marisa's graph?
6. How would the equation for Marisa's graph compare to the equation for Jerry's graph?
7. How would the equation for Jerry's graph compare to the equation for Alicia's graph?

Teacher Notes

Scaffolding Questions:

- Which person's graph has the greatest y-intercept?
- What does that mean in this situation?
- Which person's graph has the greatest x-intercept?
- What does that mean in this situation?
- How does knowing that they all traveled at 65 miles per hour help you know who traveled the greatest distance?
- Which lines appear to be parallel?
- If two lines are parallel, what will be the same in their equations?

Sample Solution:

1. The information given is that each person filled their tank to capacity. Jerry's car has the largest tank because at 0 hours his car has the greatest volume.

2. Alicia's car ran out of gas first because her car reaches a volume of 0 in the shortest amount of time.

3. Marisa's car traveled the farthest because it took her car the most time to get to a volume of 0. Each person was traveling at 65 miles per hour. Since distance traveled is the rate multiplied by the time in hours, and her time was the greatest, she traveled the greatest distance.

4. Jerry's car gets the worst gas mileage because his graph is the steepest. His rate of change is decreasing at a faster rate. The absolute value of his rate of change is the greatest. The rate of change represents the number of gallons used per hour of travel.

5. It appears that Calvin's and Marisa's lines are almost parallel, so their slopes would be about the same in the equations, but their lines have different y-intercepts.

6. Marisa's and Jerry's equations would be very different. Jerry's gas tank is larger than Marisa's, so the equation that represents Jerry's situation would have a larger y-intercept. His car is also using gas at a faster rate, so the absolute value of his rate of change is greater.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

7. Jerry's gas tank holds more gas than Alicia's gas tank so the equation that represents Jerry's situation would have a greater y-intercept. Jerry's car is using gas at a faster rate than Alicia's car; therefore, the absolute value of his rate of change is greater.

Extension Questions:

- What does the point of intersection of Jerry and Marisa's lines mean?

It represents the point in time when their gas tanks contain the same amount of gas.

- If a fifth line was added to the graph and the line was parallel to Alicia's line, what would you know about that person's car?

It used gasoline at the same rate as Alicia's car, but it would have a different tank capacity.

- Suppose everyone traveled at 55 miles per hour instead of 65 miles per hour. How would this affect their graphs?

If they were traveling at a slower rate, the amount of gas used per hour would decrease. The inclines of all the lines would be less and the x-intercepts would be greater.

- After some mechanical work, Jerry is now getting better gas mileage. How would that affect his graph?

The x-intercept on his graph would be a larger number. The graph would decline more slowly.

- Suppose Calvin's graph could be represented by the rule $V = 30 - 6t$. What information do you now know about Calvin's car? When did his car run of gas? What would be a reasonable rule for Alicia's travel?

The capacity of his tank is 30 gallons because the y-intercept of $V = 30 - 6t$ is 30. The x-intercept is 5 because if $0 = 30 - 6t$, then x is 5. This means that it takes him 5 hours to run out of gas. If he is traveling at 65 miles per hour, he would have traveled 65 times 5 or 325 miles.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
1 Developing Mathematical Models
  1.2 Valentine's Day Idea
2 Using Patterns to Identify Relationships
  2.1 Identifying Patterns

II. Linear Functions
1 Linear Functions
  1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

The rules for the other drivers could be estimated using the intercepts.

Alicia's y-intercept is about half of Jerry's or 15 gallons. If Jerry's x-intercept is 5, her x-intercept is about 4.

$$y = 15 - ax$$
$$0 = 15 - 4a$$
$$a = 3.75$$
$$y = 15 - 3.75x$$

would be a possible rule for Alicia's travel.

Graph It

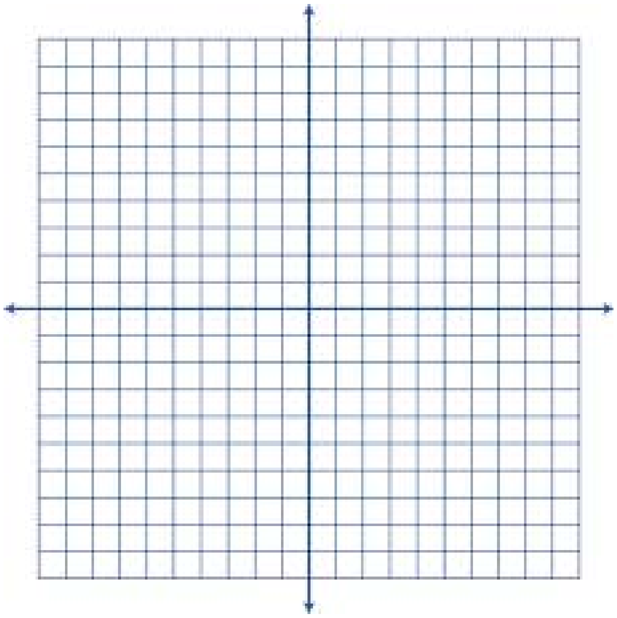
Create a graph, make a table, and write a function rule for the linear functions that meet the following conditions. Use one set of axes to graph all three lines.

1. The line has slope $-\frac{1}{2}$ and a *y*-intercept of 3.

2. The line is parallel to the line in problem 1.

3. The line has a *y*-intercept of 3 but a steeper slope than the line described in problem 1.

4. Compare and contrast the 3 lines.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

Scaffolding Questions:

- What point can be determined if you know that the *y*-intercept is 3?
- How will knowing the slope help you create a graph?
- How will knowing the slope help you create a table?
- What part of the equation must stay constant to produce a parallel line?

Sample Solution:

1. To draw the graph mark the *y*-intercept of 3 and use the slope to get another point on the graph. The slope $-\frac{1}{2}$ means that for every change in -1 of *y*, there is a change of 2 in *x*. Another point is (2,2).

   The equation of a line is of the form $y = mx + b$, where m is the slope and b is the *y*-intercept.

   For this line the equation is $y = -\frac{1}{2}x + 3$.

x	y
-4	5
-2	4
0	3
2	2
4	1
6	0

2. If two lines are parallel, they have the same slope but a different *y*-intercept. The equation for one possible parallel line is $y = -\frac{1}{2}x + 6$.

   Any line of the form $y = -\frac{1}{2}x + b$ where b is any number would be correct.

x	y
-4	8
-2	7
0	6
2	5
4	4
6	3

3. The intercept is 3, but the slope must be different. Increase the absolute value of the slope to get a steeper line. For example, let the slope be -3.

   $y = -3x + 3$.

x	y
-4	15
-2	9
0	3
2	-3
4	-9
6	-15

A positive number could also be selected. Let the slope be 3. $y = 3x + 3$.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

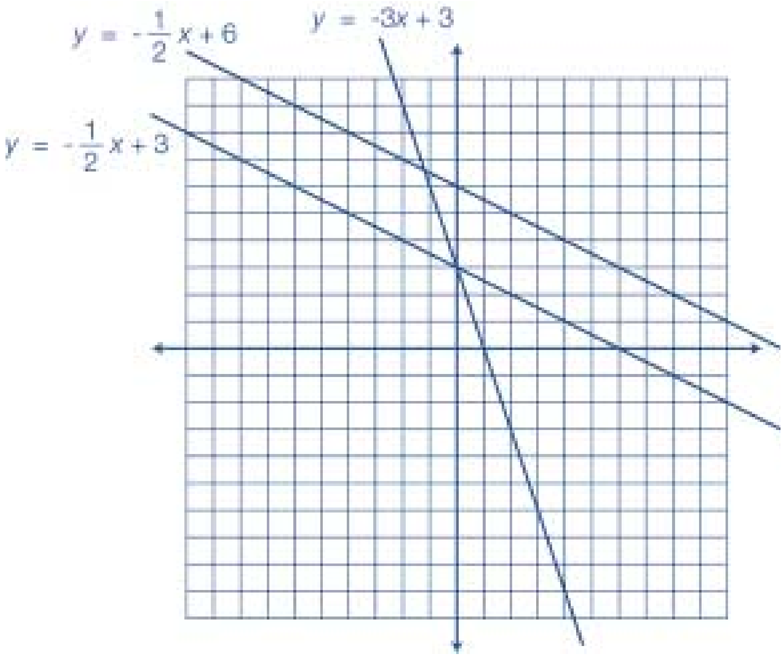
(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(C) investigates, describes, and predicts the effects of changes in m and b on the graph of y = mx + b;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.



4. The lines all have negative slopes and positive *y*-intercepts. The line for number 1 and number 2 have the same slope. They have different *y*-intercepts. They are the same distance apart. The third line intersects the other two; it has a slope of -3 and the same *y*-intercept as the line for number one.

Extension Questions:

- If another line is parallel to the first line and down 4 units, what is the equation of the new line?

The new line would have the same slope, but its y-intercept would be changed to 3 – 4 or -1. The equation would be $y = -\frac{1}{2}x - 1$.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept
1.3 Exploring Rates of Change

III. Nonlinear Functions
1 Quadratic Functions
1.3 Lines Do It Too

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions equations of lines to model problems involving real-world and mathematical situations.

- How are the table values affected by lowering the line 4 units?

All the y values would be decreased by 4.

x	$y = -\frac{1}{2}x + 3$	$y = -\frac{1}{2}x - 1$
-4	5	1
-2	4	0
0	3	-1
2	2	-2
4	1	-3
6	0	-4

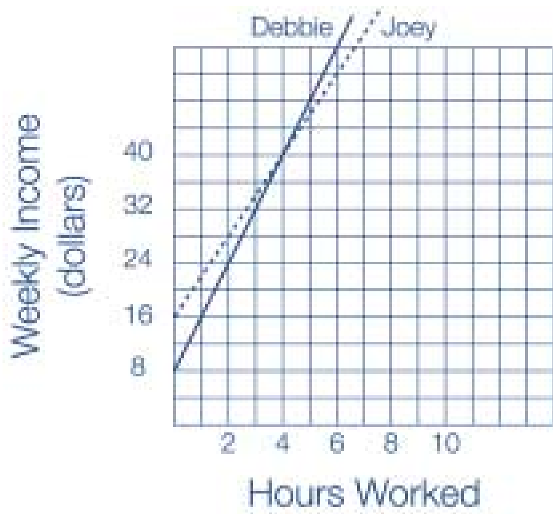
- What is the equation of a line perpendicular to the first line and with the same *y*-intercept?

A perpendicular line has the opposite, reciprocal slope. The slope of the new line is +2.

The equation of the line is y = 2x + 3.

Summer Money

Debbie and Joey decided to earn money during the summer. Each student receives a weekly allowance and has taken a job. The graphs were used to model the weekly income including the allowance as a function of the number of hours worked.



1. Write a function rule that can be used to calculate the amount of money each student will have earned in terms of the number of hours worked in a week.

2. How will an increase in each of their allowances affect the table? The graph? The function rule? Give an example to justify your thinking.

3. How will an increase in hourly wages affect the table? The graph? The function rule ?

4. If Debbie's weekly allowance is doubled, will the new income be more or less than twice the original amount?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- What are the variables?
- What do you need to know to determine the equation of a line?
- What is the y-intercept for each situation?
- What does the y-intercept mean in the context of this situation?
- Explain how to find the slope from a graph.
- Describe what the slope represents for each line.
- Describe in words how much money Debbie will earn per week.

Sample Solution:

1. Use the fact that each person has a starting amount that will be the y-intercept of the function rule or the b in $y = mx + b$. This will be the amount of allowance that each person receives. The slope of the function rule is the rate of change per hour, which is the amount of money each person will get paid per hour. The slope will also be m in $y = mx + b$.

   Debbie's starting amount is \$8, and the rate of change from the point (0,8) to the point (1,16) is \$8 for one hour. The equation for this line is $y = 8x + 8$.

   Joey's starting amount is \$16, and the rate of change from the point (0,16) to the point (2,28) is \$12 for 2 hours or \$6 for one hour. The equation for this line is $y = 6x + 16$.

2. If a person's allowance is increased, the table will show that the corresponding income earned will increase by that amount. For example, if Debbie's allowance was increased by \$4.00, each of the y values in the table is increased by \$4.00.

Debbie's Income

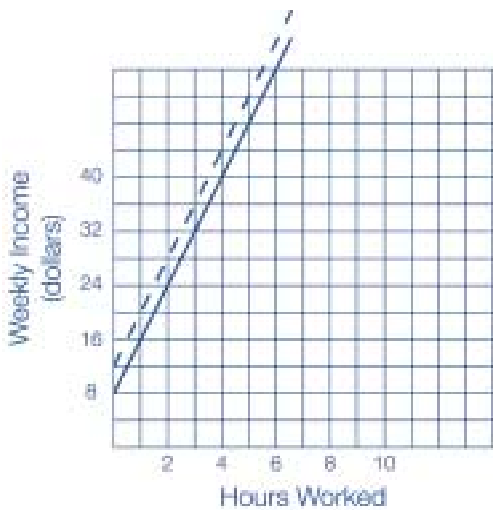
Hours Worked	Income (dollars)
0	8
1	16
2	24
3	32
4	40
5	48
6	56
7	64
8	72

Allowance Increased by $4

Hours Worked	Income (dollars)
0	12
1	20
2	28
3	36
4	44
5	52
6	60
7	68
8	76

There is a constant difference of 4 in the y-values for the same x in the table. For example, the difference between the y-values for 8 hours in the two tables is 76 – 72 or 4.

The graph of the new situation will be a straight line parallel to the original line with a y-intercept of 12.



(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of y = mx + b;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

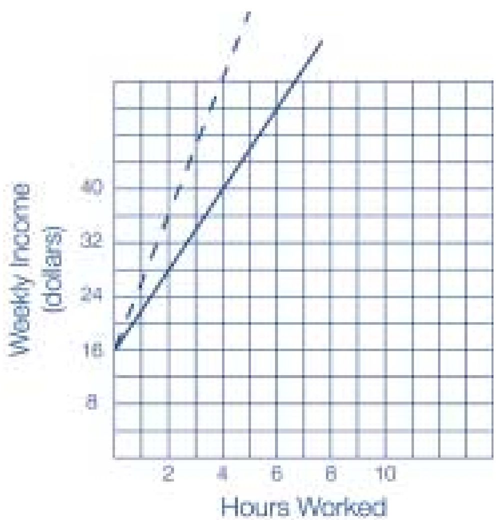
II. Linear Functions
1 Linear Functions
1.2 The Y-Intercept

Connections to Algebra End-of-Course Exam:

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

In the function rule the constant term will change. For example, if Debbie's allowance is increased by $4.00, the new allowance will be $12.00, and the equation will be $y = 8x + 12$.

3. If you increase the hourly wages in each situation, the constant rate of change per hour worked will increase. The slope of each line will get steeper, and in the function rule the coefficient of x will increase. For example, if Joey's hourly rate is increased by $4.00 per hour, the hourly rate becomes $10 per hour. His new equation becomes $y = 10x + 16$. This line will have the same y-intercept but a different slope.



Joey's Income

Hours Worked	Income (dollars)
0	16
1	22
2	28
3	34
4	40
5	46
6	52
7	58
8	64

Joey's Income with Increased Rate

Hours Worked	Income (dollars)
0	16
1	26
2	36
3	46
4	56
5	66
6	76
7	86
8	96

4. If Debbie's weekly allowance is doubled, then her function will change from $y = 8x + 8$ to $y = 8x + (2)8$.

   $y = 8x + 16$

   Her hourly wage remains the same.

   To get twice her original income we must double her original income. Since her original income is $y = 8x + 8$, then twice her original income would be $2y = 2(8x + 8) = 16x + 16$.

   The function rule for when her allowance doubled is $y = 8x + 16$. The function rule for twice her original income is $y = 16x + 16$.

   For any positive number x, $8x + 16 < 16x + 16$.

   The new income from doubling her allowance would be less than twice her original income because doubling her allowance does not affect her hourly wage, but doubling her original income increases her hourly wage.

Extension Questions:

- Which student will earn more money per week?

It depends on how many hours they work. If they work more than 4 hours, Debbie will earn more money. The y-values on Debbie's line are greater when x is greater than 4.

- What are reasonable domain values for this function?

It depends on whether the students may work portions of an hour. If they must work only whole number hours, the domain values will be whole numbers.

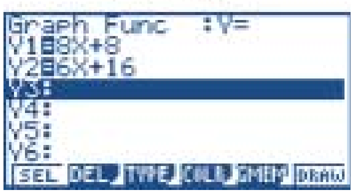
- The line is used to model the situation. Will all points on this graph represent the problem situation?

A person is usually paid for whole numbers of hours or perhaps half hours worked. The graph of the problem situation will really be sets of points and not the whole line.

- Describe the significance of the point of intersection of the two lines.

The point of intersection appears to be the point (4,40). This can be verified by examining the table or graph on the calculator or by substituting into the given functions.

Symbolic



Table

Graph

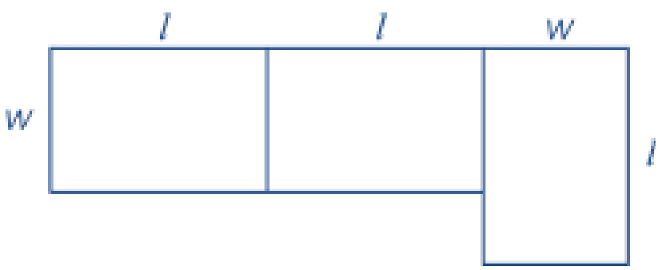
$$y = 6x + 16$$
$$40 = 6(4) + 16$$

$$y = 8x + 8$$
$$40 = 8(4) + 8$$

The point of intersection is (4,40). This means that when Debbie and Joey work for 4 hours they will both earn $40.

The Exercise Pen

Devin is planning to build exercise pens for his three horses. Because of the space that he has available, he has decided to create three rectangular pens as shown in the diagram. The three rectangular pens have the same size and shape. He has been advised that the perimeter of each of the three small pens should be 440 feet. He decides to use 1200 feet of fencing, because that is all he can afford. Use two different representations (tables, symbols, or graphs) to determine how he should construct the pen.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

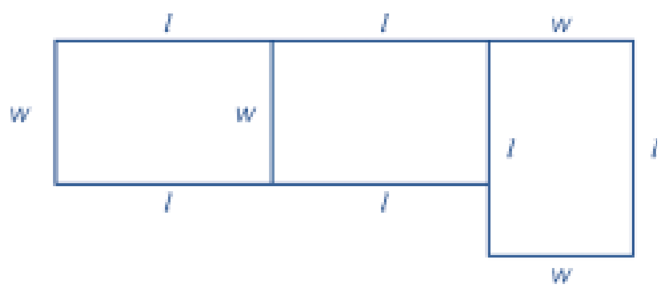
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

Scaffolding Questions:

- What are the unknown quantities in this situation?
- What are some possible dimensions of the pens?
- How many different relationships are described in the problem?

Sample Solution:

The unknown dimensions are the length and width of the pen. In the diagram, the width of each small pen is represented by w, and the length of each small pen is represented by l .



The perimeter of each of the three smaller pens must be 440 feet. The perimeter is twice the length plus twice the width.

$2l + 2w = 440$ Equation 1

Divide by 2:
$l + w = 220$

The amount of fencing must be equal to 1200 feet. There are six lengths and four widths required to fence the pens.

$6l + 4w = 1200$ Equation 2

Divide by 2:
$3l + 2w = 600$

Subtract the first equation from the second equation.

$$\begin{aligned} 3l + 2w &= 600 \\ 2l + 2w &= 440 \\ l &= 160 \end{aligned}$$

Substitute in equation 1 to solve for *w*.

$$\begin{aligned} l + w &= 220 \\ 160 + w &= 220 \\ w &= 60 \end{aligned}$$

The pen should have measurements of approximately 60 feet and 160 feet.

Another method would be to solve each equation for the length, graph the equations, and find the point of intersection.

$$\begin{aligned} l + w &= 220 \\ w &= 220 - l \\ 3l + 2w &= 600 \\ w &= \frac{600 - 3l}{2} \end{aligned}$$

Enter the equations into the graphing calculator. Let the width be the *y*-value and the length be the *x*-value. Draw the graphs on the graphing calculator, and find the point of intersection.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 4:
The student will formulate and use linear equations and inequalities.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
3 Linear Equations and Inequalities
3.4 Systems of Linear Equations and Inequalities

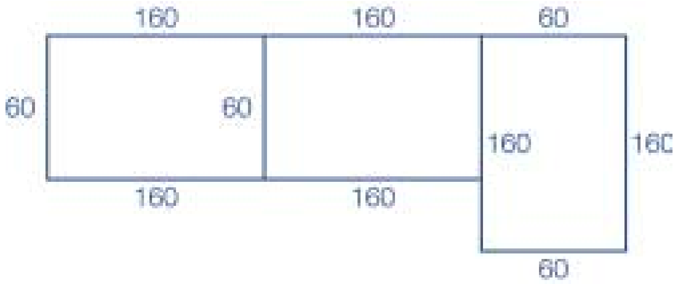
Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

The point of intersection is approximately (160,60). The *x*-value is the length of 160 feet and the *y*-value is the width of 60 feet. The fenced area would look like this diagram.



The perimeter of each small pen is 2(60) + 2(160) = 440 feet.
The total perimeter is 4(60) + 6(160) = 1200 feet.

Extension Questions:

- Two functions rules were used to create lines to represent the situation. Describe the domains and ranges for the functions and the domains and ranges for the problem situation.

The domain and range of each line function is the set of all real numbers. However, for the problem situation, the domain and range values are restricted to first quadrant values.

$y = 220 - x \quad 0 < x < 220 \quad 0 < y < 220$

$y = \dfrac{600 - 3x}{2} \quad 0 < x < 200 \quad 0 < y < 300$

For the two functions together, the domain is restricted to the intersection of the two domains $0 < x < 200$ and the range is restricted to $0 < y < 220$.

- How would your equations change if the total amount of fencing was 800 feet?

The first equation would not be different, but the total perimeter equation would become $6l + 4w = 800$ or $3l + 2w = 400$.

- Solve this system and explain the solution.

Subtract the first equation from the second equation.

$$\begin{aligned} 3l + 2w &= 400 \\ 2l + 2w &= 440 \\ l &= -40 \end{aligned}$$

Since l represents the dimension of a rectangle, it may not be negative. The limit of 800 feet is not enough fence for the smaller pens to have a perimeter of 440 feet.

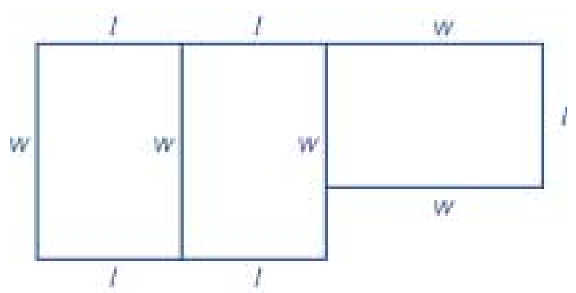
- Solve the problem if the amount of fencing is 1000 feet.

The second equation becomes $6l + 4w = 1000$ or $3l + 2w = 500$. Subtract the first equation from the second equation.

$$\begin{aligned} 3l + 2w &= 500 \\ 2l + 2w &= 440 \\ l &= 60 \\ w &= 220 - 60 = 160 \end{aligned}$$

The original diagram indicates that the longer side is the horizontal side in the diagram. The length, l, must be longer than the width, w. Thus, the values of $l = 60$ and $w = 160$ do not satisfy the conditions of the problem. The amount of fencing may not be 1000 feet.

- If the vertical measure in the diagram is longer than the horizontal measure, the diagram becomes



If the amount of fencing is 1000, what are the dimensions of the pen?

The perimeter of the smaller pen is still represented by the equation $2w + 2l = 440$ or $w + l = 220$.

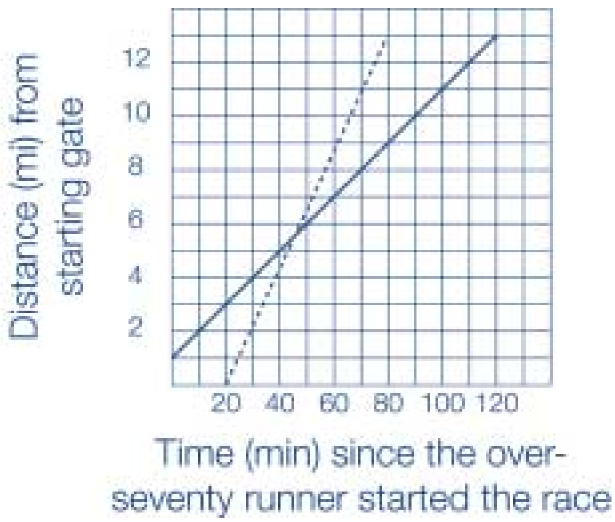
The amount of fencing for this figure is represented by $5w + 5l$. The amount of fencing is 1200. $5w + 5l = 1200$ or $w + l = 240$.

The system of equations is
$w + l = 220$
$w + l = 240.$

This system has no solution. In the diagram above, the restriction on the perimeter $w + l = 220$ means that $5w + 5l = 5(220) = 1100$ feet. He would purchase 1100 feet of fencing.

The Run

The graph below gives information about two of the people who participated in a race. The over-seventy runner left first and the wheelchair rider left second. The line segments graphed represent the relationship between the distance traveled in miles from the starting gate and the number of minutes since the over-seventy runner began the race.



1. Use the graph to describe the situation for each runner.

2. Give the function rule that models each racer's trip.

3. What does the slope of each line represent?

4. What is the point of intersection of the two lines, and what does it mean?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

Scaffolding Questions:

- Which line segment represents the wheelchair rider? How do you know?
- Explain how to determine the rate at which the wheelchair rider is traveling.
- Which segment represents the over-seventy runner and what is his rate?
- What is the rate for the wheelchair rider?
- Which runner took the least amount of time to complete the race?

Sample Solution:

1. The over-seventy runner is the person who started when the time is 0. That first point is the point (0,1). He started one mile ahead of the starting line. The points (0,1) and (10,2) may be used to find the rate at which he ran. The rate of change from the point (0,1) to the point (10,2) is

   1 mile for 10 minutes. His rate is $\frac{1}{10}$ of a mile per minute.

   The equation for his run is the starting value plus the rate times the number of minutes.

   $y = 1 + \frac{1}{10}x$

   Each of the line segments stop when y is 13; therefore, the race length is 13 miles from the starting line. The y-value is 13 when x is 60 minutes. The over-seventy runner started the race at the 1-mile mark, so he traveled 12 miles in 120 minutes.

   The second person to leave is the wheelchair rider who left 20 minutes after the over-seventy runner. The wheelchair rider started at the starting line. The first point on the graph is the point (20,0). Another point on the graph is the point (80,13). The rate of change from (20,0) to (80,13) is

   13 miles for 60 minutes. His rate is $\frac{13}{60}$ miles per minute.

   He started at 0 miles at 20 minutes and ran at $\frac{13}{60}$ miles per minute. The equation of his run is

   $y = \frac{13}{60}(x - 20) + 0.$

2. The slopes of the lines represent the rate at which each person traveled. The wheelchair rider traveled 13 miles per 60 minutes. The over-seventy runner traveled at 1 mile per 10 minutes.

3. The graphs of $y = \frac{1}{10}x + 1$ (over-seventy runner) and $y = \frac{13}{60}(x - 20) + 0$ (wheelchair rider) intersect at a point, about (45,5.5).

   The wheelchair rider was traveling at a faster rate and was ahead of the over-seventy runner for the rest of the race.

   To determine the exact values, the equation may be solved.

   $$\frac{1}{10}x + 1 = \frac{13}{60}(x - 20) + 0$$

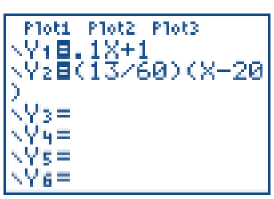
   Multiply both sides of the equation by 60.

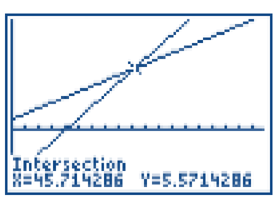
   $$6x + 60 = 13(x - 20)$$
   $$6x + 60 = 13x - 260$$
   $$-7x = -320$$
   $$x = \frac{-320}{-7} = \frac{320}{7} = 45\frac{5}{7}$$

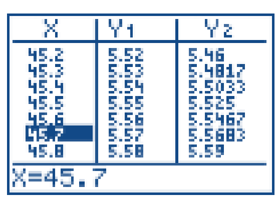
   $$y = \frac{1}{10}x + 1 = \frac{1}{10}\left(\frac{320}{7}\right) + 1 = \frac{39}{7} = 5\frac{4}{7}$$

   At $45\frac{5}{7}$ minutes the wheelchair rider overtakes the over-seventy runner at $5\frac{4}{7}$ miles from the starting gate.

   The calculator may also be used to determine the point of intersection.







The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and a y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations.

(c.4) Linear functions.
The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations and formulates systems of linear equations to solve problems;

(B) solves systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) for given contexts, interprets and determines the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

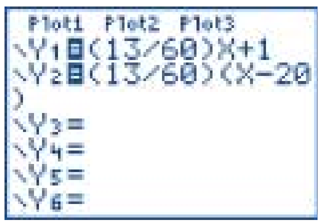
Objective 3:
The student will demonstrate an understanding of linear functions.

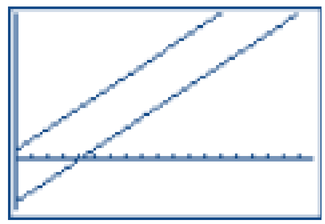
Objective 4:
The student will formulate and use linear equations and inequalities.

Extension Questions:

- Describe a way in which the over-seventy runner could have won the race.

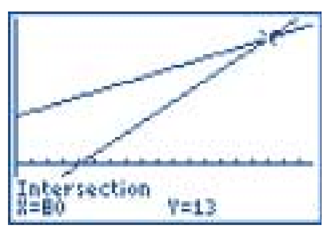
If he had traveled at the same rate as the wheelchair racer, the wheelchair racer would never catch up with him since he left first.





- If both racers traveled at their original rates, what factors or variables could be changed so that the over-seventy runner wins the race?

From the given graph it can be seen that the over-seventy runner ended 40 minutes after the wheelchair rider. If the over-seventy runner's graph is moved 40 units to the left, (x + 40) replaces x, they would end at the same time. That is the over-seventy runner needed an additional head start of 40 minutes to tie the wheelchair racer.



Thus, to win the race, the over seventy runner's rule is changed to reflect an added amount greater than 40. Then he would win. For example, replace x with x + 41.

$y = 0.1(x + 41) + 1$
$y = 0.1x + 4.1 + 1$
$y = 0.1x + 5.2$

He would need to start 5.2 miles from the starting line to beat the wheelchair rider.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundatons for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns
3 Interpreting Graphs
3.1 Interpret Distance vs. Time

II. Linear Functions
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch
3 Linear Equations and Inequalities
3.3 Systems of Linear Equations and Inequalities

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

Objective 4:
The student will formulate or solve linear equations/inequalities and systems of linear equations that describe real-world and mathematical situations.

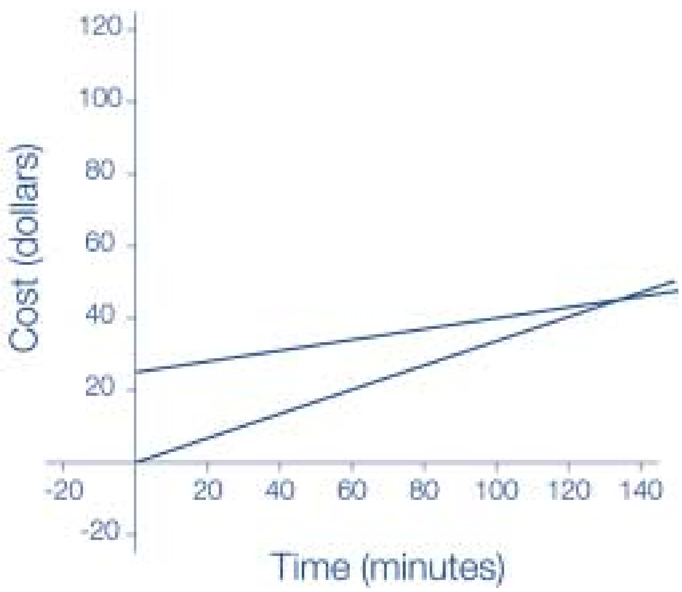
Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Which Plan is Best?

Students were given two cellular phone options to compare.

Plan 1: $C = \$0.35m$
Plan 2: $C = \$0.15m + \$25$

C represents the cost in dollars, and m represents the time in minutes. One group graphed the two plans.



1. Explain the differences between the two plans.

2. Explain the meaning of the slope for each plan.

3. Explain the meaning of the *y*-intercept for each plan.

4. Which plan do you think is a better deal? Explain your thinking.

5. If the second plan was changed to 20 cents per minute, what would be different about the graph?

6. If the first plan was changed so that the base fee was changed to $10, how would the graph have changed?

Teacher Notes

Scaffolding Questions:

- What does the slope represent for each plan?
- What does the *y*-intercept represent for each plan?
- What does the point of intersection mean in the context of this problem?

Sample Solution:

1. In Plan 1 the charge is $0.35 per minute. In Plan 2 the charge is only $0.15 per minute, but there is another fee of $25.

2. In this situation the slope is the rate of change per minute or the amount of money charged per minute in each plan. In Plan 1 the slope is $0.35. In Plan 2 the slope is $0.15.

3. In this situation the y-intercept is the charge at zero minutes. In Plan 1 the *y*-intercept is zero because there is no charge for the plan itself. In Plan 2 the *y*-intercept is 25 because there is a $25 charge to have this plan.

4. The best deal will be determined by the number of minutes you plan to use. Examine the table for the two plans:

X	Y_1	Y_2
123	43.05	43.45
124	43.4	43.6
125	43.75	43.75
126	44.1	43.9
127	44.45	44.05
128	44.8	44.2
129	45.15	44.35

X=125

If you are going to use less than 125 minutes, you should go with Plan 1. If you were going to use more than 125 minutes, the best plan for you would be Plan 2.

5. The slope of the graph would change. The new graph would have a steeper slope.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

6. The *y*-intercept of the original Plan 1 is 0. If a base fee of $10 was added, all points would rise up 10 units. The *y*-intercept would be changed to 10.

(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(A) develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations;

(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

Extension Questions:

- Suppose the cellular phone companies that are offering these plans decided to merge. Together they have come up with a new plan. The new plan charges the customer $0.25 per minute but also gives the first 40 minutes free. The customer doesn't start paying until he has used 40 minutes of airtime. What is the function rule for this new plan?

First make a chart.

Minutes	Cost
40	$0.00
41	$0.25
42	$0.50
43	$0.75
44	$1.00

The change is 25 cents per minute or $2.50 for 10 minutes. Go back $2.50 for every 10 minutes. There are negative costs for less than 40 minutes. This negative cost represents the money the customer is not paying or the money the customer is saving. Continuing to backtrack in the chart tells us that at 0 minutes the cost is -$10.00. Therefore, the customer saves $10.00 when using the 40 free minutes.

Minutes	Cost
0	-$10.00
10	-$7.50
20	-$5.00
30	-$2.50
40	$0.00
41	$0.25
42	$0.50
43	$0.75
44	$1.00

The intercept value is -\$10.00. Our function rule is C = \$0.25x – \$10.00, where x is the number of minutes and x must be greater than or equal to 40.

- How much money does the company lose per customer by giving away 40 minutes of airtime?

The company loses \$10.00 per customer.

- Describe ways to change the method of charging in Plan 1 so that it would also be a better deal than the second plan.

If they charged the same base rate as the second plan but decreased the slope, their fee would always be less. For example, they could charge 14 cents per minute with a base fee of \$25.
C = 0.14m + 25

Another plan would be to charge the same rate per minute but decrease the base fee of \$25. Their price would be less for any number of minutes. For example, they could charge 15 cents per minute with a base fee of \$20.
C = 0.15m + 20

Texas Assessment of Knowledge and Skills:

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.3 Lines Do It Too

Connections to Algebra End-of-Course Exam:

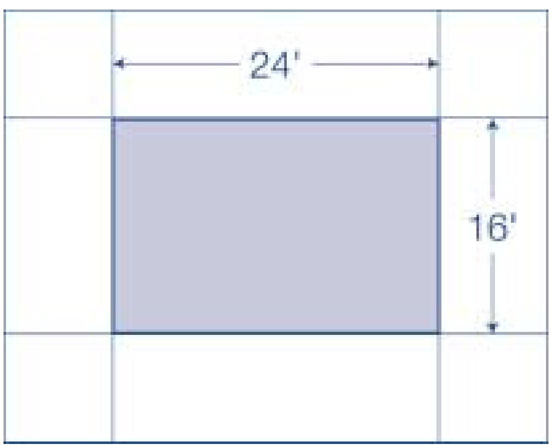
Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Chapter 9:

Quadratic Functions

A Ring Around the Posies

The Lush Landscaping Company is involved in a project for the city. The garden in front of the city hall is going to be expanded by planting a border of flowers all the way around it. The current dimensions of the garden are 24 feet long by 16 feet wide. The border will have the same width around the entire garden. The flowers that will be planted in the border will fill an area of 276 feet^2. Find the width of the border surrounding the garden using symbolic manipulation. Explain the meaning of each number and symbol that you use.



Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

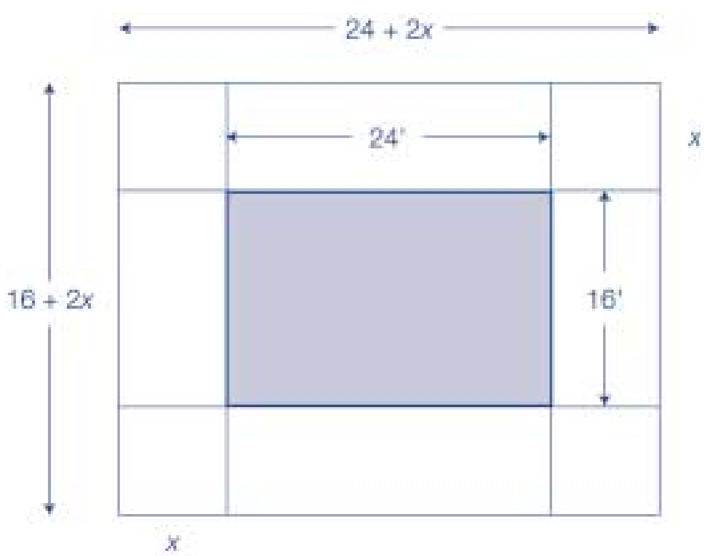
The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- How can you find the area of the existing garden?
- What are the variables in this situation?
- Name a possible length and width of a larger garden.
- Name a second possible length and width of a larger garden.
- Represent the dimensions of the larger garden algebraically.
- How can you find the area of the border?
- How does the area of 276 feet^2 relate to this situation and your equation?

Sample Solution:



The area of the original garden, having dimensions 24 feet by 16 feet, was calculated using the formula for area of rectangle = length • width.

$A = L \cdot W$
$A = 24 \cdot 16$
$A = 384$ feet^2

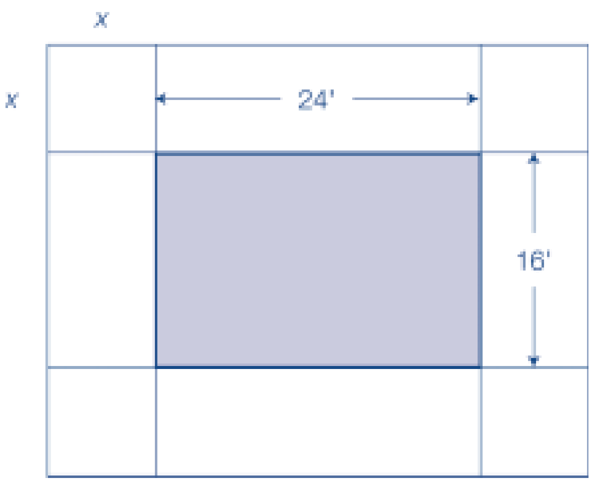
The area of the new garden with the border was calculated with the same area formula as listed above with x representing the width of the border.

$A = (24 + 2x)(16 + 2x)$
$A = 384 + 48x + 32x + 4x^2$
$A = 384 + 80x + 4x^2$

The flowers are going to be planted in the border that surrounds the 24 by 16 rectangle. To find that area, subtract the area of the small inside rectangle from the large rectangle.

Border area = large garden area – small original area
Border area = $(384 + 80x + 4x^2) - (384)$
Border area = $80x + 4x^2$

Another way to look at the border is to think of it as the sum of the rectangles that make up the border.



There are two rectangles at the top and bottom with measurements 24 and x. The two rectangles on the sides have measurements 16 and x. There are four corner squares with side length x. The border area would be

$A = 2(24x) + 2(16x) + 4x^2$
$A = 48x + 32x + 4x^2$
$A = 80x + 4x^2$

The problem stated that the flowers would fill the border that had an area of 276 feet². Substitute this value for the border area.

$276 = 80x + 4x^2$

Factor to solve this equation.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

$0 = 4x^2 + 80x - 276$
$0 = 4(x^2 + 20x - 69)$
$0 = 4(x + 23)(x - 3)$
$0 = x + 23$ and $0 = x - 3$
$x = -23, 3$

The solutions for x are -23 and 3. Since x represents the width of the border, the value cannot be negative. The only solution that makes sense is 3. The width of the border is 3 ft.

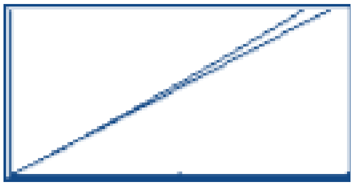
Extension Questions:

- Find the border area A as a function of border width x.

The border area is the same as illustrated in the sample solution $A = 80x + 4x^2$.

- Graph this function, and illustrate the solution to the original problem on the graph.

The top graph of this quadratic function is almost linear in the region that is relevant to the solution of this problem. The lower graph of 80x is shown for comparison.



Block That Kick

The punter on a special team unit kicks a football upward from the ground with an initial velocity of 63 feet per second. The height of the football stadium is 70 feet. The height of an object with respect to time is modeled by the equation

$$h = \frac{1}{2}gt^2 + vt + s$$

where g is -32 ft/sec ^{2}, v is the initial velocity, and s is the initial height.

1. Write a function that models this situation.

2. Sketch and describe the graph of this function.

3. At what times will the football be the same height as the top of the stadium? Explain your answer.

4. Suppose the punter's initial velocity is 68 feet per second. At what times will the football be the same height as the top of the stadium? Justify your answer.

Teacher Notes

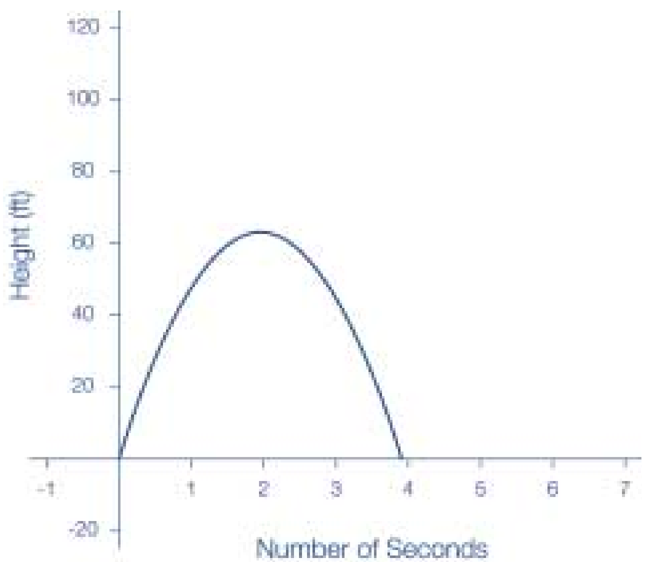
Scaffolding Questions:

- What is value of *v* in the function for this situation?
- What is the initial height?
- What do you expect the graph to look like?
- How can you tell from the graph how many solutions there will be? Explain your answer.
- How can the discriminant be used to determine the number of solutions to this problem situation?

Sample Solution:

1. The initial velocity is 63 feet per second and the initial height is 0 because the ball was kicked from ground level. The height of an object after t seconds when projected upward is modeled by the function $f(t) = -16t^2 + 63t$.

2. The graph is a parabola that opens downward.



3. To ask when the object will be at the same height as the stadium is to ask when will the height be 70 feet. From the graph it can be seen that the height will never be 70 feet.

Materials:
Graphing calculator.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

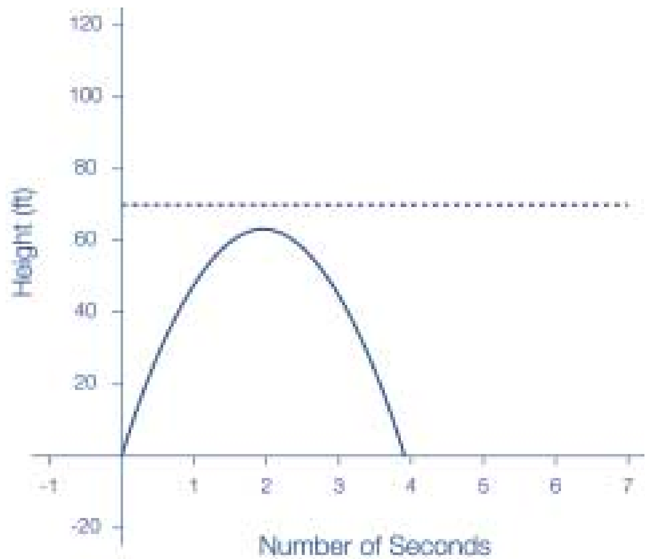
The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.



There is no time when the ball is at the height of 70 feet. Another way to look at this problem is to solve the equation $70 = -16t^2 + 63t$, or

$0 = -16t^2 + 63t - 70$

The value of the discriminant $b^2 - 4ac$ for a quadratic $ax^2 + bx + c$ verifies there is no solution because it is less than 0.

$$\begin{aligned} b^2 - 4ac &= (63)^2 - 4(-16)(-70) \\ &= 3969 - 4480 \\ &= -511 \end{aligned}$$

4. If the initial velocity is increased to 68 feet per second. The function becomes $h(t) = -16t^2 + 68t$. The equation becomes $70 = -16t^2 + 68t$, or

   $0 = -16t^2 + 68t - 70$

   The related quadratic function is $h(t) = -16t^2 + 68t - 70$

   The function will have 2 roots, because the graph crosses the x-axis twice.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

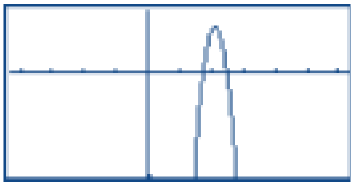
The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.



The roots of the function are at approximately 1.75 and 2.5. This means that the football will be at 70 feet on the ascent, after 1.75 seconds, and again on the descent, at 2.5 seconds.

Extension Questions:

- How many feet off the ground would the ball have to be kicked at the given velocity to reach a height of 70 feet?

The maximum value of the parabola is about 62 feet. If the ball was kicked from 8 feet off the ground, the graph of the function would be raised vertically 8 feet and would reach the height of 70 feet.

- Suppose the football just reaches the same height of the stadium one time. Predict the initial velocity needed to have a maximum height of 70 feet to the nearest thousandth.

Look at the function $f(t) = -16t^2 + vt - 70$. With the previous two problems the value would be between 63 and 68. If this equation has only one root, the discriminant must be equal to zero.

$$\begin{aligned} b^2 - 4ac &= v^2 - 4(-16)(-70) \\ 0 &= v^2 - 4(-16)(-70) \\ v^2 &= 4480 \\ v &= 66.933 \end{aligned}$$

In order to have the football reach the 70-foot level one time, the initial velocity would have to be approximately 66.933 feet per second. This value comes close to producing a discriminant with a value of zero.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

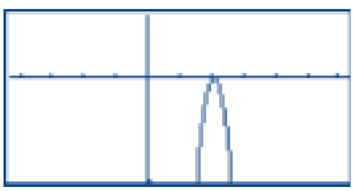
III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

The function would be f(t) = -16t² + 66.933t – 70. The graph would have one root.



BRRR!

The windchill measures how cold the temperature feels at different wind speeds. The faster the wind carries away the warm air around your body, the colder you feel. The windchill, c, at a given temperature in Fahrenheit is a reasonably good quadratic function of the wind speed in miles per hour, s.

For example, at 0 degrees Fahrenheit, the function $c = 0.028s^2 - 2.52s + 2.7$ models the chill factor with wind speeds from 0 to 45 miles per hour.

1. Graph the function and describe how the function models the situation.

2. Use the quadratic formula to find the wind speed for a windchill of -10 degrees.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

Scaffolding Questions:

- Graph the windchill function.
- Describe the graph.
- Explain why the model would not be reasonable for wind speeds above 45 miles per hour.
- Determine the roots of the function and how they relate to the given situation.

Sample Solution:

1. The graph of this function is a parabola that opens upward. According to the graph, the roots of the function are approximately at 1.0 and 88.9.

The graph doesn't make sense for wind speeds over 45 miles per hour because as the wind speed, represented by the *x* values, increases over 45 miles per hour, the y values begin to increase (warm up). Normal wind speeds don't usually go above 30 miles per hour, so the model is good for general use.

2. Windchill = -10 = *c*
$-10 = 0.028s^2 - 2.52s + 2.7$

Add 10 to each side of the equation to put it in standard form.

$0 = 0.028s^2 - 2.52s + 12.7$

Use the quadratic formula to solve, $a = 0.028$; $b = -2.52$; $c = 12.7$. Round to the nearest tenth.

$$t = \frac{-(-2.52) \pm \sqrt{(-2.52)^2 - 4(0.028)(12.7)}}{2(0.028)}$$

$$= \frac{2.52 \pm \sqrt{6.4 - 1.44}}{0.056}$$

$$= \frac{2.52 \pm \sqrt{5}}{0.056}$$

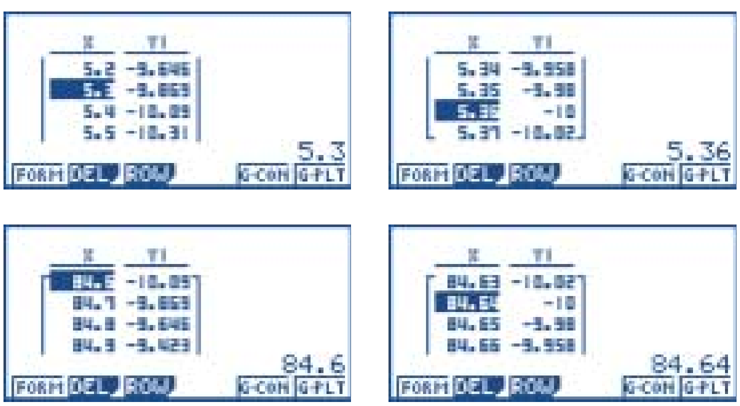
$$= 84.9, 5.4$$

Only a wind speed of 5.4 miles per hour makes sense for this situation because the value of *s* must be between 0 and 45 miles per hour.

Extension Questions:

- Describe how to solve the second problem using a table.

Examine the table of the function. Set the increments small enough to find a value close to -10. The values are 5.36 and 84.6 The value at 5.36 miles per hour is the reasonable value.



- Describe how to solve the second problem using a graph of a quadratic function.

The function is $c = 0.028s^2 - 2.52s + 12.7$.

The graph of the function is a parabola that opens upward and has two roots.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections
2.2 The Quadratic Formula

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

The values are 5.36 and 84.6. The value at 5.36 miles per hour is the reasonable value.

Calculating Cost

The marketing team for Capital Computer Company is working on a project to find the most profitable selling price for its company's new laptop computer. After much research, the team decided that the function $N = -100p^2 + 300{,}000p$ represents the expected relationship between N, the net sales in dollars, and p, the retail price of the laptop computer.

1. Show how to solve the equation $-100p^2 + 300{,}000p = 0$ symbolically.

2. What information does the solution give you about the expected sales?

3. Graph the function. What does the graph tell you about the relationship between the price and the net sales?

4. What price should the marketing team propose for the laptop computer? Explain your answer.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

Scaffolding Questions:

- What does the term net sales mean?
- Describe different methods you might use to solve the equation.
- What does the variable p represent?
- What does it mean when the net sales are equal to 0?
- Describe what the graph tells you about the net sales as the retail price increases.

Sample Solution:

1. The equation $-100p^2 + 300{,}000p = 0$ is solved by factoring:
   $-100p(p - 3000) = 0$

   $-100p = 0$, therefore $p = 0$, and
   $p - 3000 = 0$, so $p = 3000$.

2. These solutions indicate that the net sales will be 0 if the price of the laptop is \$0 or \$3000.

3. The exponent of 2 in the equation indicates the graph will be a parabola. The negative sign of the coefficient indicates the graph will open downward. The graph of the function is as follows:

   The graph shows that as the price of the laptop increases from \$0 to \$1500, the net sales increase from \$0 to \$225 million. As the price continues to increase up to \$3000, the net sales decrease back to \$0.

4. The marketing team should select \$1500 as a sales price for the laptop computer because the maximum net sales occur at that point.

Extension Questions:

- If the coefficient on p is changed to 200,000, how will the change affect the solutions?

The answer will be changed to 0 and 2000.

$-100p(p - 2000) = 0$

$-100p = 0$, therefore $p = 0$, and
$p - 2000 = 0$, so $p = 2000$.

- Without graphing, tell how you expect this change to affect the graph?

The graph will have a vertex that has an x-value halfway between 0 and 2000. The maximum net sales will occur at p = 1000.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
2 Quadratic Equations
    2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

Investigating the Effect of *a* and *c* on the Graph of $y = ax^2 + c$

1. For each of the sets of functions in Problem Sets A – D on the following pages, complete the table to compare their graphs with the graph of the parent function $y = x^2$. Describe how changing the parameters *a* and *c* affects the shape of the graph of the parent function, its intercepts, and its orientation.

2. Each of the following describe transformations on the graph of $y = x^2$. Describe how the vertex, axis of symmetry, intercepts, and symbolic function will be changed by the transformation.

   A. Vertically stretch by a factor of 4 and translate up by 2 units.

   B. Vertically compress by a factor of $\frac{1}{2}$ and translate down by 2 units.

   C. Vertically stretch by a factor of 2 and reflect over the *x*-axis.

Activity Worksheet: Problem Set A

Complete the table to compare the graphs of the given functions with the graph of the parent function $y = x^2$. Describe how changing the parameters *a* and *c* affects the shape of the graph of the parent function, its intercepts, and its orientation.

Function	Graph Show Parent graph plain, Transformed graph bold.	Table Include vertex and images of points (±1,1) and (±2,4) in table.	Description Verbal description of transformation(s)
A1. $f(x) = 2x^2$			
A2. $g(x) = (2x)^2$			
A3. $h(x) = \frac{1}{2}x^2$			
A4. $m(x) = (0.5x)^2$			

Activity Worksheet: Problem Set B

Complete the table to compare the graphs of the given functions with the graph of the parent function $y = x^2$. Describe how changing the parameters *a* and *c* affects the shape of the graph of the parent function, its intercepts, and its orientation.

Function	Graph Show Parent graph plain, Transformed graph bold.	Table Include vertex and images of points (±1,1) and (±2,4) in table.	Description Verbal description of transformation(s)
B1. $f(x) = -x^2$			
B2. $g(x) = -3x^2$			
B3. $h(x) = (-3x)^2$			
B4. $m(x) = -(3x)^2$			

Activity Worksheet: Problem Set C

Complete the table to compare the graphs of the give functions with the graph of the parent function $y = x^2$. Describe how changing the parameters *a* and *c* affects the shape of the graph of the parent function, its intercepts, and its orientation.

Function	Graph Show Parent graph plain, Transformed graph bold.	Table Include vertex and images of points (±1,1) and (±2,4) in table.	Description Verbal description of transformation(s)
C1. $f(x) = x^2 - 1$			
C2. $g(x) = x^2 + 2$			
C3. $h(x) = (x - 1)^2$			
C4. $m(x) = (x + 2)^2$			

Activity Worksheet: Problem Set D

Complete the table to compare the graphs of the given functions with the graph of the parent function $y = x^2$. Describe how changing the parameters a and c affects the shape of the graph of the parent function, its intercepts, and its orientation.

Function	Graph Show Parent graph plain, Transformed graph bold.	Table Include vertex and images of points (±1,1) and (±2,4) in table.	Description Verbal description of transformation(s)
D1. $f(x) = 2x^2 + 1$			
D2. $g(x) = -3x^2 + 12$			
D3. $h(x) = 0.5(x - 2)^2$			
D4. $m(x) = -(x + 2)^2$			

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(A) identifies and sketches the parent forms of linear ($y = x$) and quadratic ($y = x^2$) functions.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$.

Scaffolding Questions:

- What does the graph of the parent function $y = x^2$ look like? Is it a linear graph? What characterizes this graph?

For Problem Set A

- How are the functions different from the parent function?
- How does a table of values help you see the difference in the graph of the new function and the parent function?
- What does it mean to change a quantity by a scale factor?
- How is the point (1,1) in the original function affected by the scale factor?

For Problem Set B

- How are the functions different from the parent function?
- How are they different from the functions in Set A?
- What effect does the negative coefficient have on the graph?

For Problem Set C

- How are the functions different from the parent function?
- How does comparing a table of values for the parent function and these four functions help you see what is happening to the graph of the parent function?

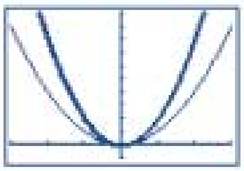
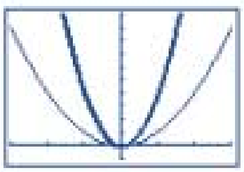
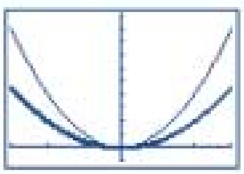
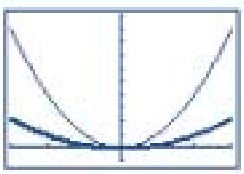
For Problem Set D, you are working with combinations of transformations on the parent function.

- For each function, how would you describe the sequence of transformations performed on the parent function that give the new function?

Sample Solution:

In each set, to compare the parent function with each of the four transformed functions, we can use graphs and tables to tell us what effect each transformation has on the graph of the parent function.

Set A:

Function	Graph	Table	Description
A1. $f(x) = 2x^2$		x | y -2 | 8 -1 | 2 0 | 0 1 | 2 2 | 8	Vertically stretch $y = x^2$ by a factor of 2. The vertex remains at (0,0).
A2. $g(x) = (2x)^2$		x | y -2 | 16 -1 | 4 0 | 0 1 | 4 2 | 16	Horizontally stretch x by a factor of 2. Then square. It is equivalent to vertical stretch by 4. The vertex remains at (0,0).
A3. $h(x) = \frac{1}{2}x^2$		x | y -2 | 2 -1 | 0.5 0 | 0 1 | 0.5 2 | 2	Vertically compress $y = x^2$ by a factor of 0.5. The vertex remains at (0,0).
A4. $m(x) = (0.5x)^2$		x | y -2 | 1 -1 | 0.25 0 | 0 1 | 0.25 2 | 1	Horizontally compress x by a factor of 0.5 and then $0.5x$ is squared. It is equivalent to a vertical compression by a factor of 0.25. The vertex remains at (0,0).

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

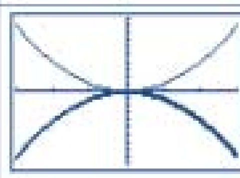
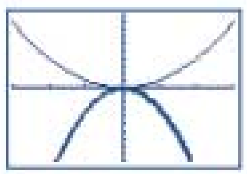
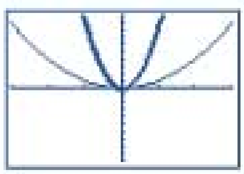
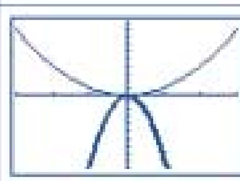
III. Nonlinear Functions
1 Quadratic Functions
 1.2 Transformations

Connections to Algebra End-of-Course Exam:

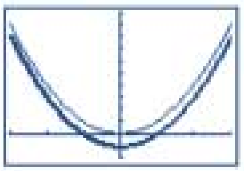
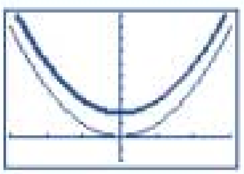
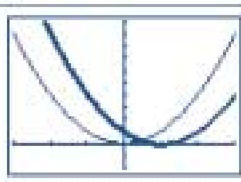
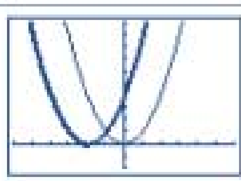
Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

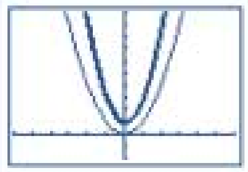
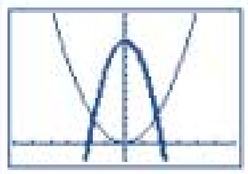
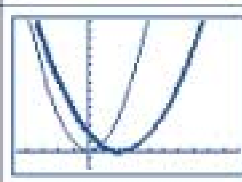
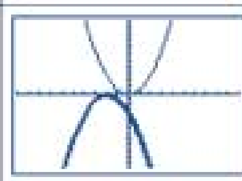
Set B:

Function	Graph	Table	Description
B1. $f(x) = -x^2$		x: -2, -1, 0, 1, 2 y: -4, -1, 0, -1, -4	Reflect graph of parent function over x-axis. The vertex remains at (0,0).
B2. $g(x) = -3x^2$		x: -2, -1, 0, 1, 2 y: -12, -3, 0, -3, -12	Vertically stretch parent graph by a factor of 3. Reflect over x-axis. The vertex remains at (0,0).
B3. $h(x) = (-3x)^2$		x: -2, -1, 0, 1, 2 y: 36, 9, 0, 9, 36	Stretch x horizontally by a factor of 3 and then the product is squared. Reflect over y-axis. It is equivalent to a vertical stretch by 9. The vertex remains at (0,0).
B4. $m(x) = -(3x)^2$		x: -2, -1, 0, 1, 2 y: -36, -9, 0, -9, -36	Stretch x horizontally by a factor of 3. Square to get y. Reflect over x-axis. The vertex remains at (0,0).

Set C:

Function	Graph	Table	Description
C1. $f(x) = x^2 - 1$		x | y -2 | 3 -1 | 0 0 | -1 1 | 0 2 | 3	Translate vertically down one unit. Vertex moves to (0,-1). y-intercept = (0,-1). x-intercepts are (-1,0), (1,0).
C2. $g(x) = x^2 + 2$		x | y -2 | 6 -1 | 3 0 | 2 1 | 3 2 | 6	Translate vertically up two units. Vertex moves to (0,2). y-intercept = (0,2). No x-intercepts.
C3. $h(x) = (x - 1)^2$		x | y -2 | 9 -1 | 4 0 | 1 1 | 0 2 | 1	Translate right one unit. Vertex moves to (1,0). y-intercept = (0,1). x-intercept = (1,0).
C4. $m(x) = (x + 2)^2$		x | y -2 | 0 -1 | 1 0 | 4 1 | 9 2 | 16	Translate left two units. Vertex moves to (-2,0). y-intercept = (0,4). x-intercept = (-2,0).

Set D:

Function	Graph	Table	Description
D1. $f(x) = 2x^2 + 1$		x: -2, -1, 0, 1, 2 y: 9, 3, 1, 3, 9	Vertically stretch parent graph by a factor of 2. Then translate up one unit. The vertex becomes (0,1). There are no x-intercepts.
D2. $g(x) = -3x^2 + 12$		x: -2, -1, 0, 1, 2 y: 0, 9, 12, 9, 0	Vertically stretch parent graph by a factor of 3. Reflect over x-axis. Translate up 12 units. The vertex becomes (0,12). The x-intercept is (±2,0).
D3. $h(x) = 0.5(x - 2)^2$		x: -2, -1, 0, 1, 2 y: 8, 4.5, 2, 0.5, 0	Translate parent graph right two units. Vertically compress by a factor of 0.5. The vertex becomes (2,0). The y-intercept is (0,2).
D4. $m(x) = -(x + 2)^2$		x: -2, -1, 0, 1, 2 y: 0, -1, -4, -9, -16	Translate parent graph left two units. Reflect over x-axis. The vertex becomes (-2,0). The y-intercept is (0,-4).

2A. If the graph of $y = x^2$ is vertically stretched by a factor of 4 and translated up 2 units, the new function is $y = 4x^2 + 2$. The original vertex (0,0) translates to (0,2). The axis of symmetry is still the y-axis. The y-intercept is the vertex. There are no x-intercepts since the equation $4x^2 + 2 = 0$ has no real solution.

2B. If the graph of $y = x^2$ is vertically compressed by a factor of 1/2 and translated down 2 units, the new function is $y = \frac{1}{2}x^2 - 2$. The original vertex (0,0) translates to (0,-2). The axis of symmetry is still the y-axis. The

y-intercept is the vertex. Solving the equation $\frac{1}{2}x^2 - 2 = 0$ implies $x = \pm 2$, so the x-intercepts are (-2,0) and (2,0).

2C. If the graph $y = x^2$ is vertically stretched by a factor of 2 and reflected over the x-axis, the new function is $y = -2x^2$. The vertex is still (0,0). The axis of symmetry is still the y-axis. The y-intercept and x-intercepts are both 0.

Extension Questions:

- What kinds of transformations on the graph of $y = x^2$ can be performed so that the resulting graph continues to be that of a function?

Since a function is a relation between x and y that generates exactly one output value, y, for each input value, x, the only transformations on $y = x^2$ we can consider are dilations, translations, and reflections over the axes.

- What parameter causes a dilation on the graph of $y = x^2$? What is another way of saying "dilation"? What else does this parameter tell you?

The parameter, a, in $y = ax^2$ causes dilation. This dilation is a vertical stretch or a vertical compression on the graph of $y = x^2$. The parameter, a, also causes a reflection over the x-axis if a is negative. As the magnitude of $|a|$ increases, the graph of $y = ax^2$ becomes steeper.

- What parameter causes a translation and what kind of translation?

The parameter, c, in $y = ax^2 + c$ causes a vertical translation. The parent graph is translated c units up if c is positive and -c units down if c is negative.

- What does the "Order of Operations" sequence tell you about the sequence of transformations performed on the parent graph to generate the graph of $y = ax^2 + c$?

If $a > 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a.

Then translate vertically c units.

If $a < 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Next, reflect the graph over the x-axis, and then translate vertically c units.

- What is the difference between the transformations $y = ax^2$ and $y = (ax)^2$?

The first transformation is a vertical stretch or compression of the graph of $y = x^2$. The second transformation is a horizontal stretch or compression on x before squaring. It could also be described as a vertical stretch or compression, but the dilation factor is a^2 since $(ax)^2 = a^2x^2$.

- What is the difference between the transformations $y = x^2 + c$ and $y = (x + c)^2$?

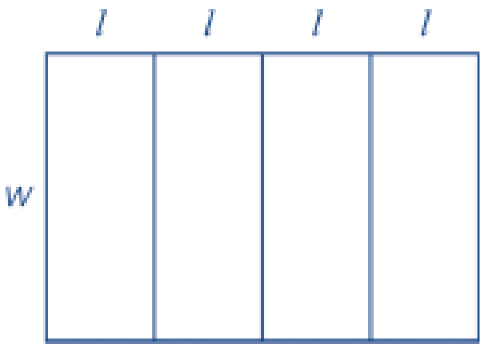
The first transformation is a vertical translation c units up if c is positive and -c units down if c is negative. The second transformation is a horizontal translation c units left if c is positive and -c units right if c is negative.

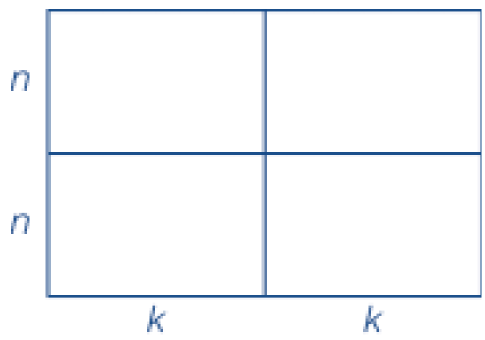
Ostrich Pen

A rancher who raises ostriches has 60 yards of fencing to enclose 4, equally sized, rectangular pens for his flock of ostriches. He is considering two options:

A. Arrange the smaller pens in a line with adjacent pens sharing one common fence.

B. Arrange the four smaller pens so that they share exactly two common fences as shown in the diagram.





1. Define the functions for the total area of each of the two options.

2. Create a graph of each function. Compare the domains and ranges for the options.

3. For each option, describe the dimensions that give maximum total area of the pens. How do the total areas of the two options compare? How do the individual pen areas in the two options compare?

4. If the total amount of fencing for the pens was doubled, how would this change your responses to Parts 1, 2, and 3?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

Scaffolding Questions:

- What is an expression for the length of the large pen for possibility A?
- What is an expression for the width of the large pen for possibility A?
- What is an expression for the length of the large pen for possibility B?
- What is an expression for the width of the large pen for possibility B?
- What expression in l and w describes the total perimeter for the first set of pens?
- What expression in n and k describes the total perimeter for the second set of pens?
- Describe the restriction on the perimeter.
- What equation expresses this restriction?
- With each of the equations, how can you express one variable in terms of the other?
- Using the given variables, what area function will you write for the total area, A, for each set of pens?
- How can you express the area, A, for each set of pens just in terms of one of the variables?
- What representation(s) will best help you describe the domain and range for each set of pens? How does the situation restrict the domains and ranges?
- By looking at the graph, how can you determine the maximum area?

Sample Solution:

1. For each set of pens the total area, A, is given by the product of the length and the width, but the total amount of fencing to be used, 60 yards, can be used to relate the two variables.

   For the first set of pens,

   $$5w + 8l = 60$$
   $$l = \frac{60 - 5w}{8}$$

   For the second set of pens,

   $$6n + 6k = 60$$
   $$k = \frac{60 - 6n}{6} = \frac{6(10 - n)}{6}$$

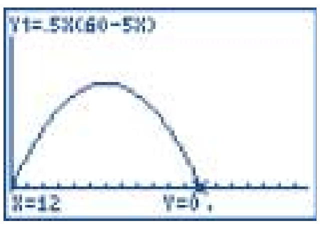
   Area equals width times length. The area functions, for the first and second sets of pens, respectively, are

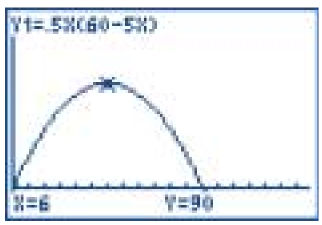
$$A_1 = w \cdot 4l$$
$$= w \cdot 4\left(\frac{60 - 5w}{8}\right)$$
$$= \frac{4}{8} w(60 - 5w)$$
$$= 0.5w(60 - 5w)$$

$$A_2 = 2n \cdot 2k$$
$$= 2n \cdot 2(10 - n)$$
$$= 4n \cdot (10 - n)$$

2. The domains and ranges for the two area functions are easily seen by building a table or by graphing. For the first set of pens, rewrite the area function using the variables y and x.

   $y = 0.5x(60 - 5x)$ where y is the area and x is the width.

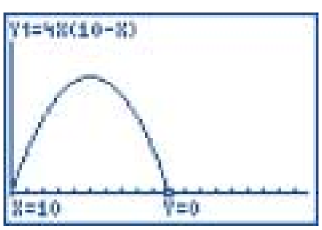


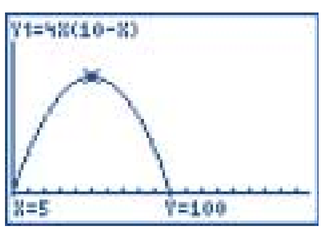


   The domain for the first set of pens is the set of all values, $0 < x < 12$. These are the only values that make sense in the situation, since the area must be positive.

   The range would be the set of all values, A, $0 < A \leq 90$, because the maximum value seen on the graph is 90.

   Similarly, the domain and range for the second set of pens may be determined by examining the graph of $y = 4x(10 - x)$.





   The domain for the first set of pens is the set of all values, $0 < x < 10$.

   The range would be the set of all values, A, $0 < A \leq 100$ because the maximum value seen on the graph is 100.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
  1.1 Quadratic Relationships
2 Quadratic Equations
  2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

3. The maximum area occurs at the vertex of the parabola, which is the graph of each area function. The maximum area for situation *A* is 90 square units when *x* is 6. *x* represented the width, *w*, in the original diagram.

$$5w + 8l = 60$$

$$l = \frac{60 - 5w}{8} = \frac{60 - 5(6)}{8} = 3.75$$

The dimensions of the large pen with a total area of 90 square yards are *w* and 4 *l* , or 6 yards and 4(3.75), or 15 yards.

For situation B the maximum area is 100 square units at $x = 5$.
x represented the variable, *n,* in the original diagram.

The dimensions of the pen are 2*n* and 2*k*.

$$6n + 6k = 60$$

$$k = \frac{60 - 6n}{6} = \frac{6(10 - n)}{6} = 10 - n = 10 - 5 = 5$$

The dimensions of the large pen for the area of 100 square yards are 2(5) = 10 yards and 2(5) = 10 yards.

Dividing the total area by 4 gives the area of each pen. Each pen in the first set has an area of $\frac{90}{4}$ or 22.5 square yards and has a dimension of 6 yards by 3.75 yards.

Each pen in the second set has an area of $\frac{100}{4}$ or 25 square yards and has a dimension of 5 yards by 5 yards. Therefore, a square arrangement of the pens gives the maximum area.

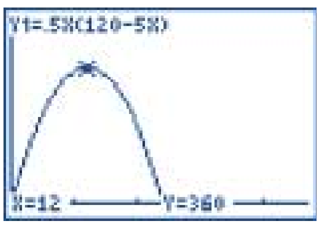
4. If the total amount of fencing to be used were doubled or tripled, this would change the area functions and increase domains and ranges. This can be investigated easily using the table of values for the functions or the graphs.

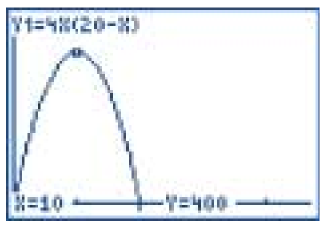
Doubling the fencing changes the 60 feet of fencing to 120 feet of fencing. The first area function changes.

$$5w + 8l = 120 \qquad 6n + 6k = 120$$

$$l = \frac{120 - 5w}{8} \qquad k = \frac{120 - 6n}{6} = \frac{6(20 - n)}{6} = 20 - n$$

$$\begin{aligned} A_1 &= 4lw \\ &= 4w\left(\frac{120 - 5w}{8}\right) \\ &= 0.5w(120 - 5w) \end{aligned} \qquad \begin{aligned} A_2 &= 2n \cdot 2k \\ &= 2n \cdot 2(20 - n) \\ &= 4n(20 - n) \end{aligned}$$





The parabola vertices are (12,360) and (10,400).

The square arrangement of pens still gives the greater area. Doubling the fencing doubles the dimensions of the pens and increases the area by a factor of 4.

Extension Questions:

- What quantities vary in this situation, and how do these quantities affect the total area, *A*?

The dimensions, x and y, of the pens vary, and the arrangement of the pens varies. This means that initially the area, A, is a function of the two variables, x and y. The fixed perimeter lets us relate the variables x and y, and this relation depends on the arrangement of the pens. With this relation, we can express y in terms of x, and then the total area, A, as a function of x only.

- What type of function is *A*(*x*), and how do you know this function has a maximum value? How does this help you determine the dimensions of the pens that give the maximum area?

The function A(x) is a quadratic function with a negative quadratic coefficient. Therefore, its graph is a parabola opening downward. The vertex will be the highest point on the graph, and its coordinates tell you the value of x that gives the maximum area and what that maximum area will be. Once x is known, using that value in the perimeter equation gives the corresponding value of y.

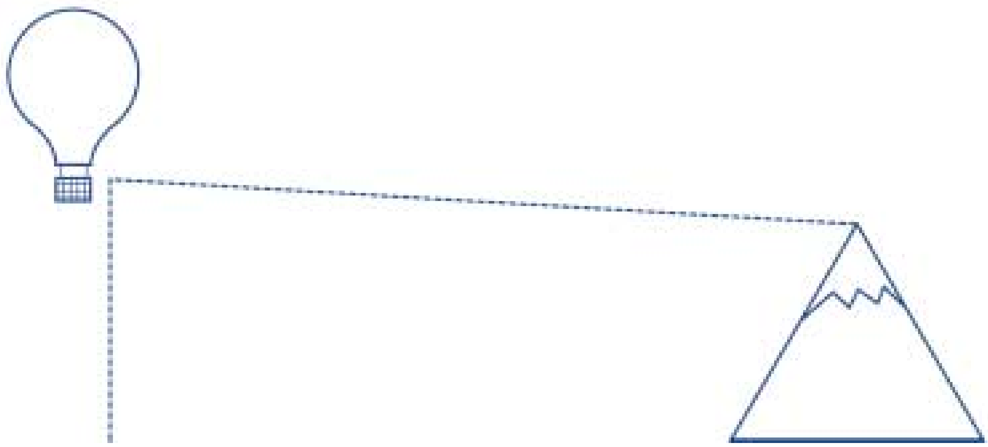
- How do you know which arrangement of pens gives the greater maximum area? Which arrangement is this, and is it realistic? What other factors might need to be considered?

By comparing the parabolas that are the graphs of the area functions, we can determine which has the higher vertex. The graph of the function for the second arrangement has the higher vertex, with a total maximum area of 100 square yards. This is realistic, but it may not be practical. The pens are square, and that may not be the best shaped pen for this animal. It depends on the space ostriches need for exercise.

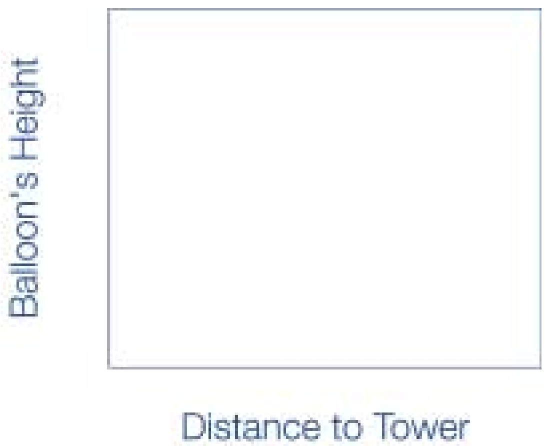
Seeing the Horizon

At the county fair, you get to take a tour of the area in a hot-air balloon. As the balloon rises from the ground, you keep your eye on the top of Mount Franklyn, which is visible on the horizon. The balloon is flying away from the mountain. In order to know the balloon's position, you use a device called a GPS (Global Positioning System), to measure the distance in kilometers, *d*, from the balloon to the tower on top of Mount Franklyn, as well as the balloon's height in meters, *h*. The table below compares the two measures.

Distance (km)	10	20	30	40	50	60	70
Height (m)	8	32	72	128	200	288	392



1. How does the balloon's height vary with the distance you can see to the tower? Without actually plotting points, sketch a graph to describe this relationship. Justify your sketch.



2. Determine a function that describes the height of the balloon in terms of distance to the tower.

3. If the distance from the balloon to the tower is 80 kilometers, what is the height of the balloon?

4. When the balloon's height is 1000 meters, what is the distance from the balloon to the tower?

Teacher Notes

Scaffolding Questions:

- How is the distance changing in the table?
- How is the height changing in the table?
- Is the relationship of height to distance linear? Explain why or why not.
- How does this information help you sketch a graph without actually plotting points?
- How can you use your graphing calculator to check your thinking about the graph?
- Besides a linear function, what other types of functions have you studied in algebra that you could try as a model for this situation?

Sample Solution:

1. To sketch a graph relating the balloon's height to its distance to the tower, compare the change in the distance, *d*, with the change in the height, *h*.

10 10 10 10 10 10

Distance (km)	10	20	30	40	50	60	70
Height (m)	8	32	72	128	200	288	392

24 40 56 72 88 104

The distance is increasing in equal amounts while the height is increasing more and more (faster and faster). This means the graph is not linear and must show for successive increases of 10 kilometers in distance a greater vertical change in the height. Therefore, the graph should be increasing and curving up (concave up).

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

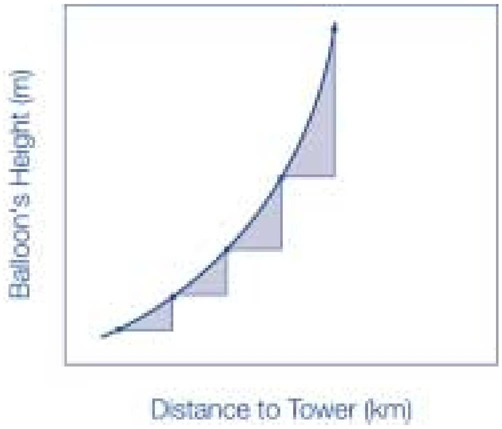
(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(B) given situations, looks for patterns and represents generalizations algebraically.



(d.1) **Quadratic and other nonlinear functions.**
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(d.2) **Quadratic and other nonlinear functions.**
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods.

2. The graph appears to be the graph of a quadratic function. Pattern analysis of the table shows that (0,0) is a correct mathematical entry in the table.

Differences in distance: 10, 10, 10, 10, 10, 10, 10

Distance, d (km)	0	10	20	30	40	50	60	70
Height, h (m)	0	8	32	72	128	200	288	392

First Order Differences: 8, 24, 40, 56, 72, 88, 104

Second Order Differences: 16, 16, 16, 16, 16, 16

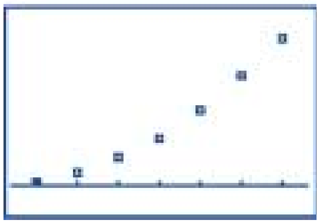
The function for this situation should have the form $y = ax^2$. Using a data point, such as (10,8), substitute for x and y and solve to get $a = 0.08$.

Check the model, $y = 0.08x^2$, against some of the other data points:

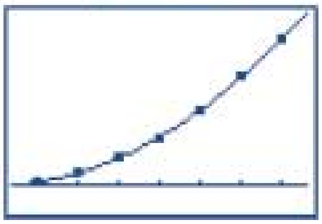
$x = 20 \rightarrow y = 0.08 \cdot 20^2 = 32$
$x = 30 \rightarrow y = 0.08 \cdot 30^2 = 72$
$x = 40 \rightarrow y = 0.08 \cdot 40^2 = 128$

Also verify the model by using the graphing calculator to make a scatterplot and fit a function to the scatterplot.

scatterplot



scatterplot with regression equation, $y = 0.08x^2$



3. If the balloon's distance from the tower is 80 kilometers, evaluate the function $y = 0.08x^2$, where $x = 80$.

   $y = 0.08(80^2) = 512$

   The height of the balloon will be 512 meters.

4. To find the balloon's distance to the tower when the balloon's height is 1000 meters, solve the equation

$$\begin{aligned} 0.08x^2 &= 1000 \\ 8x^2 &= 100000 \\ x^2 &= 12500 \\ x &= 111.803 \end{aligned}$$

   The answer is 111.803 kilometers.

Extension Questions:

- If the height, instead of the distance, increased at a constant amount, how would this change your response to Question #1?

The distance would increase by smaller and smaller amounts. To see this, build a table. Select values for the height, and solve the resulting equations for the distance.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
  1.2 Transformations

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

16 10 8 6

Distance, *d* (km)	11	27	37	45	51
Height, *h* (m)	10	60	110	160	210

50 50 50 50

- What are the domain and range of the function?

The domain is the set of all real numbers, and the range is the set of all nonnegative real numbers.

- What are the domain and range for the situation?

The domain of the situation would not include negative numbers because you cannot have a negative distance for the balloon that is moving away from the top of the mountain. The distance to the tower would be zero only when the balloon is on the top of the mountain. The balloon started at the ground, so the minimum height is 0. There is a maximum height because there is a limit to how high the balloon can rise.

The domain could be described as $0 \leq h \leq$ maximum height of the balloon. The range could be described as the numbers from 0 to the maximum distance.

Student Work

① As the balloon gets higher you can see further in the distance

Since we are looking at the distance to the horizon as the steady rate, and the height of the balloon as the dependent variable, The Graph would be a curved line due to the ever larger and faster balloon's height.

② By looking at the charts and seeing how the heights were all twice of a square root, I figured out that whatever my formula was it would be multiplied by two. Then I took the numbers which were squared and figured that my numbers would have to be squared then multiplied by two. The next part was to link the Distance to the remaining number and figured that the number that had to be squared then multiplied by two first had to be divided by Five. That therefore results in this formula.

$$\left(\frac{d}{5}\right)^2 \cdot 2 = h$$

③

$\left(\frac{d}{5}\right)^2 \cdot 2 = h$	First start with the formula
$\left(\frac{80}{5}\right)^2 \cdot 2 = h$	Substitute everything in
$(16)^2 \cdot 2 = h$	Divide inside the parentheses
$356 \cdot 2 = h$	Do the squaring
512 meters high	Multiply it out to get the answer

④

$\left(\frac{d}{5}\right)^2 \cdot 2 = h$	Start with the formula
$\left(\frac{d}{5}\right)^2 \cdot 2 = 1000$	Do the substitutions
$\left(\frac{d}{5}\right)^2 = 1000$	Divide everything by two
$\left(\frac{d}{5}\right) = 22.360$	Do the square root to both sides
$d = 111.8$ km high	Multiply both sides by Five

Sky Diving

An airplane is flying at an altitude of 1000 meters. A skydiver jumps from the airplane with no upward speed.

1. Use the vertical motion formula $h = \frac{1}{2}(-9.8)t^2 + vt + s$ to write her height during free fall as a function of the time since she jumped. (h = new height; t = time in seconds; v = initial velocity; s = starting height). The initial velocity of a free fall is zero. Graph your function, and identify its roots. Relate the roots to the problem situation.

2. If the skydiver has fallen approximately 100 meters, how many seconds have passed? Explain how to use your graph to estimate the solution. Show how to find the number of seconds algebraically.

3. If the skydiver has fallen 400 meters, approximately how many seconds have passed?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Scaffolding Questions:

- If the skydiver has no upward speed, what is the initial velocity?
- Determine the domain and range for this situation.
- Which values are not reasonable for the domain and/or range? Explain.

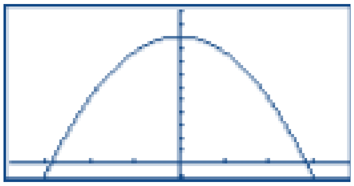
Sample Solution:

1. Using the vertical motion formula $h = \frac{1}{2}(-9.8)t^2 + vt + s$, the skydiver's height during free fall as a function of the time since she jumped, substitute values into the given formula. Given h = new height; t = time in seconds; s = starting height, v = initial velocity.

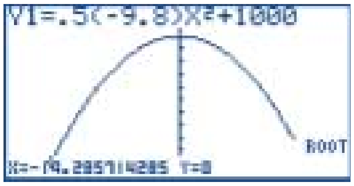
   The initial velocity for this situation is zero, therefore $v = 0$. The starting height is the altitude of 1000 meters, so $s = 1000$. The function is

   $h = \frac{1}{2}(-9.8)t^2 + 1000$ or $h = -4.9t^2 + 1000$.

   The graph of the function is a parabola that opens downward.



   The roots of the function are at approximately -14.29 and 14.29. Only the positive values make sense in the situation because this value represents the number of seconds from the time the skydiver jumps to the time she lands. The number of seconds cannot be negative.



2. The graph can be used to estimate the amount of seconds that have passed when the skydiver has fallen approximately 100 meters. Using the trace function on the calculator, one can find that the value of 900 meters for the height will yield a value of about 4.5 seconds.

Another way to determine the number of seconds is to solve algebraically. Substitute the value 900 for the height into the original function and solve for *t*.

$$\begin{aligned} h &= -4.9t^2 + 1000 \\ 900 &= -4.9t^2 + 1000 \\ 0 &= -4.9t^2 + 100 \end{aligned}$$

Use the quadratic formula to solve.
a = -4.9 b = 0 c = 100

$$\begin{aligned} t &= \frac{0 \pm \sqrt{0^2 - 4(-4.9)(100)}}{2(-4.9)} \\ &= \frac{\pm\sqrt{-4(-4.9)(100)}}{2(-4.9)} \\ &= \frac{\pm\sqrt{1960}}{-9.8} \\ &= \pm 4.5 \end{aligned}$$

The solutions are approximately -4.5 and 4.5. The number of seconds cannot be negative, so only the positive value is reasonable.

3. To find the elapsed time if the skydiver has fallen 400 meters, one can use the table of values. If she started at 1000 meters, her height at that time will be 600 meters from the ground. Look for a table value of approximately 600.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

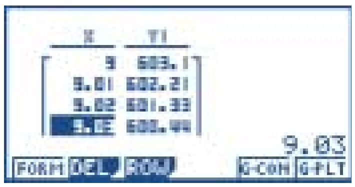
III. Nonlinear Functions
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

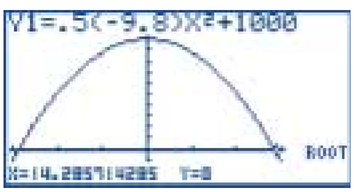
Approximately 9 seconds have passed if the skydiver has fallen 400 meters.



Extension Questions:

- Is it possible for the skydiver to wait 15 seconds before pulling the parachute cord? Justify your answer.

The skydiver cannot wait 15 seconds to open the parachute. The skydiver will land back on the ground at approximately 14.28 seconds after the jump. Waiting 15 seconds to open the parachute would be tragic.



- At the same time the skydiver jumped her friend jumped from an airplane flying at an altitude of 2500 meters. How much longer after landing must the skydiver wait for her friend to land?

First we must determine the roots of the vertical motion formula for the friend. The formula is $h = -4.9t^2 + 2500$*; the only change is the starting height, which is now 2500 meters.*

To find the roots we want to know when this formula equals zero.

$$
\begin{aligned}
0 &= -4.9t^2 + 2500 \\
4.9t^2 &= 2500 \\
t^2 &= \frac{2500}{4.9} \\
t^2 &\cong 510.20 \\
t &\cong \pm 22.59
\end{aligned}
$$

So the friend lands after 22.59 seconds. Disregard the negative solution because time will not be negative. The first skydiver lands after 14.29 seconds. Therefore the first skydiver must wait 22.59 – 14.29 = 8.3 seconds for her friend to land.

- Pretend that the skydiver could skydive on the moon. The moon's gravity is 1/6 that of the Earth. How long will it take the skydiver to land if she jumped from an altitude of 1000 meters on the moon?

First change the vertical motion formula to reflect the moon's gravity. Our formula for earth is $H = -4.9t^2 + 1000$*. For the moon it will be:*

$$H = \frac{1}{6}(-4.9)t^2 + 1000$$

$$H = -0.817t^2 + 1000.$$

To determine when the skydiver will land, determine the roots of this new formula. We set the formula equal to zero.

$$
\begin{aligned}
0 &= -0.817t^2 + 1000 \\
0.817t^2 &= 1000 \\
t^2 &= \frac{1000}{0.817} \\
t^2 &\cong 1223.99 \\
t &\cong \pm 34.99
\end{aligned}
$$

Therefore, if skydiving on the moon, the skydiver will land after 34.99 seconds.

Supply and Demand

Each year, the senior class sponsors a Star Trek Day when they will show 10 favorite Star Trek episodes. Last year, they charged $3 per ticket and sold 2500 tickets. Based on a survey of the student body, they know that for every 10¢ price increase, they would sell 50 fewer tickets. As the senior class president, you must help your classmates decide how much to charge per ticket for this year's Star Trek Day.

A. Write a function for the amount of money, *A* dollars, that would be collected in terms of *x*, the number of 10¢ price increases.

B. Obtain a reasonable graph of your function in Part A, and write a verbal description of what the graph tells you about the situation.

C. Suppose the senior class must collect at least $7800. What range of price increases would allow them to do this? How many tickets would they need to sell to meet this goal?

D. The vice president of the senior class conducts an independent survey. Based on her survey, she predicts that if the class charges $2.50 per ticket they can expect to sell 2700 tickets. Each 10¢ increase in price per ticket would result in 30 fewer tickets being purchased. If she is correct, how would the money collected under this plan compare with the previous situation?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Scaffolding Questions:

- What do you need to know to compute the amount of money that would be collected?
- What expression would represent the ticket price, depending on x, the number of 10¢ price increases?
- What expression would represent the number of tickets sold, depending on x, the number of 10¢ price increases?
- Now what expression would represent the amount of money collected, depending on x, the number of 10¢ price increases?
- By experimenting with your graphing calculator window, what are a reasonable domain and range for the situation?
- What does the vertex of the parabola in the graph of the situation tell you?
- What graph can you add to model the class collecting at least $7800? How does this help you answer the question asked in Part C?
- How do the graphs of the original situation and the situation in Part D compare? How do the graphs help you compare the two situations?

Sample Solution:

A. Since x is the number of 10¢ price increases and 50 fewer tickets are sold per price increase, the number of tickets sold would be given by the expression $2500 - 50x$, and the price per ticket would be given by the expression $3.00 + 0.10x$.

The amount of money collected would equal the price per ticket times the number of tickets sold, so

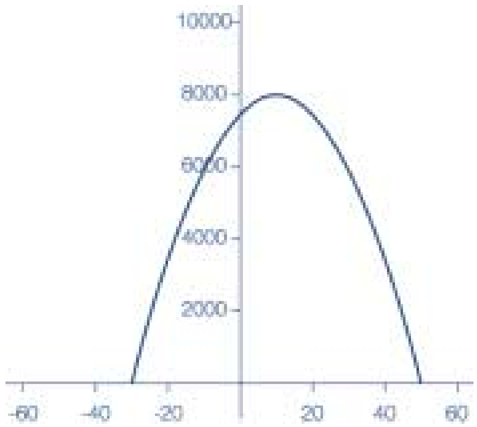
$$\begin{aligned} A &= (3.00 + 0.10x)(2500 - 50x) \\ &= 7500 + 100x - 5x^2 \end{aligned}$$

B. The domain for A is easily seen when A is in factored form and helps in setting a window for the graph. Clearly, $x \geq 0$, since x counts the number of price increases. Also, $x \leq 50$. Otherwise, $2500 - 50x$ would represent a negative number of tickets. To determine the range for A locate both intercepts, -30 and 50; find the x-coordinate of the vertex by determining the midpoint of the intercepts, $x = 10$. Evaluate the area function for $x = 10$.

$$\begin{aligned} A &= 7500 + 100(10) - 5(10)^2 \\ &= 8000 \end{aligned}$$

The range values for the function must be less than or equal to 8000. For the problem situation the range values must also be greater than or equal to 0.

The following graph represents the money collected as a function of the number of 10¢ price increases:



The intercepts that make sense for this situation are (0,7500) and (50,0). The number of 10¢ price increases can range from none ($x = 0$) to 50 ($x = 50$). The ticket prices are represented by $3.00 + 0.10x$. The ticket prices can range from $3.00 + 0.10(0)$ or \$3 to $3.00 + 0.10(50) = \$8$. When $x = 10$, the maximum amount of money, \$8000, is collected. By examining the graph it can be seen that the amount of money collected is increasing if the number of 10¢ price increases is between 0 and 10. They collect the most when the ticket price is $3.00 + 0.10(10) = \$4$, and they sell $2500 – 50(10)$ or 2000 tickets.

C. To see what range in the number of price increases the class must consider to collect at least \$7800, you could add the graph of $y = 7800$ to the money collected graph and determine the x-coordinates of the points of intersection of the two graphs:

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

(d.2) Quadratic and other nonlinear functions.
The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student:
(A) solves quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) relates the solutions of quadratic equations to the roots of their functions.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

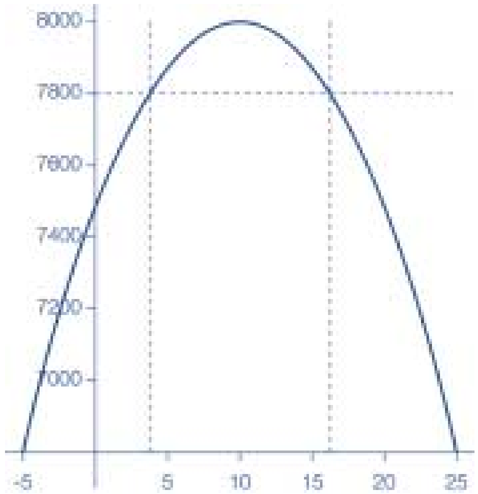
Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships
2 Quadratic Equations
2.1 Connections

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.



The number of 10¢ price increases must fall between 4 and 16. This means the class should keep the ticket price between 3.00 + 0.10(4) = \$3.40 and 3.00 + 0.10(16) = \$4.60 inclusive, and they need to sell between 2500 – 4(50) = 2300 and 2500 – 16(50) = 1700 tickets.

D. In this second situation the number of tickets is changed to 2700, and the price per ticket is changed from the original situation to \$2.50.

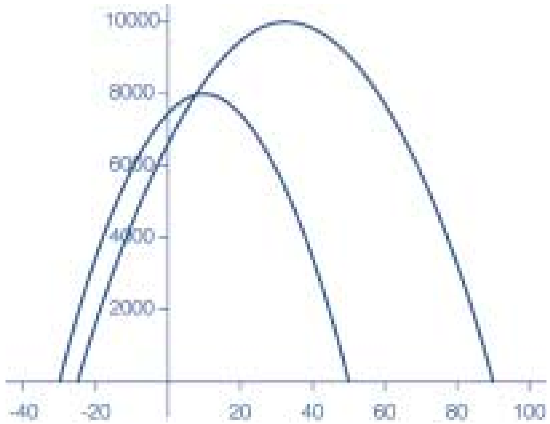
The number of tickets sold would be given by the expression $2700 - 30x$, and the price per ticket would be given by the expression $2.50 + 0.10x$.

The money collected function for this second situation would be

$$\begin{aligned} A &= (2.50 + 0.10x)(2700 - 30x) \\ &= 6750 + 195x - 3x^2 \end{aligned}$$

where the first factor in the expression for A is the price per ticket per 10¢ increase, and the second factor is the number of tickets sold.

The following graph compares the two situations:



For the second situation, the maximum amount of money that can be collected is when the number of 10¢ price increases per ticket is between 32 and 33. At 32 the ticket price is 2.50 + 0.10(32) = \$5.70, and at 33 the ticket price is 2.50 + 0.10(33) = \$5.80.

If the function is evaluated at both 32 and 33, the value of the function is \$9918.

$A(32) = 6750 + 195(32) - 3(32)^2 = 9918$
$A(33) = 6750 + 195(33) - 3(33)^2 = 9918$

The fact that the vertex of the parabola for the second situation is higher and further to the right than that of the first situation shows that the second situation will result in more tickets sold and more money made.

The *y*-intercept for the first graph is (0,7500), while the *y*-intercept for the second graph is (0,6750). This means with no increase in ticket price the money collected for the first situation would be \$7500, and the money collected for the second situation would be \$6750. By finding the intersection of the two graphs, we know that the first situation is better up to an increase of 69¢ per ticket. With an increase of more than 69¢, the second situation will make more money for the class.

The wider spread of the graph of the second situation also shows that more students are willing to accept the greater number of 10¢ price increases.

Extension Questions:

- In this situation, what decisions must the senior class make in order to determine the possible amount of money they can collect with their fundraiser?

They know that for each 10¢ increase in ticket price they will sell 50 fewer tickets. Therefore, the price per ticket depends on the number of 10¢ increases in price per ticket and the number of tickets sold also depends on the number of 10¢ increases they might make. The amount of money they can collect would be price per ticket times number of tickets sold. Since both of these quantities depend on the number of 10¢ increases in price per ticket, the amount of money to be collected will also be affected by the number of 10¢ price increases.

- What properties of the graph of the money collected function help you to draw conclusions about this situation?

Since the price per ticket will have a positive rate of change (10¢ increase per ticket) and the number of tickets sold (50 fewer per 10¢ increase per ticket) will have a negative rate of change, their product will produce a negative quadratic coefficient. Therefore, the graph of the quadratic function for the money collected will be a parabola opening downward. The vertex of the parabola will tell us the maximum number of 10¢ price increases per ticket to make in order to collect the most amount of money.

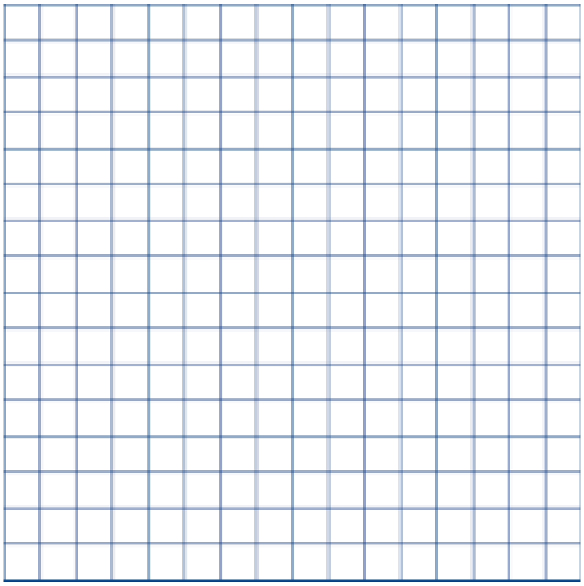
The Dog Run

The owner of a kennel that raises Saint Bernards needs to build a dog run for a new litter of puppies. He has 22 meters of chain link fence to enclose all four sides.

1. Construct a table of values (at least five entries) relating the area of the run in square meters, A, to the length of the pen in meters, l.

Length, l, in meters	Area, A, in square meters

2. Write a function rule relating A and l, and construct a graph.



3. What length maximizes the area of the pen? Should the kennel owner build the pen to maximize the area? Why or why not?

4. How will your function change if the perimeter of the dog run is 24 meters? 26 meters? 28 meters?

5. Describe how you can find the maximum area and corresponding length for any given perimeter.

Teacher Notes

Scaffolding Questions:

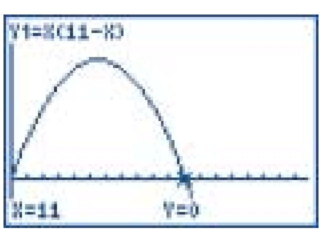
- What lengths make sense in this situation?
- If the length were 1 meter, what would the width be?
- If you are to organize the information in a table, what columns might you have in the table?
- What is the relationship between the length and the width?
- What are the corresponding areas?
- What function type describes the area in terms of the dog run's length?
- What is the parent for this function?
- What does the graph of the parent function look like?
- What will the graph of your function look like?
- How does your table and/or graph help you find the maximum area and corresponding length?

Sample Solution:

1. Since the perimeter of the dog run is to be 22 meters, it takes 11 meters to fence a width and a length. The width of the pen will be 11 minus the length. If the length is represented by l, the width may be represented by $11 - l$ meters.

Length (m)	Width (m)	Area Process	Area (m^2)
1	10	1(10)	10
3	8	3(8)	24
5	6	5(6)	30
7	4	7(4)	28
9	2	9(2)	18
l	$11 - l$	$l(11 - l)$	

2. The function is $A = l(11 - l)$ where $0 < l < 11$. The following is a graph of the situation:



Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations;

(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

3. The maximum area occurs when the length equals the x-coordinate that is halfway between the x-intercepts, 0 and 11. Therefore, the length that gives the maximum area is $l = 5.5$ meters, and the maximum area is $A = 5.5(11-5.5)$ or 30.25 square meters.

   The shape of the dog run that maximizes the area is a square. Usually, a dog run is longer than it is wide so that the dog has plenty of room to run. The kennel owner may decide not to build a square run. He should research what could be the best length to width ratio for the run and use that information to decide on the dimensions of the dog run.

4. The sum of the length and the width must be one-half of the total amount of fencing. The table below gives area, A, as a function of length, l, for varying perimeters:

Perimeter	Area as a function of Length
22	$A = l(11 - l)$
24	$A = l(12 - l)$
26	$A = l(13 - l)$
28	$A = l(14 - l)$

   In general, for any given perimeter, P, the area as a function of length is

$$A = l\left(\frac{P}{2} - l\right)$$

5. To find the maximum area for any given perimeter, you can trace along the graph or look at the table. The maximum area will occur when the length equals the width. The dog run with maximum area is square in shape. The length and the width would be one-fourth of the total perimeter.

$$w = l = \frac{P}{4}$$

$$A = w \cdot l = \frac{P}{4} \cdot \frac{P}{4} = \frac{P^2}{16}$$

Extension Questions:

- Suppose the dog owner investigates and determines that the best pen for the dog is one in which the ratio of the length to the width of the pen is 2:1. What is the area of this pen?

If the ratio of the length to width ratio is 2:1, then $l = 2w$ and $l + w = 11$.

$$\begin{aligned} 2w + w &= 11 \\ 3w &= 11 \\ w &= \frac{11}{3} = 3\frac{2}{3} \\ l &= 2\left(3\frac{2}{3}\right) = 7\frac{1}{3} \end{aligned}$$

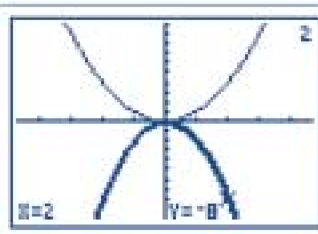
The dimensions of the dog run would be $3\frac{2}{3}$ meters and $7\frac{1}{3}$ meters.

- If a given perimeter is doubled, how will this pen's maximum area and its corresponding value be related to the maximum area and its corresponding value for the given perimeter?

Let the original perimeter be represented by P. The area for the original perimeter is given by the rule $A = l\left(\frac{P}{2} - l\right)$. When the perimeter is doubled, the area becomes $A = l\left(\frac{2P}{2} - l\right) = l(P - l)$. For the original perimeter the maximum area occurs at $l = \frac{P}{4}$. The area would be $\frac{P}{4} \cdot \frac{P}{4}$ or $\frac{P^2}{16}$. For the new perimeter the maximum area occurs at $\frac{P}{2}$ which is twice the value for the original perimeter. The maximum area would be $\frac{P}{2} \cdot \frac{P}{2}$ or $\frac{P^2}{4}$. This value is four times the maximum area for the original perimeter.

Transformations of Quadratic Functions

In this activity, your task is to investigate, describe, and predict the effects of the changes in the parameters *a* and *c* on the graph of $y = ax^2 + c$ as compared to the graph of the parent function $y = x^2$. One representation of the parameter change is given, and you are to complete the others. Include in the verbal description the images of (0,0) and (1,1) under the transformation. For each problem, graph the parent function and the transformed function on the same grid. Draw the parent function in red and the transformed function in blue.

<table>
<tr><th>Function</th><th>Description
Verbal description of transformation(s)</th><th>Graph
Graph should show the vertex and x- and y-intercepts.</th><th>Table
Include vertex and images of (±1,1) and (±2,4).</th></tr>
<tr><td>1a.
$y = 3x^2$</td><td></td><td></td><td>
<table><tr><td>$y = x^2$</td><td>$y =$</td></tr><tr><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table>
</td><td>
<table><tr><th>x</th><th>y</th></tr></table>
</td></tr></table>
</td></tr>
<tr><td>1b.</td><td>Vertically compress $y = x^2$ by a factor of $\frac{1}{2}$.</td><td></td><td>
<table><tr><td>$y = x^2$</td><td>$y =$</td></tr><tr><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table>
</td><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>2</td></tr><tr><td>-1</td><td>0.5</td></tr></table>
</td></tr></table>
</td></tr>
<tr><td>1c.</td><td></td><td>
</td><td>
<table><tr><td>$y = x^2$</td><td>$y =$</td></tr><tr><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table>
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<table><tr><th>x</th><th>y</th></tr></table>
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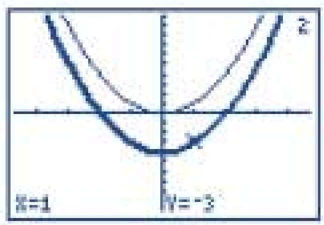
<tr><td>1d.</td><td></td><td></td><td>
<table><tr><td>$y = x^2$</td><td>$y =$</td></tr><tr><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table>
</td><td>
<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>16</td></tr><tr><td>-1</td><td>4</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>16</td></tr></table>
</td></tr></table>
</td></tr>
</table>

Function	Description Verbal description of transformation(s)	Graph Graph should show the vertex and *x*- and *y*-intercepts.	Table Include vertex and images of (±1,1) and (±2,4).
2a. $y = x^2 + 1$			$y = x^2$: x: -2, -1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x | y (blank)
2b.			$y = x^2$: x: -2, -1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x | y (blank)
2c.	Vertically stretch $y = x^2$ by a factor of 2 and translate down 2 units. (0,0) ⟶ (0,-2) (1,1) ⟶ (1,0)		$y = x^2$: x: -2, -1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x | y (blank)
2d.			$y = x^2$: x: -2, -1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x: -2, -1, 0, 1, 2; y: 1, 4, 5, 4, 1

3. Write a summary of the effects of the changes in the parameters *a* and *c* on the graph of $y = ax^2 + c$. Include a description of the effects of the transformations on the vertex, axis of symmetry, and the intercepts.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(A) identifies and sketches the parent forms of linear (y = x) and quadratic ($y = x^2$) functions.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(B) investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$;

(C) investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$; and

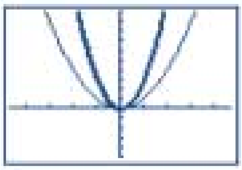
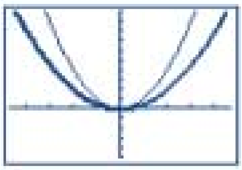
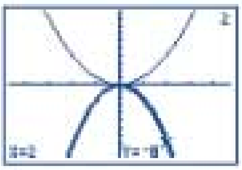
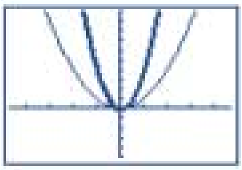
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

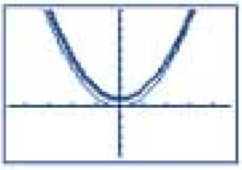
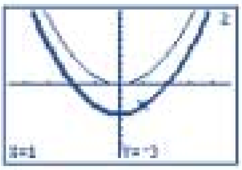
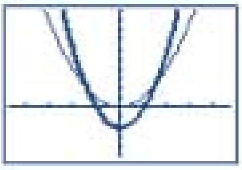
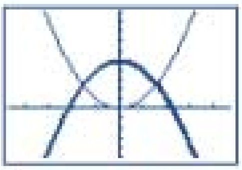
Scaffolding Questions:

- What points help you graph the parent function? What are its vertex, axis of symmetry, and intercepts?
- What does the graph show you about the effect of *a* on the shape and orientation of the graph of $y = ax^2$?
- Describe how the value of *y* varies in your tables as the value of *a* varies.
- What does the graph show you about the effect of *c* on the position of the graph of $y = ax^2 + c$?
- Describe how the value of *c* affects the value of *y* in your tables.
- What is the order of operations in the expression $ax^2 + c$?

Sample Solution:

Note: The lighter lined graph is the graph of $y = x^2$. The darker graph is the transformed graph.

Function	Description	Graph	Table
1a. $y = 3x^2$	The parent graph is vertically stretched by a factor of 3. (0,0) → (0,0) (1,1) → (1,3)		x: -2, -1, 0, 1, 2 y: 12, 3, 0, 3, 12
1b. $y = \frac{1}{2}x^2$	Vertically compress $y = x^2$ by a factor of $\frac{1}{2}$. (0,0) → (0,0) (1,1) → (1,0.5)		x: -2, -1, 0, 1, 2 y: 2, 0.5, 0, 0.5, 2
1c. $y = -2x^2$	Vertically stretch $y = x^2$ by a factor of 2 and reflect over the x-axis. (0,0) → (0,0) (1,1) → (1,-2)		x: -2, -1, 0, 1, 2 y: -8, -2, 0, -2, -8
1d. $y = 4x^2$	Vertically stretch $y = x^2$ by a factor of 4. (0,0) → (0,0) (1,1) → (1,4)		x: -2, -1, 0, 1, 2 y: 16, 4, 0, 4, 16

Function	Description	Graph	Table
2a. $y = x^2 + 1$	Translate $y = x^2$ up one unit. (0,0) → (0,1) (1,1) → (1,2)		x: -2, -1, 0, 1, 2; y: 5, 2, 1, 2, 5
2b. $y = x^2 - 4$	Translate $y = x^2$ down four units. (0,0) → (0,-4) (1,1) → (1,-3)		x: -2, -1, 0, 1, 2; y: 0, -3, -4, -3, 0
2c. $y = 2x^2 - 2$	Vertically stretch $y = x^2$ by a factor of 2 and vertically translate down 2 units. (0,0) → (0,-2) (1,1) → (1,0)		x: -2, -1, 0, 1, 2; y: 6, 0, -2, 0, 6
2d. $y = 5 - x^2$	Reflect $y = x^2$ over x-axis and translate up 5 units. (0,0) → (0,5) (1,1) → (1,4)		x: -2, -1, 0, 1, 2; y: 1, 4, 5, 4, 1

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
1 Quadratic Functions
1.2 Transformations

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

3. Summary: The effect of a on the graph of $y = ax^2$ depends on the signed value of a and the magnitude of a. If $a > 0$, the graph of the transformed function still opens up. If $a < 0$, the graph of $y = x^2$ is reflected over the x-axis, and the transformed function opens downward. If the magnitude of a is greater than one, the graph of $y = x^2$ is vertically stretched by a factor of the magnitude of a. If a is less than one in magnitude, the graph of $y = x^2$ is vertically compressed by a factor of the magnitude of a; in particular, $(1,1) \rightarrow (1,|a|)$. The vertex and the axis of symmetry are preserved under this transformation.

   The effect of c on the graph of $y = x^2 + c$ is to translate the graph of $y = x^2$ vertically c units. If $c > 0$, the graph of $y = x^2$ is translated up c units. If $c < 0$, the graph of $y = x^2$ is translated down $|c| = -c$ units. The x-coordinate of the vertex is still 0, and the axis of symmetry is still $x = 0$. But the vertex has been translated to the point $(0, c)$. This is the new

y-intercept. The new x-intercepts are $(\pm \sqrt{-c}, 0)$, since to get the x-intercepts, we solve $x^2 + c = 0$.

Extension Questions:

- What kinds of transformations on the graph of $y = x^2$ can be performed so that the resulting graph continues to be that of a function?

Since a function is a relation between x and y that generates exactly one output value, y, for each input value, x, the only transformations on $y = x^2$ we can consider are dilations, translations, and reflections over the axes.

- What parameter causes a dilation on the graph of $y = x^2$? What is another way of saying "dilation?" What else does this parameter tell you?

The parameter, a, in $y = ax^2$ causes dilation. If $|a| > 1$, the dilation is a vertical stretch. If $|a| < 1$, then there is a vertical compression on the graph of $y = x^2$. The parameter, a, also causes a reflection over the x-axis if a is negative.

- What parameter causes a translation and what kind of translation?

The parameter, c, in $y = x^2 + c$ causes a vertical translation. The parent graph is translated c units up if c is positive and $|c|$ units down if c is negative.

- What does the "Order of Operations" sequence tell you about the sequence of transformation performed on the parent graph to generate the graph of $y = ax^2 + c$?

If $a > 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Then translate vertically $|c|$ units. If $a < 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Next, reflect the graph over the x-axis, and then translate vertically $|c|$ units.

- If $a > 0$, what is the difference between the transformations $y = ax^2$ and $y = (ax)^2$?

The first transformation is a vertical stretch or compression of the graph of $y = x^2$. The second transformation is a horizontal stretch or compression on x before squaring. It could also be described as a vertical stretch or compression, but the dilation factor is a^2 since $(ax)^2 = a^2x^2$.

- What is the difference between the transformations $y = x^2 + c$ and $y = (x + c)^2$?

The first transformation is a vertical translation c units up if c is positive and -c units down if c is negative. The second transformation is a horizontal translation c units left if c is positive and -c units right if c is negative.

What is the Best Price?

Laura makes earrings to sell at craft fairs. Because of her expenses she has decided that the cheapest price at which she can sell them is $15. She has tried different selling prices at several different fairs and has recorded the data in a table.

Selling Price ($)	Number Sold
15	118
16	115
17.50	110
19	102
20	99
21.50	93
22	91
24	79
25	75
27.50	62
28.50	56
30	51
35	27

She thinks the number sold seems to depend on the selling price. The revenue, the amount of money she receives from the sales, depends on the selling price and the number sold.

1. Use a graphing calculator to create a scatter plot of the data. Determine a model for the number of earrings sold as a function of the selling price.

2. If she had set the selling price at $32, how many might she have expected to sell?

3. Revenue is the amount of money received from sales. For example, if you sold 118 items for $15, the revenue would be $1770. Make a table comparing the selling price and the revenue. Create a scatter plot of the points (revenue, selling price).

4. Use the function rule you found for the number of items to find a function for the revenue in terms of the selling price.

5. Evaluate the revenue function for the selling price of $32.

6. Explain what you think the selling price should be to have the greatest revenue. Justify your reasoning using algebraic representations and tables or graphs.

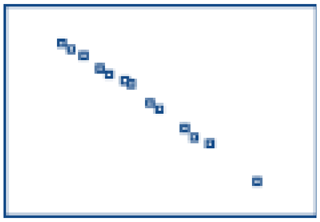
Teacher Notes

Scaffolding Questions:

- What are the variables in this situation? Which one is the dependent variable?
- Examine the table and determine the rate of change. What is the average rate of change in the scatterplot?
- How could you determine the *y*-intercept for the linear model?
- What is the function rule that shows the relationship between the number sold and the selling price?
- What do you need to know to determine the revenue?
- What is the function rule for the revenue?

Sample Solution:

1. The points determined by the table were plotted with the number sold depending on the selling price. The graph appears to be linear.



The finite differences were computed to determine the rate of change.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgements.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(D) for problem situations, analyzes graphs of quadratic functions and draws conclusions.

	Selling Price ($)	Number Sold	
1	15	118	-3
1.5	16	115	-5
1.5	17.50	110	-8
1	19	102	-3
1.5	20	99	-6
0.5	21.50	93	-2
2	22	91	-12
1	24	79	-4
2.5	25	75	-13
1	27.50	62	-6
1.5	28.50	56	-5
5	30	51	-24
	35	27	

The rates of change are found by finding the ratios of the differences.

$\frac{-3}{1} = -3$ $\frac{-5}{1.5} = -3.33$ $\frac{-8}{1.5} = -5.33$

$\frac{-3}{1} = -3$ $\frac{-6}{1.5} = -4$ $\frac{-2}{0.5} = -4$

$\frac{-12}{2} = -6$ $\frac{-4}{1} = -4$ $\frac{-13}{2.5} = -5.2$

$\frac{-6}{1} = -6$ $\frac{-5}{1.5} = -3.33$ $\frac{-24}{5} = -4.8$

The ratios are -3, -3.33, -5.33, -3, -4, -4, -6, -4, -5.2, -6, -3.33, -4.8.

The average of this set of numbers is found by adding up these rates and dividing by 12. The average is -4.33. This number may be used as the rate of change of the linear function that models the set of data. The function rule is of the form $y = -4.33x + b$. Use one of the given points, (20,99), and solve for b.

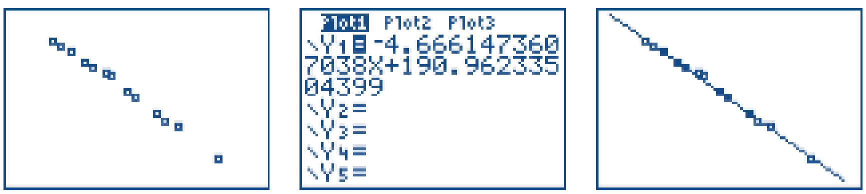
$$99 = -4.33(20) + b$$
$$b = 185.6$$

The number sold, n, as a function of the selling price, p, is

$n = -4.33p + 185.6$

Note that using a different data point would give a different y-intercept and thus a different rule. This is an approximate value. Students may find other approximations.

It is also possible to use the regression line from a graphing calculator.



$y = -4.66x + 190.96$ where y is the number sold, and x is the selling price.

2. If she had set a selling price of $32, the function must be evaluated for $p = 32$.

   $n = -4.33(32) + 185.6 = 47.04$.

   She could expect to sell about 47 items using the first model. Using the second model

   $y = -4.66x + 190.96 = -4.66(32) + 190.96 = 41.84$

   she would sell 41 items.

3. To find the revenue, multiply the selling price by the number sold.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
1.1 The Linear Parent Function
2 Interpreting Relationships Between Data Sets
2.1 Out for the Stretch

III. Nonlinear Functions
1 Quadratic Functions
1.2 Transformations

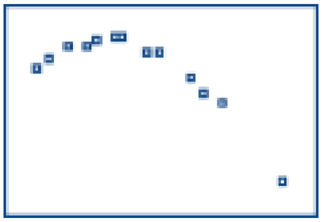
Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

Selling Price ($)	Number Sold	Revenue ($)
15	118	1770
16	115	1840
17.50	110	1925
19	102	1938
20	99	1980
21.50	93	1999.5
22	91	2002
24	79	1896
25	75	1875
27.50	62	1705
28.50	56	1596
30	51	1530
35	27	945

The scatter plot of selling price and the revenue.



To develop the symbolic representation, remember that revenue = number sold times selling price.

$R = np$

Method 1:
$R = (-4.33p + 185.6)p$
$R = -4.33p^2 + 185.6p$

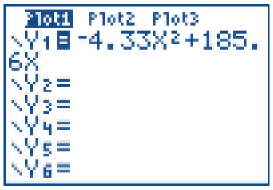
Method 2 (linear regression):
$R = (-4.66p + 190.96)p$
$R = -4.66p^2 + 190.96p$

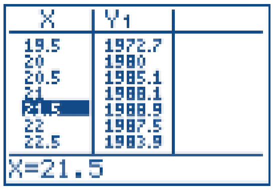
5. The value of the function at $32 is

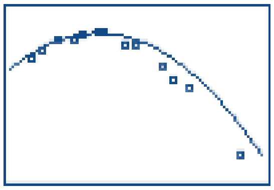
   Method 1: -4.33(32)² + 185.6(32) or $1505.28
   Method 2: -4.66(32)² + 190.96(32) or $1338.88

6. Enter the rule into the calculator and examine the table to determine the selling price that will give the highest revenue.

   Method 1:

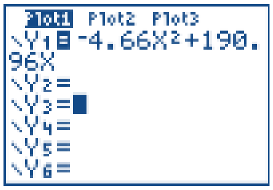


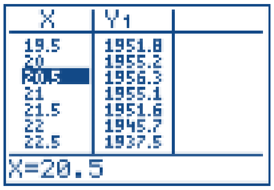


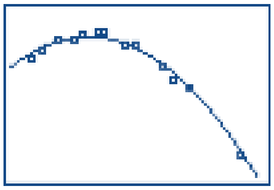


   The table gives a maximum revenue of $1988.90 at the selling price of $21.50.

   Method 2:







   The table gives a maximum revenue of $1956.30 at the selling price of $20.50.

   The answers may vary slightly depending on the model selected. However, the regression equation gives the model that more accurately matches the data.

Extension Questions:

- Describe the domain of the linear function used to model the situation. Compare the domain of the function to the domain for the problem situation.

The domain of this linear function is all real numbers, but the domain of this problem situation requires that p be a selling price in dollars and cents, thus it must be a positive rational number with at most two decimal places. Further restrictions given in the problem require that p be greater than or equal to 15. The x-intercept is between 42 and 43. For any integer greater than 42, the value of n will be negative. Since the number sold may not be negative, $15 \leq x \leq 42$.

- Explain how the domain of the revenue function compares to the domain of the linear model.

There are no restrictions for the function rules on x, but for the problem situation the same restrictions apply to x, $15 \leq x \leq 42$.

- What other conditions might be considered in this situation?

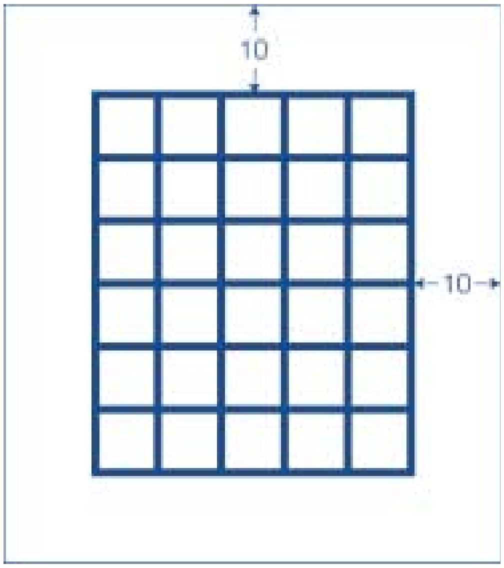
The cost of the production of the goods would affect how many earnings she would make. The amount of time required to produce the product would also affect her production.

- What is the meaning of the slope in the linear equation?

For every 4.33 dollars decrease in price, one more of the items is sold.

Window Panes

The window shown below is made up of squares: wooden strips surrounding each square, and a border that frames all of the squares. The individual white squares have dimensions of *x* inches by *x* inches. The width of the wooden strips surrounding each of the squares is *y* inches. The width of the border that frames all of the squares is 10 inches.



1. Write expressions in terms of *x* and *y* for the dimensions of the entire window (including the border).

2. Write an expression in terms of *x* and *y* for the perimeter of the window. Simplify the expression.

3. Write an expression in terms of *x* and *y* for the area of the window. Simplify the expression.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

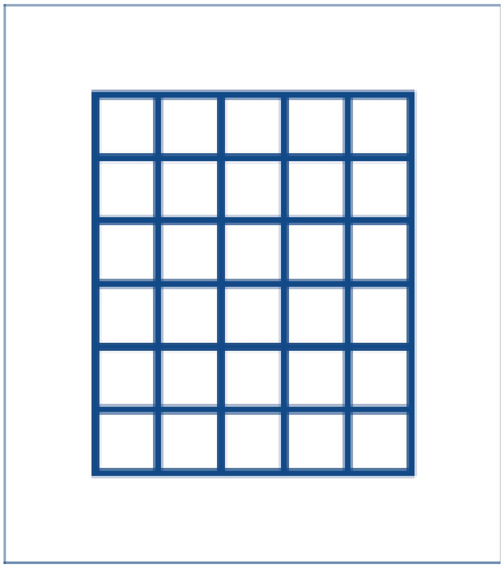
The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations; and

(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Scaffolding Questions:

- If the width of the border is 10 inches, each wooden strip surrounding a square is y inches wide, and the side of each square is x inches, how can you find the width of the entire window? The length?
- Describe the shape of the window.
- How can you determine the perimeter of a rectangle?
- How can you determine the area of a rectangle?

Sample Solution:



1. Along the horizontal side there are 5 widths of the window plus 6 wooden strip widths and 2 border widths. The width is represented by $5x + 6y + 20$.

   The height of the window is 6 widths of the window plus 7 wooden strip widths and 2 border widths. The height is $6x + 7y + 20$.

   The dimensions of the window are $(5x + 6y + 20)$ by $(6x + 7y + 20)$.

2. The perimeter is found by using the formula $P = 2$ (length) + 2 (width).

$P = 2(5x + 6y + 20) + 2(6x + 7y + 20)$
$P = 10x + 12y + 40 + 12x + 14y + 40$
$P = 22x + 26y + 80$

3. The area of the window would be found by multiplying the length and width.

   $A = (5x + 6y + 20)(6x + 7y + 20)$
   $A = 30x^2 + 35xy + 100x + 36xy + 42y^2 + 120y + 120x + 140y + 400$
   $A = 30x^2 + 71xy + 220x + 42y^2 + 260y + 400$

Extension Question:

- Suppose you want your window to be 98 inches wide by 113 inches long. Write and solve a system of equations to find the widths of the squares (x) and the width of the wooden strips surrounding the squares (y).

The expression for the width is 5x + 6y + 20. This amount must equal 98 inches.

$5x + 6y + 20 = 98$

The expression for the length is 6x +7y + 20. The length must equal 113 inches.

$6x + 7y + 20 = 113$

Subtract 20 from each side of each equation.

$5x + 6y = 78$
$6x + 7y = 93$

Multiply the first equation by 6, the second by -5.

$6(5x + 6y + 20 = 98)$ $\quad$ $30x + 36y = 468$
$-5(6x + 7y + 20 = 113)$ $\quad$ $-30x - 35y = -465$
$y = 3$

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2.1 Using Patterns to Identify Relationships

Connections to Algebra End-of-Course Exam:

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

Substitute back into one of the original equations to find x:

$$
\begin{aligned}
5x + 6y &= 78 \\
5x + 6(3) &= 78 \\
5x + 18 &= 78 \\
5x &= 60 \\
x &= 12
\end{aligned}
$$

The width of the square window (x) is 12 inches, and the width of the wooden strips (y) is 3 inches.

Chapter 10:

Inverse Variations, Exponential Functions, and Other Functions

College Tuition

In 1980 the average annual cost for tuition and fees at two-year colleges were $350. Since then, the cost of tuition has increased an average of 9% annually.

1. Create a function rule that models the annual growth in tuition costs since 1980. Identify the variables, and describe the dependency relationship.
2. Determine the average annual cost of tuition for 2001. Justify your answer using tables and graphs.
3. Predict the cost of tuition for the year you plan to graduate from high school.
4. When will the average cost be double the 1980 cost?
5. When will the average cost reach $1000?

Teacher Notes

Scaffolding Questions:

- Identify the variables in this situation.
- Represent the annual growth factor as a decimal.
- What is the starting value?
- Describe how you might create a table to help you determine the rule for this situation.
- Describe how tuition amounts change with each additional year.
- Create a scatterplot of the data in your table and describe the graph.
- Determine the function rule that shows the relationship between the cost of tuition and the number of years since 1980.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

Sample Solution:

1. The cost of tuition depends on the number of years since 1980. Each year the tuition is the previous year's tuition plus nine percent of the previous year's tuition. This may be thought of as 100% + 9% of the previous year's tuition. This is equivalent to multiplying by 1 + .09 or 1.09. To find the cost after one year, 1.09 is multiplied with the starting amount ($350). To find the cost after 2 years, multiply 1.09 by itself and then multiply that result by the starting amount ($350).

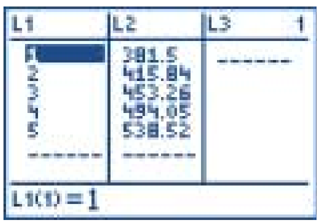
Number of Years Since 1980	Pattern	Tuition Cost
0	350	350
1	$350 \cdot 1.09^1$	381.50
2	$350 \cdot 1.09^2$	415.84
3	$350 \cdot 1.09^3$	453.26
4	$350 \cdot 1.09^4$	494.05
5	$350 \cdot 1.09^5$	538.52
n	$350 \cdot 1.09^n$	$350 \cdot 1.09^n$

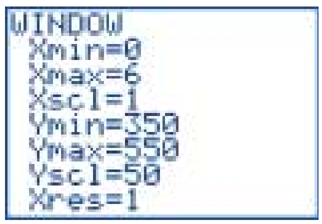
The function created by this situation is $T = 350 \cdot 1.09^n$.

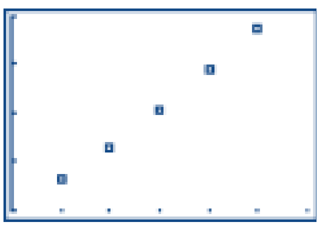
The n values represent the number of years since 1980. The T values represent the cost of tuition. The minimum for the range is $350.

There was no constant rate of change in the table. Therefore, the relationship is nonlinear.

2. The situation may be represented graphically and tabularly using a graphing calculator. The data may be plotted using the statistic feature on the calculator. Let *y* represent tuition cost and *x* represent the number of years.







The graph is an exponential curve showing growth. The starting amount is $350, and the growth factor is 1.09. The growth factor shows 100% of the initial cost plus 9% of the cost. To graph the function, let *y* represent the tuition cost and let *x* represent the number of years.

Use the table feature on the calculator to explore cost increases since 1980.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(A) uses patterns to generate the laws of exponents and applies them in problem-solving situations.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
3 Exponential Functions and Equations
 3.1 Exponential Relationships
 3.2 Exponential Growth and Decay
 3.3 Exponential Models

Connections to Algebra End-of-Course Exam:

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

Objective 8:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving one-variable or two-variable situations.

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

X	Y_1
18	1651
19	1799.6
20	1961.5
21	2138.1
22	2330.5
23	2540.3
24	2768.9

X=21

The function can also be used to find the cost in the year 2001. 21 years would be used for the value of *x* minus the number of years since 1980. The cost would be $\$350 \cdot 1.09^{21} = \$2138.08$.

3. Answers will vary.

4. To determine when the average cost will be doubled, find the table value that is least 2 times 350.

X	Y_1
5	538.52
6	586.99
7	639.81
8	697.4
9	760.16
10	828.58
11	903.15

X=8

The costs will be doubled by the ninth year.

5. Look on the table for *y* values that are at least 1000.

X	Y_1
9	760.16
10	828.58
11	903.15
12	984.43
13	1073
14	1169.6
15	1274.9

X=13

The cost will reach $1000 by the 13th year or by 1993.

Extension Questions:

- Describe the domain and range for this function.

The domain is the set of whole numbers. The domain represents the number of years since 1980. The years can only be represented in whole numbers because the increase is calculated on a yearly basis. The range represents the cost of the tuition. For this problem, the year 1980 is a starting point, and the tuition that year was $350. The cost will continue to increase at a rate of 9% per year for as long as the school is operating. The range for this problem is y greater than or equal to 350.

- How would the graph change if the annual cost increase was 12%? How would this affect costs?

The graph would be "skinnier" indicating a steeper rise per year. Cost of tuition would increase at a faster rate.

- Predict the annual cost of tuition in the year 2010 at the current growth rate.

$y = 350 \cdot 1.09^{30}$ *would equal approximately $4643.69 per year.*

Constructing Houses

Robert is part of a volunteer crew constructing houses for low-income families. It always takes 200 individual workdays to complete one house. For example, if a crew of 20 people can complete a house in 10 days, it has taken 20 times 10 individual workdays.

1. Working at the same rate, how long should it take a crew of 40 people to build the house?

2. Express the number of workdays as a function of the crew size. Define the variables, and explain how you created your function. What type of relationship is formed in the situation?

3. Write a verbal description of the effect of the crew size on the number of construction days.

4. About how long would it take a crew of 32 to complete a house?

5. If a crew can complete a house in 12.5 days, how big was the crew?

6. Express crew size as a function of the number of workdays. Compare the domain, range, and graph with your original function in question.

Teacher Notes

Scaffolding Questions:

- How long will it take one person to complete the home by him/herself?
- Explain how a crew of 20 can complete the home in 10 days.
- Identify the variables.
- Describe how you might create a table to help you determine the rule for this situation.
- How long will it take a crew of 2 to complete the house?
- How long for a crew of 4 to complete the house?

Sample Solution:

1. Use a table to determine how long it will take 40 people to complete the house.

Crew Size (x)	Construction Days (y)	Individual Workdays (t)
2	100	200
4	50	200
8	25	200
10	20	200
20	10	200
40	5	200

The number of construction days, *y*, can be found by dividing the total number of days by the value of *x*, the crew size. The product of the crew size and the construction days always equals 200 individual workdays.

By entering the values for the crew size, *x*, and construction days, *y*, into two lists on the graphing calculator, the points can be plotted, revealing a nonlinear graph.

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

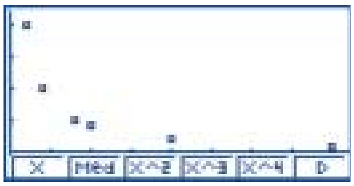
(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

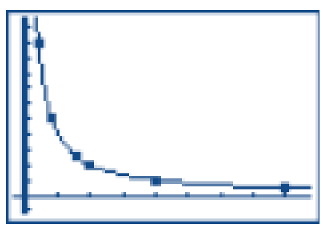
(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.



2. The function for this situation is $xy = 200$ or $y = \frac{200}{x}$, where x represents the crew size and y represents the number of construction days. The graph of this relationship is nonlinear.

3. As one quantity increases, the other decreases. The product of the quantities remains constant (200 total individual workdays) and forms an inverse variation. This constant product is called the constant of variation.



4. The table feature on the calculator allows exploration of the number of days it will take to build the house with various size crews.

It should take a crew of 40 people 5 days to build the house. The bigger the crew, the less time it will take to build the house. It will take 32 crew members about 6.25 days to build the house.

These values might also be verified on the home screen of the calculator by substituting into the function for each situation.

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(B) analyzes data and represents situations involving inverse variation using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations of Functions
1 Developing Mathematical Models
1.1 Variables and Functions

Connections to Algebra End-of-Course Exam:

Objective 9:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving probability, ratio and proportions, or graphical and tabular data.

5. If the number of days, y, is 12.5, the equation may be used to solve for x, the number of crew members.

$$x\,(12.5) = 200$$
$$x = \frac{200}{12.5} = 16$$

It will take 16 crew members to complete the work in 12.5 days.

6.

$$y = \frac{200}{x}$$
$$xy = 200$$
$$x = \frac{200}{y}$$

y is the independent variable, the number of workdays, and x is the dependent variable, the number of crew members.

The graph of the functions are the same as for $y = \frac{200}{x}$. However, for the problem situation, the x values must be whole numbers, since one may not use a portion of a person to build the houses. The y values will be determined by the whole number values. For example, for the original function $y = \frac{200}{x}$ the following values would be determined by the rule and the problem situation.

Crew Size	Number of Workdays
1	200
2	100
3	$200 \div 3 = 66.7$
4	50
5	40
6	$200 \div 6 = 33.3$
7	$200 \div 7 = 28.6$

Extension Questions:

- Describe how the values of *y* change as the values of *x* increase.

As the value of x increases, the value of y decreases.

- How do the values of the crew size and the construction days relate to the total number of workdays?

The product of the crew size and the number of construction days equals the total workdays, 200.

- Can your rule be written another (equivalent) way?

$xy = t$ or $y = t/x$

- Describe the graph of this relationship.

The graph is nonlinear. The graph is a firs-quadrant graph that begins at (1,200) and decreases to curve along the x-axis near (200,1).

- If the total number of construction workdays was 160, how would the equation have been written?

$xy = 160$ or $y = \frac{160}{x}$

Student Work

1. $20 \cdot 10 = 200$

$40x = 200$

$x = 5$

5 days

2. d = number of days

p = number of people

The amount of days multiplyed by the number of people always equals 200. This is a dependent function.

$dp = 200$

$d = \frac{200}{p}$

3. As the number of people increases the number of "days" decreases.

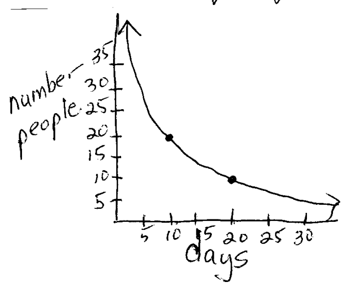
4. $32d = 200$

6.25 days

$32\overline{)200}$ $6\frac{1}{4}$

5. $12.5x = 200$

16 people



6. $p = \frac{200}{d}$

The domain, range, and graph are the same as the original function #2.

Exploring Exponential Functions

A rectangular sheet of notebook paper is folded in half. The fold divides the paper into 2 rectangles. The folded paper is then folded in half again. When it is opened, there are 4 rectangles formed by the folds. Take a sheet of notebook paper and repeat this process. Record your answers in the table similar to the one shown below. Continue folding until you cannot make another fold.

Number of Folds	Number of Rectangles
0	1
1	2
2	4
3	
4	
5	

1. Identify the variables for this relationship and describe the domain and range for this situation. Describe how the number of rectangles changes as the number of folds increase.

2. Express symbolically the relationship between the variables.

3. If your paper had 128 rectangles, how many folds would you have made? Explain your answer.

4. Describe the graph of your data.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

Scaffolding Questions:

- What is the relationship between the original number of rectangles and the number of rectangles after one fold?
- What is the relationship between the number of rectangles after one fold and the number of rectangles after two folds?
- What do you think the relationship will be between the number of rectangles after two folds and the number of rectangles after 3 folds?
- What pattern do you notice in the number of rectangles as the number of folds increase?
- What would you have to do to the original number of rectangles to get the second number of rectangles?
- How do the exponents relate to the values in your table?
- Suppose you could continue to fold the paper and extend your table to include 10 folds. How many rectangles would there be?

Sample Solution:

The paper was folded and the number of rectangles formed after each fold was recorded in the table. There was no constant change in the number of rectangles, therefore, the situation was not be linear. The values for the rectangles were all multiples of 2. The number of factors was the same as the number of folds, and a pattern with exponents emerged.

Successive ratios were constant.

Folds	Rectangles	Process
0	1	
1	2	$2 = 2^1$
2	4	$2 \cdot 2 = 2^2$
3	8	$2 \cdot 2 \cdot 2 = 2^3$
4	16	$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
5	32	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

Ratios
2/1 = 2
4/2 = 2
8/4 = 2
16/8 = 2
32/16 = 2

1. The *n*-values would represent the number of folds, and the *r*-values would represent the number of rectangles. The domain represents the number of folds, so n = 0, 1, 2, 3, 4, 5 …. There will be a limit to the number of

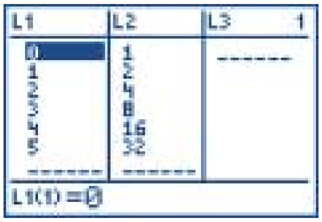
folds, depending on the type of paper. The range represents the number of rectangles formed by the folds. The range is r =1, 2, 4, 8, 16, 32….

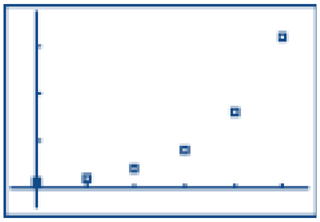
2. The pattern involves repeated multiplication by 2. You could show the pattern as powers of 2.

   $r = 2^n$

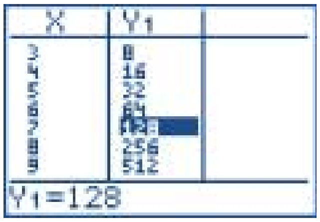
   As the value of n increases by 1, the value of r increases by a factor of 2.

3. In the graphing calculator let the y-values represent the number of rectangles. The x-values represent the number of folds.





The function rule and the table can help find the number of folds when the number of rectangles (y) was 128. It took 7 folds to get 128 rectangles.



(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(A) finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(A) uses patterns to generate the laws of exponents and applies them in problem-solving situations;

(C) analyzes data and represents situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.2 Identify More Patterns

III. Nonlinear Functions
3 Exponential Functions and Equations
3.1 Exponential Relationships

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

4. The graph is nonlinear because the rate of change is not constant. The rate of change of successive terms is a constant ratio of 2. The graph is in the first quadrant and curves upward. As the value of x increases, the value of y increases exponentially.

Extension Questions:

- Describe how the situation would be different if the paper had been folded into thirds each time instead of halves.

The multiplier would be 3. The equation would be $r = 3^n$.

- Suppose that you began with a square sheet of paper that was 2 feet on a side. What is the relationship between the number of folds and the area of the rectangle after each fold?

The area of the sheet of paper is 4 square feet. Each time the paper is folded the area is multiplied by one-half.

Fold Number	Area of Rectangle (ft^2)
0	4
1	$4\left(\frac{1}{2}\right)$
2	$4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
3	$4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
n	$4\left(\frac{1}{2}\right)^n$

The area, A, would be a function of the number of folds.

$$A = 4\left(\frac{1}{2}\right)^n$$

- Explain how the graph of this function, $A = 4\left(\frac{1}{2}\right)^n$, would be different from the graph of the original function, $A = 2^n$.

The graph of the function $A = 4\left(\frac{1}{2}\right)^n$ decreases as n gets larger. The starting value of the function is 4. The function, $A = 2^n$, increases as n increases. It has a starting value of 1.

Function Families

1. For each of the following sets of functions, compare the domains and ranges of the functions in that set. What representation best helps you see the domain and range of each function? Explain.

Set A:	Set B:	Set C:
$f(x) = 2x - 5$	$f(x) = \frac{1}{2}x^2$	$f(x) = 2^x$
$g(x) = 4$	$g(x) = -3x^2$	$g(x) = \left(\frac{1}{2}\right)^x$
$h(x) = 8 - \frac{1}{2}x$	$h(x) = 2x^2 + 5$	$h(x) = -3^x$

2. Describe the family of functions you see in each set and its parent function. Explain how each function is obtained from its parent function.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(B) determines the domain and range values for which linear functions make sense for given situations; and

(C) translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations.

Scaffolding Questions:

- What do the functions in each set have in common?
- How are the functions in each set different?
- What values, if any, make each function undefined?
- What would the graph of each function look like?
- What are the intercepts for each function?
- How are the functions from Set A to Set B to Set C different?

Sample Solution:

1. The domains and ranges of the functions in the problem sets could be seen by graphing the functions or by analyzing the expressions defining the functions.

   Set A:
   Since the functions are all linear, the domains are the set of all real numbers. There is no value of x that makes the function undefined. The ranges for functions f and h are also the set of all real numbers. If any real number is chosen as a function value, the resulting equation can be solved for the x-value that generates that function value. The function, g, however, is a constant function and has only one value in its range, 4. The function means "$g(x) = 4$ for any x-value you choose."

   Set B:
   These functions are all quadratic. Each domain is also the set of all real numbers because there is no x-value that makes the function undefined. The range for f is the set of all nonnegative real numbers, since squaring any real number and multiplying it by $\frac{1}{2}$ gives a nonnegative real number.

   x = any real number $\rightarrow x^2 \geq 0$, making $\frac{1}{2}x^2 \geq 0$.

   The range for g is the set of all nonpositive real numbers, since squaring any real number gives a nonnegative real number. Multiplying that by -3 gives a nonpositive real number.

   x = any real number $\rightarrow x^2 \geq 0$, making $-3x^2 \leq 0$.

The range for h is the set of all real numbers greater than or equal to 5, since $2x^2$ is nonnegative for any number x. Adding 5 to the equation gives a number greater than or equal to 5.

x = any real number $\rightarrow x^2 \geq 0$, $2x^2 \geq 0$, and $2x^2 + 5 \geq 5$.

Set C:
These functions are exponential, and, since any real number can be used as an exponent, the domain for each function is the set of all real numbers. The range for both functions f and g is the set of positive real numbers. However, the range for h is the set of negative real numbers, since using any real number x as an exponent of 3 gives a positive real number. Multiplying that by -1 gives a negative real number.

$-3^x = -1 \cdot 3^x$ and $3^x > 0 \rightarrow -1 \cdot 3^x < 0$.

The representation that best shows the domain and the range of each function is the graph of the function, because the graph gives a complete picture of input and output, intercepts, minimum or maximum points, where the function increases or decreases, and where it has minimum or maximum values.

2. *Set A:*
Since all the functions in this set are linear, the parent function is $y = x$.

- To obtain $f(x) = 2x - 5$ the parent function is vertically stretched by 2 and translated down 5 units. The slope of this line is positive 2; therefore, the line goes from quadrant III to quadrant I.

- $g(x) = 4$ is a horizontal line through $y = 4$.

- $h(x) = 8 - \frac{1}{2}x$ is obtained by vertically compressing the parent function by $\frac{1}{2}$ and translating up 8 units. The slope of this line is negative $\frac{1}{2}$; therefore, the line goes from quadrant II to quadrant IV.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5:
The student will demonstrate an understanding of the quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
  1.1 The Linear Parent Function
  1.2 The Y-Intercept

II. Nonlinear Functions
1 Quadratic Functions
  1.2 Transformations

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Set B:
All of the functions in this set are quadratic, so the parent function is $y = x^2$.

- $f(x) = \frac{1}{2}x^2$ is formed by vertically compressing the parent function by $\frac{1}{2}$.

- $g(x) = -3x^2$ is obtained by vertically stretching the parent function by 3 and then reflecting it over the x-axis.

- $h(x) = 2x^2 + 5$ is made by vertically stretching the parent function by 2 and translating up 5 units.

Set C:
The functions in this set are exponential of the form $f(x) = a^x$. There is not a parent function for this set of functions. Each of these functions could be called parent functions.

Extension Questions:

- How will knowing the parent function help you determine the domains and ranges of the functions in each set?

By knowing what the parent function looks like, we can see its domain and range. We can then analyze the transformed functions in each set to see how the domain and range might need to change. In other words, we can analyze the parameter changes and the effects of those changes on the domain and/or range.

- Consider the parent function for Set A and the point (2,2) on its graph. How does $f(x) = 2x - 5$ transform this point?

The point (2,2) is transformed to the point (2,-1) because 2x "stretches" (2,2) to (2,4) and subtracting 5 translates (2,4) down to (2,-1).

- In Set A, if we relabel $f(x) = 2x - 5$ as $y = 2x - 5$, how can we show symbolically that this function generates every real number as a y-value?

We need to show that for any y-value we choose, we can find an x-value that generates it. To see what that x-value should be, we can solve $y = 2x - 5$ for x:

$$\begin{aligned} y &= 2x - 5 \\ y + 5 &= 2x \\ \frac{y+5}{2} &= x \end{aligned}$$

No matter what value we choose for y, we can find the value for x that will generate it. We can also see that the expression $\frac{y+5}{2} = x$ is always defined, no matter what y is.

- In Set B, how does the coefficient of x^2 affect the parent function?

The coefficicient of x^2 multiplies the y values of the function. The graph is stretched if the absolute value of the multiplier is greater than 1. If the absolute value of the multiplier is less than one, the graph is compressed.

- For Set C, for $f(x) = 2^x$, how would the graph of $g(x) = \frac{1}{2}2^x$ compare to $f(x)$?

The values of y would be one-half of the values of f(x).

Mathematical Domains and Ranges of Nonlinear Functions

I. For the following problems:

A. Sketch a complete graph for the given function. Show the coordinates of any intercepts.

B. Describe the domain and range for each mathematical situation.

Function	Graph or Table	Domain and Range
1. $f(x) = \frac{1}{2}x^2$		Domain: Range:
2. $y = x^2 + 3$		Domain: Range:
3. $y = -3x^2$		Domain: Range:
4. $y = x(5 - x)$		Domain: Range:

Function	Graph or Table	Domain and Range
5. $h(x) = 3^x$		Domain: Range:
6. $m(x) = \left(\frac{1}{3}\right)^x$		Domain: Range:
7. $g(x) = \frac{4}{x}$		Domain: Range:

II. Write a summary comparing the functions. Compare their domains and ranges and their graphs.

III. Describe a practical situation that the functions in #4, #5, #6 might represent. What restrictions will the situation make on the mathematical domain and range of the function? How will the situation affect the graph of the mathematical function?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations;

(C) interprets situations in terms of given graphs or creates situations that fit given graphs.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

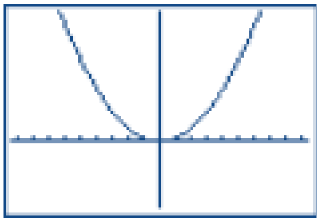
The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations.

Scaffolding Questions:

- What type of function relates the variables?
- What is the dependent variable?
- What is the independent variable?
- What are the constants in the function? What do they mean?
- What restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?

Sample Solution:

I. 1.



The *y*-intercept and the *x*-intercept for this function are the same, (0,0).

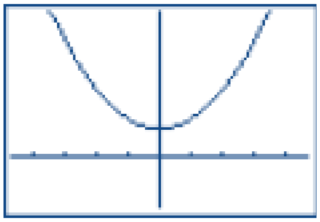
The domain for this function is the set of all real numbers, since any value for *x* can be squared and multiplied by $\frac{1}{2}$.

The range for this function is the set of all nonnegative numbers, which is the result of squaring any number and multiplying by $\frac{1}{2}$.

$x^2 \geq 0$

$\frac{1}{2}x^2 \geq 0$

I. 2.



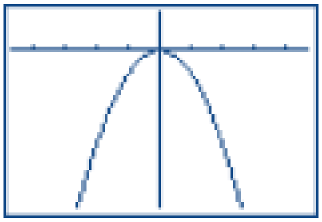
The y-intercept for this function is (0,3). There is no x-intercept because the graph does not intersect the x-axis.

The domain for this function is the set of all real numbers, since any value for x can be squared and increased by 3.

The range for this function is the set of all numbers greater than or equal to 3, since

$x^2 \geq 0$
$x^2 + 3 \geq 3 \geq 0$

I. 3.



The y-intercept and the x-intercept for this function are the same, (0,0).

The domain for this function is the set of all real numbers, since any value for x can be squared and multiplied by -3.

The range for this function is the set of all nonpositive numbers, which is the result of squaring any number and multiplying by -3.

$x^2 \geq 0$
$-3x^2 \leq 0$

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
 2.2 Identify More Patterns

III. Nonlinear Functions
1 Quadratic Functions
 1.1 Quadratic Relationships
2 Quadratic Equations
 2.1 Connections
 2.2 The Quadratic Formula
3 Exponential Functions and Equations
 3.1 Exponential Relationships
 3.2 Exponential Growth and Decay
 3.3 Exponential Models

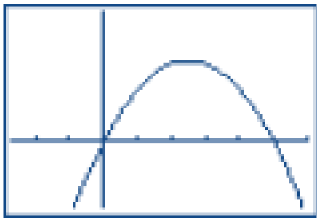
Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

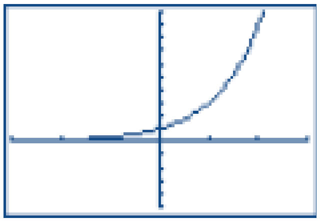
I. 4.



The y-intercept for this function is (0,0). The graphs cross the x-axis at two points so that the x-intercepts are (0,0) and (5,0). The domain for this function is the set of all real numbers, since the expression $x(5 - x)$ is always defined for any value of x.

To determine the range, trace along the graph to find the largest y value. It occurs at the point (2.5,6.25). The range for this function is the set of all numbers less than or equal to 6.25. This maximum y-value occurs when at $x = 2.5$, which is the average of the two x-intercept values, 0 and 5.

I. 5.

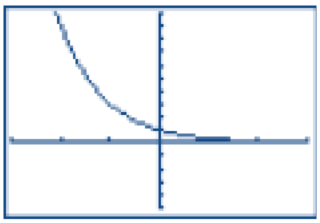


The y-intercept for this function is (0,1) because the value of the function at 0 is 1. There are no x-intercepts, since no power of 3 equals zero. The graph does not cross the x-axis.

The domain for this function is the set of all real numbers, since any real number can be used as an exponent on 3.

The range for this function is the set of all positive real numbers, since powers of 3 are always positive. Negative powers of 3 give y values between 0 and 1, and positive powers of 3 give values greater than 1.

I. 6.

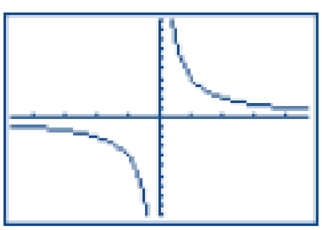


This function is the reflection of the graph in problem 5 about the *y*-axis. The *y*-intercept for this function is still (0,1). There are no *x*-intercepts, since no power of one equals zero.

The domain for this function is the set of all real numbers, since any real number can be used as an exponent.

The range for this function is the set of all positive real numbers, since powers of a positive number are always positive. Negative powers of a positive number give values more than 1, and positive powers of a negative number give values between 0 and 1.

1. 7.



The graph does not cross either axis. This function has no *y*-intercept, since division by 0 is undefined. There is no *x*-intercept, since $0 = \frac{4}{x}$ does not have a solution. The domain for this function is the set of all real numbers except 0.

The range for this function is the set of all real numbers except 0.

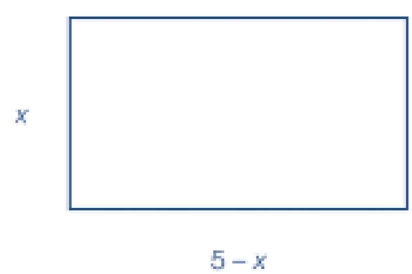
II. Comparison of the functions:

The first four of these functions are quadratic functions. They all have the same domain, the set of all real numbers. Their ranges vary because the expression ax^2 is nonnegative if $a > 0$ and is nonpositive if $a < 0$. The expression $x^2 + c$ is greater than or equal to c for any c.

The functions in #5 and #6 are exponential functions. They have the same domain and the same range.

The function in #7 is an inverse variation function with both domain and range being the set of all real numbers except zero.

III. A practical situation that $y = x(5 - x)$ could represent is the area of a rectangle.



The domain would be all numbers between 0 and 5 because the numbers less than 0 or greater than 5 result in a negative product, and the area may not be negative. The range would be the numbers between 0 and the maximum value 6.25. 6.25 would be included in this range. The graph would be that part of the parabola above the x-axis.

A practical situation that $h(x) = 3^x$ could represent is the number of rectangles formed by folding a rectangular sheet of paper repeatedly into thirds. The domain would be the number of folds made, which must be the set of whole numbers {0,1,2,3,4….}. The range would be the number of rectangles formed by the folds. The number of rectangles is a power of 3, {1,3,9,27,….}. The graph would be only those points on the original graph with nonnegative integer coordinates.

For a practical situation that might represent $m(x) = \left(\frac{1}{3}\right)^x$ consider the area of the original sheet of paper as one square unit; the function

$m(x) = \left(\frac{1}{3}\right)^x$ would represent the area of each rectangle resulting from the folds. The domain is the set of the possible number of folds {0,1,2,3,4…}, and the range is $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$. The graph would be the points on the original graph where x equals a nonnegative integer.

Extension Questions:

- Compare the range of $f(x) = \frac{1}{2}x^2$ and $f(x) = \frac{1}{2}x^2 + 3$.

The range of the first function is the set of all numbers greater than or equal to 0. If three is added to all values of the function, the range will be the set of all numbers greater than or equal to 3.

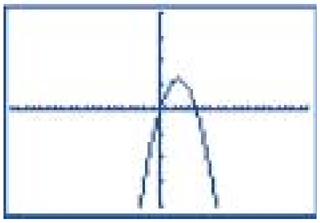
- Compare the range of $f(x) = \frac{1}{2}(x - 3)^2$ to the range of $f(x) = \frac{1}{2}x^2$.

The function will be shifted 3 units to the left. The vertex will change, but the range values will still be all the numbers greater than or equal to 0.

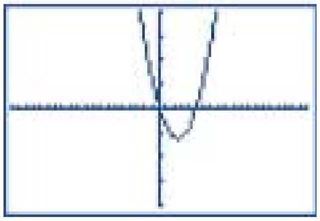
- If the function $y = x(5 - x)$ is multiplied by -1, how will the domain and range be affected?

The domain of y = x(5 – x) and y = -1x(5 – x) will both be the set of all real numbers because these products are defined for any x. However, the ranges will be different. The range of the first function is y < 6.25. The range of the second function will be y > -6.25 because all values of the function are multiplied by -1.

$y = x(5 - x)$

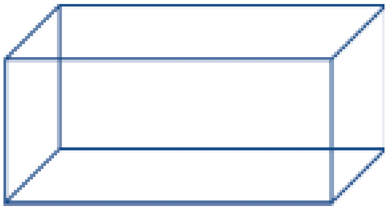


$y = -1x(5 - x)$



Paper Boxes

Marcy used a sheet of 8-inch by 11-inch paper to make an open box. She cut x by x squares out of each corner, and folded up the sides. The diagram below shows the finished box. Simplify your expressions and justify your answers to each of the questions.



1. Draw a diagram of the sheet of paper showing the fold lines needed to make the box. Label your diagram with the dimensions of the cut out pieces and the lengths of the fold lines. Use your diagram to help you find the dimensions of the open box.

2. Marcy decides to put a ribbon around the bottom edge of the box. She will need to determine the perimeter of the base. Write a polynomial to represent the perimeter of the base of the box, simplify the expression, and explain how you determined your answer.

3. Write a polynomial to represent the area of the base of the box. Explain how you found the area of the base.

4. Write a polynomial to represent the volume of the box. Justify your answer.

5. Suppose the length of the box is increased by 3 units. How will this affect the area of the base?

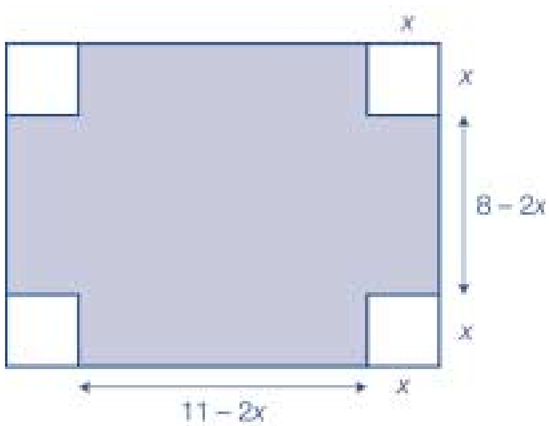
Teacher Notes

Scaffolding Questions:

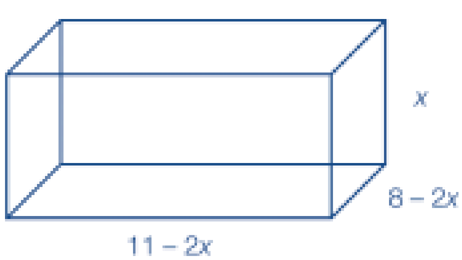
- Identify the dimensions of the base.
- How can you find the perimeter of the base?
- What would you do to find the area of the base?
- What is the formula for the volume of a rectangular prism?

Sample Solution:

1. The sheet of paper is 8 inches by 11 inches. The length of the side of the square that will be cut from each corner may vary. Let the length of the side of the square in inches be represented by x. The length of each side will be decreased by $2x$. The length of the fold lines will be $8 - 2x$ and $11 - 2x$.



The height of the open box will be the same as the length of the side of the square that was cut out.



Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

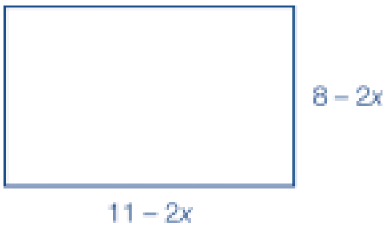
(E) interprets and makes inferences from functional relationships.

(b.3) Foundations for functions.
The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student:
(A) uses symbols to represent unknowns and variables; and

(B) given situations, looks for patterns and represents generalizations algebraically.

2. The perimeter is the distance around a figure. The base of the box is a rectangle having dimensions of $11 - 2x$ and $8 - 2x$. Opposite sides of a rectangle are congruent, so there are two sides with length $11 - 2x$ and two sides with length $8 - 2x$.



Using the formula for finding the perimeter [P = 2(length) + 2(width)], the perimeter can be calculated as follows:

$P = 2(11 - 2x) + 2(8 - 2x)$
$P = 22 - 4x + 16 - 4x$
$P = 38 - 8x$

3. The base of the box is a rectangle. The area can be found using the formula for the area of a rectangle (A = length times width).

$A = (11 - 2x)(8 - 2x)$
$A = 88 - 22x - 16x + 4x^2$
$A = 88 - 38x + 4x^2$
$A = 4x^2 - 38x + 88$

4. A polynomial that represents the volume of the original box would be represented by the formula V = (Length)(Width)(Height).

The value of the length times the width was calculated when the area of the base was determined. That area was $4x^2 - 38x + 88$. The height of the box is represented by x. The volume of the original box is:

V = (Length)(Width)(Height)
$V = (4x^2 - 38x + 88)(x)$
$V = 4x^3 - 38x^2 + 88x$

(b.4) Foundations for functions.
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student:
(B) uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Texas Assessment of Knowledge and Skills:

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.2 Identifying Patterns
2.3 Identifying More Patterns

Connections to Algebra End-of-Course Exam:

Objective 6:
The student will perform operations on and factor polynomials that describe real-world and mathematical situations.

5. If the length of the base is increased by 3 units, the box will have dimensions with a width of $8 - 2x$ and the length of $14 - 2x$. Using the formula A = length times width, the area of the base with an increased length of 3 units can be represented by:

$A = (14 - 2x)(8 - 2x)$
$A = 112 - 28x - 16x + 4x^2$
$A = 112 - 44x + 4x^2$
$A = 4x^2 - 44x + 112$

Extension Questions:

- Which box would hold more, a box that had an x value of 2 or a box that had an x value of 3? Justify your solution.

To find the solution, substitute the values of 2 and 3 into the volume formula to find the larger volume. If $x = 2$, the volume of the box will be:

V = (Length)(Width)(Height)
$V = (4x^2 - 38x + 88)(x)$
$V = 4(2)^3 - 38(2)^2 + 88(2)$
$V = 56$ in.^{3}

If $x = 3$, the volume of the box will be:

V = (Length)(Width)(Height)
$V = (4x^2 - 38x + 88)(x)$
$V = 4(3)^3 - 38(3)^2 + 88(3)$
$V = 30$ in.^{3}

Therefore, the smaller x-value of 2 yields the larger volume.

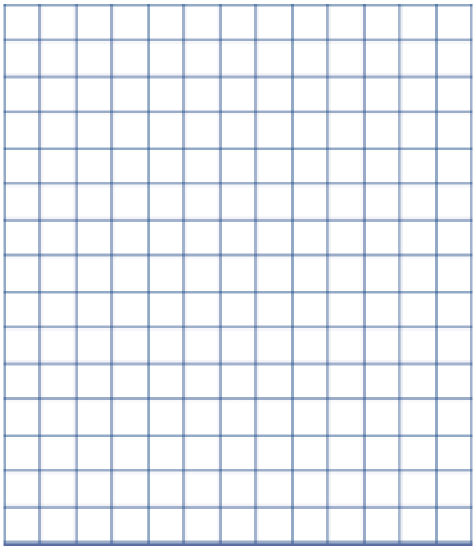
- What is the largest value of *x* that fits this situation? The smallest?

The sides of the piece of paper are 8 inches and 11 inches. If a square of side length x is cut out, the value of x must be less than 4 inches. The 8-inch side has two squares removed when the box is formed. If the sides of the squares are each 4 inches, it will cause the dimension of that side of the box to be zero. Therefore, the value for the side of the square, represented by x, must be greater than zero, and less than 4 inches.

Rebound Height

Three teams of students independently conducted experiments to relate the rebound height of a ball to the rebound number. The table below left gives the average of the three teams' results.

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44



1. Construct a scatterplot of the data. Describe the functional relationship between the rebound height of the ball and the rebound number verbally and symbolically.

2. Predict the rebound height of the ball on its 10th rebound.

3. Suppose the ball stops rebounding and begins to roll across the floor when it reaches a rebound height of 3 centimeter. How many times has the ball rebounded?

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(B) gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

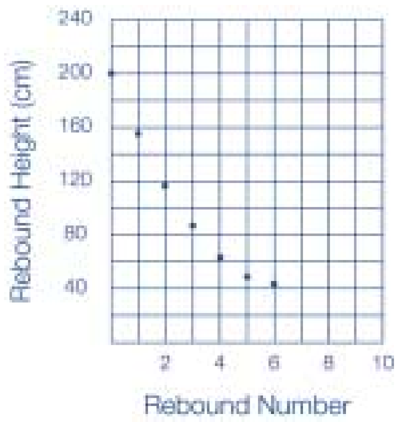
The student:
(D) in solving problems, collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

Scaffolding Questions:

- How will you organize the data that is collected?
- What will you need to consider to construct a scatterplot of the data?
- For the graph what will be the dependent variable? The independent variable?
- What will you need to consider to determine a reasonable interval of values and scale for each of the axes?
- What function type (linear, quadratic, exponential, inverse variation) appears to best represent your scatterplot?
- What do you need to know to determine a particular function model for your scatterplot?

Sample Solution:

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44



1. The plot is clearly nonlinear and suggests that each rebound height is a fraction of the previous rebound height. The table also supports this statement, since the rate of change from one bounce to the next is not constant. The ratios of rebound height to previous rebound height are:

$$\frac{155}{200} = 0.775 \qquad \frac{66}{88} = 0.750$$

$$\frac{116}{155} = 0.748 \qquad \frac{50}{66} = 0.758$$

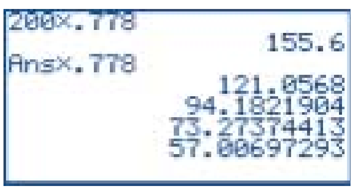
$$\frac{88}{116} = 0.759 \qquad \frac{44}{50} = 0.880$$

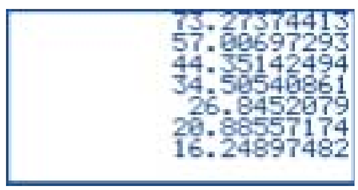
The average of these ratios is 0.778. The value at $x = 0$ is 200. A function for exponential growth is equal to (starting value) times $(\text{ratio})^x$. A function that could model this situation is $h(x) = 200(0.778)^x$, where x is the rebound number and $h(x)$ is height in centimeters.

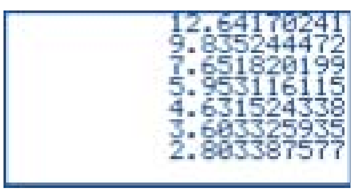
2. To predict the rebound height of the ball on its 10th rebound, substitute 10 for x in the function and evaluate: $h(10) = 200(0.778)^{10} = 16.249$.

   On the 10th rebound the ball will bounce to a height of 16.249 centimeters.

3. We need to solve the equation $200(0.778)^x = 3$. The calculator may be used to compute the values.







The value for the 16th bounce is 3.603. The value for the 17th bounce is 2.803.

This can also be done by entering the function and using the graph or a table.

(d.3) Quadratic and other non linear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(C) analyzes data and represents situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Connections to Algebra I: 2000 and Beyond Institute:

III. Nonlinear Functions
3 Exponential Relationships
3.2 Exponential Growth and Decay
3.3 Exponential Models

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

On the 17th bounce, the ball reaches a height of about 3 centimeters and rolls across the floor.

Extension Questions:

- What do the data and scatterplot suggest about the functional relationship between rebound height and rebound number?

Since the rebound numbers increase by one, the rate of change between rebounds is simply the change in rebound height. Since these are not constant, the function cannot be linear. This is also obvious by looking at the scatterplot. The scatterplot appears to be quadratic or exponentially decreasing. The ratios of consecutive rebound heights are nearly constant, suggesting the functional relationship is exponential.

- How will the scatterplot change if the original height for rebound number 0 is increased? How will this change your function?

The y-intercept will be higher, and so will each rebound height. The initial value of 200 in the function will change to the new "original height."

- Suppose the height of 200 was the height of the ball on the first rebound. How can you use your table, scatterplot, or function to predict the height before this rebound?

You could divide 200 by 0.778 to get a height of about 257. This is equivalent to evaluating the expression $h(-1) = 200(0.778)^{-1} = 257.07$.

- If the general function rule $h(x) = Ha^x$ is given to represent the rebounding ball situation, what could H, a, and x represent?

H would represent the initial height, a the fraction of the previous height by which it bounces each time, and x is the bounce number.

What is Reasonable?

For each of the following situations, determine a function that represents it. Describe the mathematical domain and range of the function and a reasonable domain and range for the situation.

A. A rectangular garden plot is to be enclosed with 40 meters of fencing. The area of the garden is a function of the dimensions of the rectangle.

B. You fold a 12-inch square piece of paper repeatedly in half. The number of rectangles formed by the folds is a function of the number of folds you make.

C. The time it takes to proofread a certain book varies inversely with the number of people assigned to the proofreading task. Suppose 5 people proofread the book in 30 hours.

Teacher Notes

Materials:
One graphing calculator per student.

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(b.2) Foundations for functions.
The student uses the properties and attributes of functions.

The student:
(B) for a variety of situations, identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(d.1) Quadratic and other nonlinear functions.
The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student:
(A) determines the domain and range values for which quadratic functions make sense for given situations.

Scaffolding Questions:

For each situation,

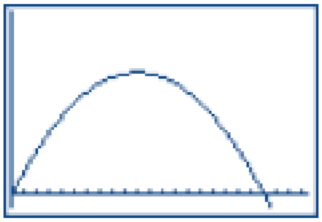
- What are the constants?
- What is the dependent variable?
- What type of function (linear, quadratic, exponential, inverse variation) relates the variables?
- What restrictions does the function place on the independent variable?
- Should you use all real numbers for the domain? Why or why not?
- What representation would best help you see the domain and range?

Sample Solution:

A. Since there are 40 meters of fencing for the perimeter of the garden, there are 20 meters of fencing for the semi-perimeter (half-way around the rectangle). The dimensions of the garden may be represented by x and $20 - x$, and the area of the garden is the product of the length and the width. The area may be expressed as a function of the width.

$A(x) = x(20 - x)$

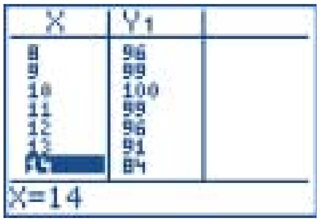
The mathematical domain for this function is all real numbers since no value for x makes the expression $x(20 - x)$ undefined. The mathematical range for the function is all real numbers less than or equal to 100.

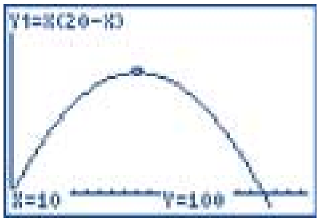


The graph shows that x can be any number (negative, zero, or positive) and that the range includes all values for y less than or equal to 100.

However, the domain of the situation is the set of all numbers between 0 and 20, because the x value must be less than 20 and greater than 0 to give a positive area. For the garden to exist, the length of a side, x, must be greater than 0 but less than half the amount of fencing, 40 meters, available for the whole garden.

The range can be determined by examining a table or a graph. The greatest value of *y* is 100.





The range for the function is the set of all numbers less than or equal to 100. For the problem situation the area must be positive and less than or equal to the maximum possible area, 100 square meters.

B. To determine the function, experiment with a piece of paper and record the information in a table.

Number of Folds	Number of Rectangles
0	1
1	2
2	4
3	8

Each time the paper is folded the number of rectangles is doubled or multiplied by 2. The number of rectangles is a power of 2.

Number of Folds	Number of Rectangles
0	2^0
1	2^1
2	2^2
3	2^3
4	2^4

(d.3) Quadratic and other nonlinear functions.
The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student:
(A) uses patterns to generate the laws of exponents and applies them in problem-solving situations;

(B) analyzes data and represents situations involving inverse variation using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 5:
The student will demonstrate an understanding of quadratic and other nonlinear functions.

Connections to Algebra I: 2000 and Beyond Institute:

I. Foundations for Functions
2 Using Patterns to Identify Relationships
2.1 Identifying Patterns

II. Nonlinear Functions
1 Quadratic Functions
1.1 Quadratic Relationships
3 Exponential Functions and Equations
3.1 Exponential Relationships
3.2 Exponential Growth and Decay
3.3 Exponential Models

The function is $r = 2^n$, where n = the number of folds made and r = the number of non-overlapping rectangles formed.

The mathematical domain for this function is the set of all real numbers since no value of n makes the function undefined. The range is the set of all positive real numbers since no power of 2 gives 0 or a negative value.

The domain of the situation is $\{0,1,2,3,...,n\}$ where n is the maximum number of folds you can make. This would depend on the dimensions of the piece of paper and its thickness.

The resulting range of the situation is $\{1,2,4,8,...,2^n\}$.

C. If two quantities vary inversely, their product is a constant. In this case the product is 5 times 30 or 150. $nh = 150$ or $h = \frac{150}{n}$, where n is the number of people proofreading the book, and h = the number of hours to proofread the book.

The mathematical domain and range are both the set of all real numbers except zero. If $n = 0$, then h would be undefined, and $h = \frac{150}{n}$ will never equal 0. The quotient of 150 and a number will never be zero.

The domain and range for the situation are best described in a table. The number of people proofreading the book must be positive integer values.

Number of People proofreading	Number of Hours to proofread
1	150
2	75
3	50
4	37.5
5	30
...	...
149	1.01
150	1

Connections to Algebra End-of-Course Exam:

Objective 1:
The student will demonstrate an understanding of the characteristics of graphing in problems involving real-world and mathematical situations.

Objective 5:
The student will formulate or solve quadratic equations that describe real-world and mathematical situations.

Objective 7:
The student will use problem-solving strategies to analyze, solve, and/or justify solutions to real-world and mathematical problems involving exponents, quadratic situations, or right triangles.

Extension Questions:

- In Situation A, describe how the domain and range would change if you change the amount of fencing you use to enclose the garden.

If you decrease the amount of fencing, both domain and range will decrease since both the dimensions and area of the garden will decrease. If you increase the amount of fencing, both the domain and range will increase since the dimensions and area of the garden will increase.

- In Situation B, determine the function that would relate the area of each of the rectangles formed in the folding process to the number of folds. Describe the domain and range of this function. Compare the area function with the "Number of Rectangles" function.

The initial area is 12^2 *or 144 square inches. Each fold produces a new rectangle that is half as large as the previous rectangle. The function is*

$$A = 144\left(\frac{1}{2}\right)^n$$

The domain is the set {0,1,2,...,n} where n is the maximum number of folds you can make.

The range is $\{144, 72, 36, 18, ..., 144\left(\frac{1}{2}\right)^n\}$.

The area function is a decreasing exponential function, while the Number of Rectangles function is an increasing exponential function. Both functions have finite domains and ranges.

- In Situation C, suppose 6 people complete the proofreading task in 30 hours and that the time to complete the task must be measured in whole hours. How will this change your function and the domain and range for the situation?

The function becomes $h = \frac{180}{n}$ *since the task now requires 6 times 30 or 180 people-hours to complete. The domain and range now consist of factor pairs of 180, since we are measuring by both whole people and whole hours. There are 18 factor pairs for the new situation.*

Number of People	Number of Hours
1	180
2	90
3	60
4	45
5	36
6	30
9	20
10	18
12	15

Number of People	Number of Hours
15	12
18	10
20	9
30	6
36	5
45	4
60	3
90	2
180	1

- Describe, in general, how changing the number of people affects the time to complete the proofreading task.

As the number of people increases, the time to complete the task decreases, and this occurs at a nonlinear rate. This can be seen by building a table where x, the number of people, increases by a constant amount, and comparing the corresponding change in the time y, to complete the task.

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Texas Version

Algebra I Assessments