Student Activity: Investigate Completing the Square

Overview: Students investigate completing the square with algebra tiles.

Objective: **Algebra I TEKS**
(d.2.A) The student solves quadratic equations using concrete models, tables, graphs, and algebraic methods.

Terms: complete the square

Materials: algebra tiles, graphing calculator

Procedures: Have students work through Exercises 1 – 4 in groups. As a whole group, discuss their answers using the following.

Note: This activity assumes that students have prior experience with representing, adding and subtracting polynomials with algebra tiles (area model), with using algebra tiles to model monomial and binomial multiplication and with modeling factoring trinomials with algebra tiles.
Note: This activity uses a concrete model to lay the foundation for the algebraic work of completing the square that students will do in Algebra II.

1. a. You need 9 unit tiles to complete the square.
   b. The dimensions of the completed square are $(x+3)$ by $(x+3)$.
   c. $x^2+6x+9=(x+3)^2$
2. a. You need 16 unit tiles to complete the square.
   b. The dimensions of the completed square are $(x-4)$ by $(x-4)$.
   c. $x^2-8x+16=(x-4)^2$
3. The number of unit tiles needed to complete the square is the square of half of the coefficient of x. For x^2+Bx, the number of unit tiles needed is $\left(\frac{B}{2}\right)^2$.
4. a. $x^2-3x+\frac{9}{4}=\left(x-\frac{3}{2}\right)^2$
   b. $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x-\frac{b}{2}\right)^2$

Do the following 2 examples with students.
The first example is to write the equation, $y=x^2+4x+5$ in vertex form.
Complete the square.

$y=x^2+4x+4-4+5$

$y=\left(x^2+4x+4\right)-4+5$

$y=(x+2)^2+1$

Have students quickly sketch a graph.

The second example is to solve a quadratic equation, $0 = x^2 - 10x + 19$, by completing the square.
Complete the square:

$0 = x^2 - 10x + 19$
$0 = x^2 - 10x + 25 - 25 + 19$
$0 = (x^2 - 10x + 25) - 25 + 19$
$0 = (x-5)^2 - 6$
$6 = (x-5)^2$
$\pm\sqrt{6} = x - 5$
$x = 5 \pm \sqrt{6}$

Have students complete the rest of the Activity.

5. $y = x^2 + 6x + 4$
$y = x^2 + 6x + 9 - 9 + 4$
$y = (x^2 + 6x + 9) - 9 + 4$
$y = (x+3)^2 - 5$

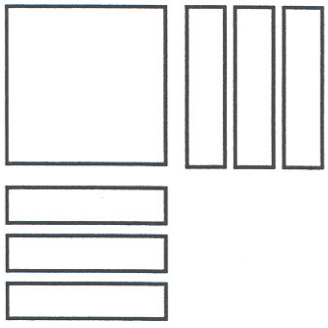
6. $3(x+1)^2 - 6 = 0$
$3(x+1)^2 = 6$
$(x+1)^2 = 2$
$x + 1 = \pm\sqrt{2}$
$x = -1 \pm \sqrt{2}$

7. $x^2 - 4x - 8 = 0$
$x^2 - 4x + 4 - 4 - 8 = 0$
$(x^2 - 4x + 4) - 4 - 8 = 0$
$(x-2)^2 - 12 = 0$
$(x-2)^2 = 12$
$x - 2 = \pm\sqrt{12}$
$x - 2 = \pm 2\sqrt{3}$
$x = 2 \pm 2\sqrt{3}$

Summary: Using algebra tiles to complete the square based on the area model of multiplication gives students a geometric approach to understanding the algebraic steps to complete the square.

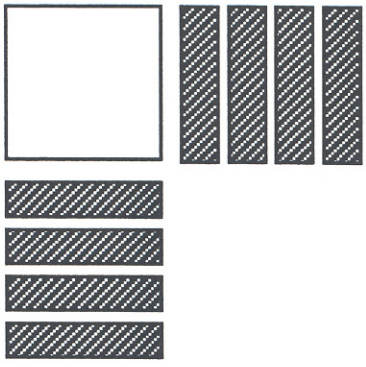
Student Activity: Investigate Completing the Square

1. Create a partial square with algebra tiles to represent $x^2 + 6x$ as shown.



   a. How many unit tiles do you need to complete the square?
   b. What are the dimensions of the completed square?
   c. $x^2 + 6x + ? = (x + ?)^2$

2. Create a partial square with algebra tiles to represent $x^2 - 8x$ as shown.



   a. How many unit tiles do you need to complete the square?
   b. What are the dimensions of the completed square?
   c. $x^2 - 8x + ? = (x - ?)^2$

3. How does the number of unit tiles to complete the square compare to each respective coefficient of x?

4. Based on the above, complete the two square diagrams.
   a. $x^2 - 3x + ? = (x - ?)^2$ b. $x^2 + bx + ? = (x - ?)^2$

x^2	$-1.5x$
$-1.5x$	?

x^2	$\frac{b}{2}x$
$\frac{b}{2}x$	?

5. Write the function in vertex form by completing the square $y = x^2 + 6x + 4$. Then sketch a graph.

6. Solve the quadratic equation written in vertex form. $3(x+1)^2 - 6 = 0$

7. Solve the equation by completing the square. $x^2 - 4x - 8 = 0$