1.2 Graphs and Trends

Overview: Data in two variables may or may not show a trend. If there is a trend, then we can use graphical models of the data to make predictions. We will work to develop an intuitive sense for reading graphs and finding different trends

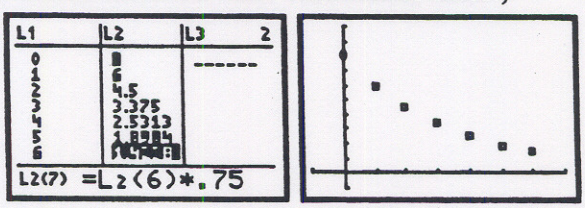
Objective: **Mathematical Models with Applications TEKS:**
2A, 2C, 3C

Terms: Increasing, decreasing, rate of change, periodic, cyclic, logistic, exponential, linear, and quadratic

Materials: 2 transparencies for Activity 1, basketball, graphing calculator, and Internet access for Reflect and Apply

Procedures: Begin with the demonstration described below:

Hold up a basketball that is inflated to regulation. A basketball that is inflated to regulation has 75% rebound height. (Referees use the following rule: When you drop the ball from your highest reach, it should hit your elbow when your arm is raised square on the return.) Enter the bounce number versus the height for a regulation basketball dropped from a height of 8 feet in your calculator. Use mental math to do the first few, and then use the calculator to figure the rest. Graph the scatter plot. (This is a demonstration and will introduce scatter plots. We will set up scatter plots and have participants create them on their calculators later.)



- What kind of relation is this? [Repeated multiplication gives us an exponential relation.]

Activity 1: Interpreting Graphs

In many of the following, the data may suggest one or more models: Linear trend analysis often works well over the short haul but may not always be reliable. In general, avoid suggesting a "correct" model; rather, let participants discuss the merits of different models, justifying their choices.

1. Height vs. Age:
   - Ask participants to label the points with the name of the person represented.

Have a few participants explain their reasoning for their choices. Point out that just as this graph is "weird" to read because height is along the x-axis, many graphs are as strange to students until they have practice reading graphs. This is especially true when the "look" of the graph does not match the appearance of the data. For instance, the tall person's point on the graph is far to the right, not at the top of the graph.

- Have participants graph the data with the axes reversed.

Have a participant present his/her answers. Use a second transparency of the activity to show the reflection of the points over the line $y = x$. Discuss the idea of inverse relations briefly.

2. Certificates of Deposit: Look over the CD graph.
   - What is the general trend of the graph? [Both CD's go up and down and seem to do so together.]
   - Use the graph to estimate the rate for a six-month CD in 1998. [Answers will vary. While a linear model may work here, interest rates can change quickly and may not always work well.]
   - Use the graph to estimate the rate for a five-year CD in 1984. [Answers will vary.]
   - When did a six-month and a five-year CD offer the same rate? [1989]
   - When did a six-month CD offer a rate of about 5%? [early and late 1993]
3. A Ball Bouncing: Study and discuss the graph.
   - What is the general trend of the graph?
   - Where is the ball at one second? [5 feet]
   - How many times did the ball hit the ground in the first second? The second second? The third second? The forth second? [once; twice; once; twice]

   Take a ball and have participants simulate the bouncing ball. Try having one person clap every second while another "bounces" the ball the appropriate number of times.
   - What is the model for each bounce? [quadratic]
   - Is each bounce half as high as the one before? [No, especially if it was a regulation basketball. Remember the 75% rebound rule.]
   - What do you think the relationship is between the bounce number and the maximum height of each bounce? [It is a decreasing exponential.]
4. Orbit Time for the Moons of Saturn: Study the graph and discuss its general trend.
   - Estimate the orbiting time if the radius of the orbit of a moon is 500,000 km. [about four and one-fourth days]

- How large is the radius of the orbit of a moon that takes 3.5 days to orbit? [about 425,000 km]
- What is a model for this data? [Answers vary. It does not look linear.]

5. Personal Savings Account Rates: Study the graph and discuss its general trend.
   - Estimate the average personal savings rate in the year 2000 if the trend continues. [Answers will vary.]
   - When was the rate about 5%? [1987 and 1989]
   - What is a good model for this data? [It looks like linear decay but may go back up.]

6. Sun Rise Time in Texas: Study the graph and discuss its general trend.
   - Estimate the sunrise time in February. [6:45 a.m.]
   - Estimate the sunrise time in June. [5:00 a.m.]
   - When does the sun rise the earliest? [mid-June]
   - When does the sun rise the latest? [late Dec/early Jan]
   - When does the sunrise about 6:00 am? [mid-March and mid-October]
   - Use the graph to sketch the average sunset time. [Answers will vary.]

7. Logistic Graph: Discuss the general trend of the graph.
   - Where is the graph changing fastest? The slowest? [The graph is changing the fastest between time four and five and the slowest between ten and eleven.]
   - What could have caused the changes in the rate? [Answers will vary.]

8. Telephone Calling Cards: Look over the first two graphs of calling card prices per minute.
   - Describe the similarities and differences between the two calling cards. [The first card charges $0.15 a minute. The second card charges $0.30 for the first minute and $0.10 for each minute thereafter.]
   - Which card do you want to use to make a quick call? How quick? [The Call-With-Us card is cheaper for the first three minutes.]
   - Which card do you want to use to make a long call? [One Rate card]

Activity 2: Using a Trend Line

1. Study the Cable CPI (Consumer Price Index) graph using Transparency 1: CPI Graph.
   - What are the variables in the graph? [time in years and the amount of the CPI and cable CPI]
   - How does the CPI compare with the cable CPI? [The general CPI is lower than the cable CPI.]

Point out that someone did a trend line for the cable CPI and compared it to the CPI.

- What does the trend line tell you about the cable CPI? [The trend line shows that cable CPI was out pacing the CPI.]

After the trend line was done, cable regulation started; hence, there was a reduction in the cable CPI.

- Have participants draw their own trend line for the CPI and extend it to the year 2000. What do you expect the CPI to be in 2000? [about 148]

2. Look at the graph Family Households in the U.S. "A 'household' comprises all persons who occupy a 'housing unit,' that is, a house, an apartment or other group of rooms, or a single room that constitutes 'separate living quarters.' A household includes the related family members and all the unrelated persons, if any, such as lodgers, foster children, wards, or employees who share the housing unit. A person living alone or a group of unrelated persons sharing the same housing unit is also counted as a household" (*Statistical Abstract of the United States 1997*, US Department of Commerce). There were 2.65 people per household in 1996.

   a. For the first graph, have participants:
      - Identify the variables. [years and number of households in millions]
      - Describe the situation in a sentence or two.
      - Use a trend line to predict the number of family households in the U.S. in the year 2000. [around 72 million households]

   b. The next graph shows the number of female households in the U.S. Point out that this data is a subset of the Family Households (the top graph). Note that the scales are different.
      - Identify the variables. [years and number of female householders in millions]
      - Describe the situation in a sentence or two.
      - Use a trend line to predict the number of female households in the U.S. in the year 2000. [about 13.5 million female householders]

3. Look at the first two graphs; both are increasing. Discuss how to compare the first two graphs with ratios. Have participants:
   a. Graph the ratios of female households to total households using the data from the two graphs above. [See Transparency 2.]
   b. Identify the variables and describe the situation in a sentence or two.
   c. Use a trend line to predict the ratio of female households to total households in the U.S. in the year 2000. [about 19.5-20 million]

d. Take the ratios of the year 2000 values you predicted in exercise 2. How does this ratio compare to the ratio you predicted with the trend line in part c?

Transparency 2: Percent of Total Households Headed by Females shows the female households as a percent of the total households. Talk about the graph and what it means. You are looking at the percent change from 1960 to 1995. Have participants compare their graphs to this graph.

- The graph is increasing. Will it continue to increase? [No, percents top out at 100%.]

Summary The big idea is to look at how variables relate, interpret graphs, build intuition for rate of change, and begin to make predictions with hand-drawn trend lines and other models where appropriate.

Activity 1: Interpreting Graphs

1.

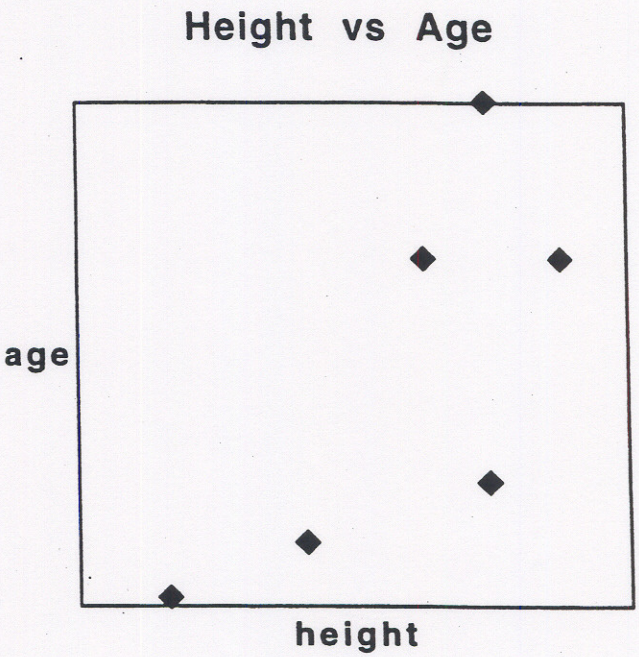
Gretchen and Ivan, mother and baby

Claudio, grandfather

Charles, teenage son

Malcolm, father

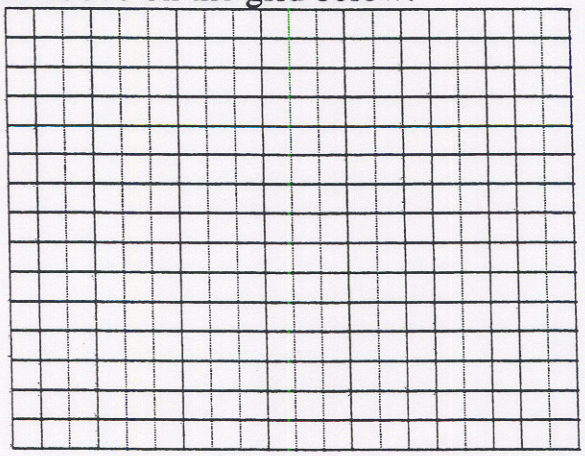
Maria, young daughter



a. Label the above graph with the name of the person each data point represents.

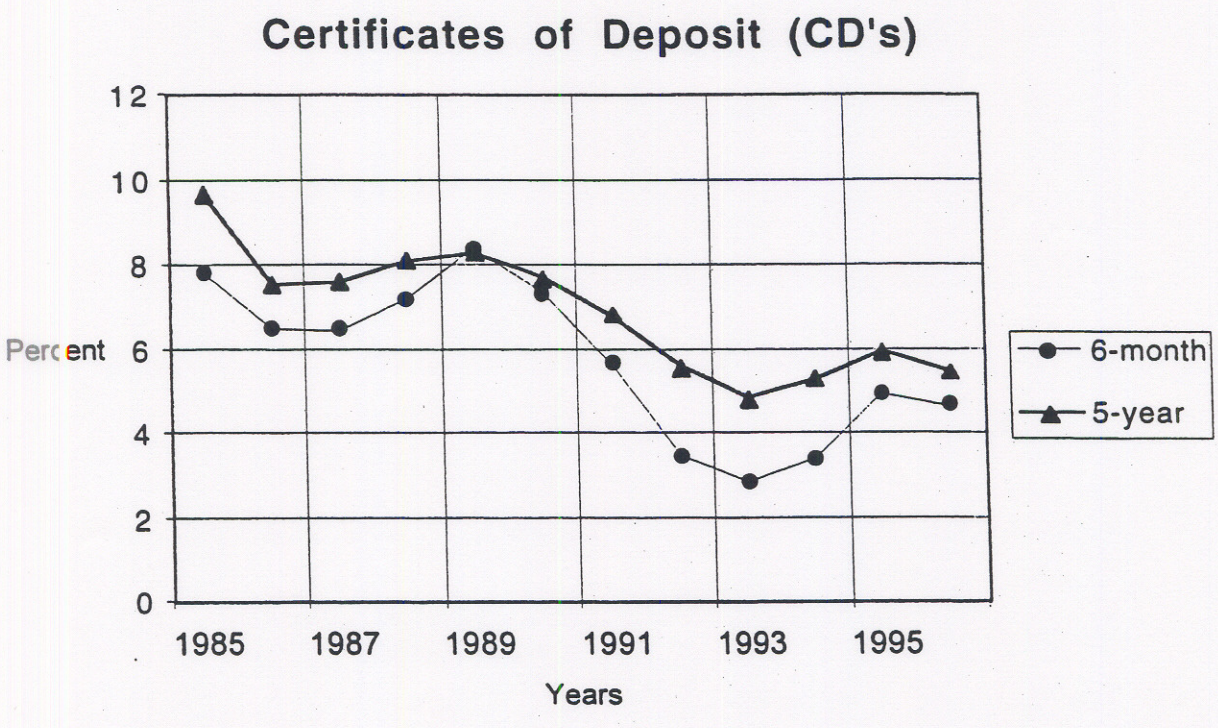
b. Graph the data above on the grid below.

height



age

2. The following graph shows interest rates for six-month and five-year certificates of deposit (CD's) from 1985 to 1996.



a. What is the general trend of the graph?

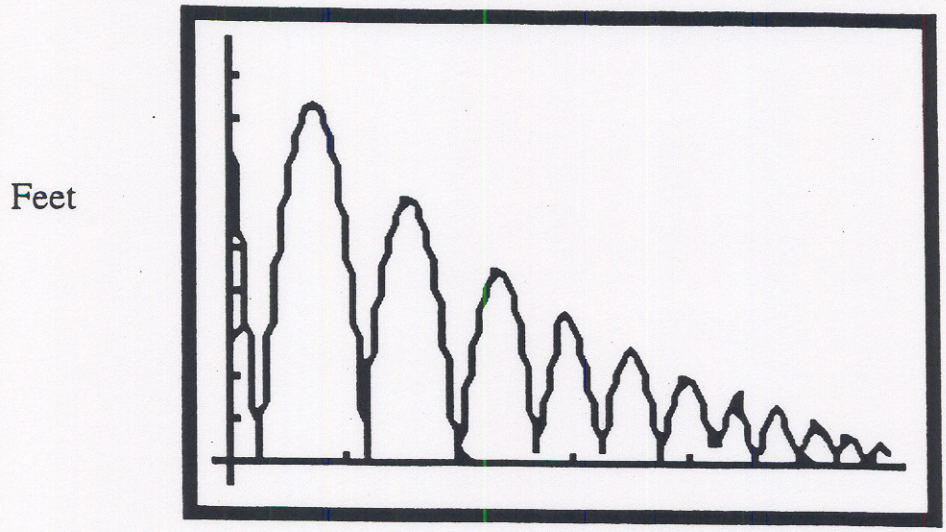
b. Use the graph to estimate the rate for a six-month CD in 1998.

c. Use the graph to estimate the rate for a five-year CD in 1984.

d. When did a six-month and a five-year CD offer the same rate?

e. When did a six-month CD offer a rate of about 5%?

3. The following graph shows the height of a ball bouncing versus time.



a. What is the general trend of the graph?

b. Where is the ball at one second?

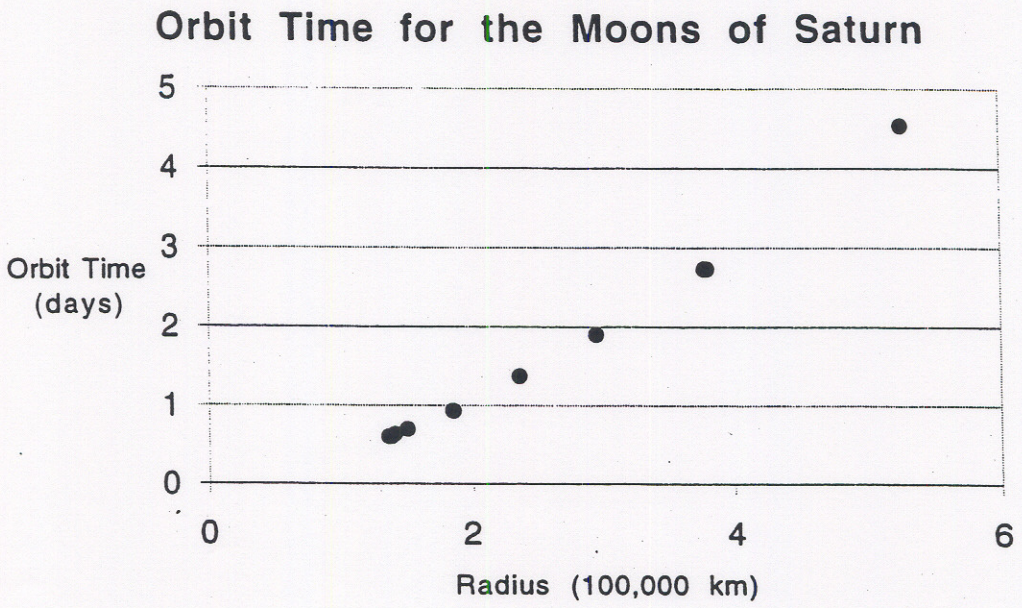
c. How many times did the ball hit the ground in the first second? The second second? The third second? The fourth second?

d. What is the model for each bounce?

e. Is each bounce half as high as the one before?

f. What do you think the relationship between the bounce and the maximum height of each bounce is?

4. The orbit times for the moons of Saturn are shown in the graph below. There is a relationship between the radius of the orbit of a moon and the time it takes to complete one orbit.

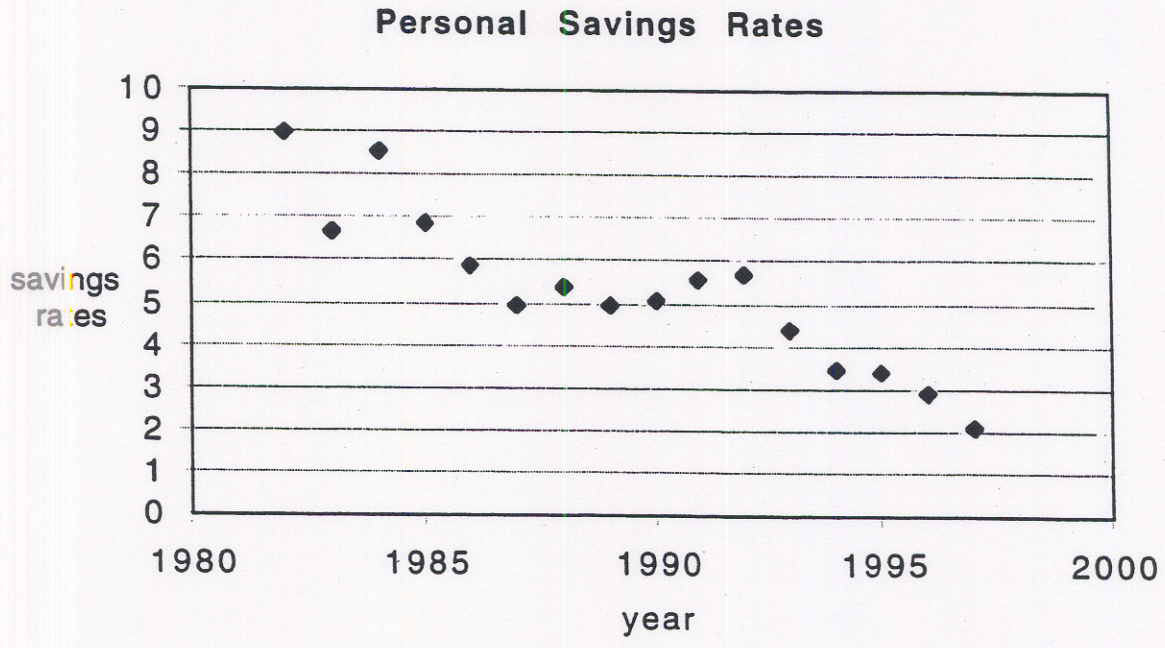


a. Estimate the orbiting time if the radius of the orbit of a moon is 500,000 km.

b. How large is the radius of the orbit of a moon if the moon takes 3.5 days to orbit Saturn?

c. What could be a model for this data?

5. The following graph shows the personal savings rates (percent of income saved) from 1980 to 2000.

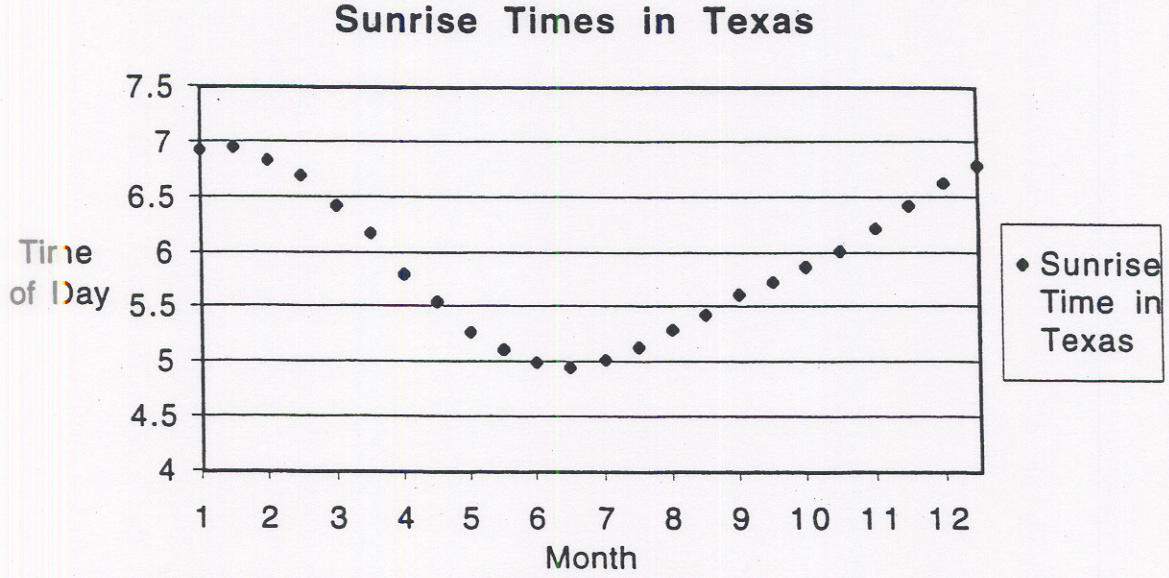


a. Estimate the average personal savings rate in 2000 if the trend continues.

b. When was the rate about 5%?

c. What could be a model for this data?

6. The graph below shows the sunrise time in Texas for a one-year period.



a. Estimate the sunrise time in February

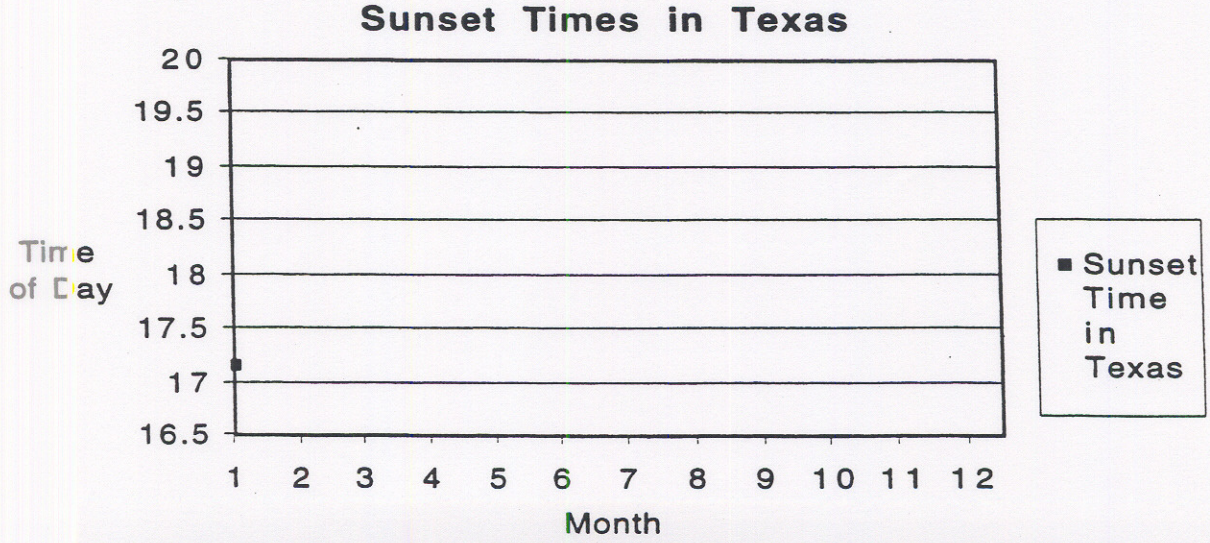
b. Estimate the sunrise time in June.

c. When does the sun rise the earliest?

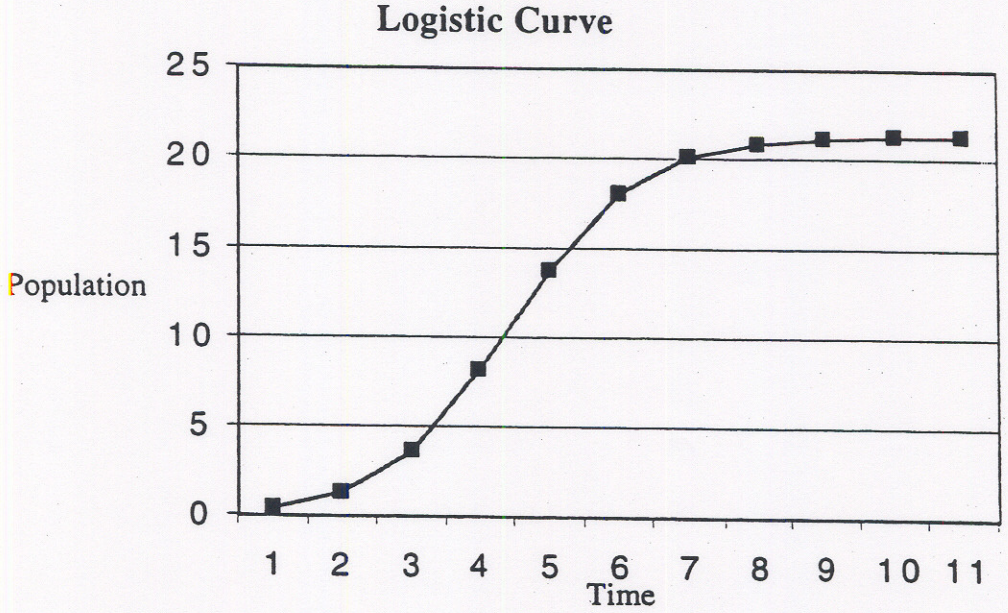
d. When does the sun rise the latest?

e. When does the sunrise about 6:00 am?

f. Use the graph below to sketch the average sun set time in each month.



7. The following logistic curve shows the change in population over time.

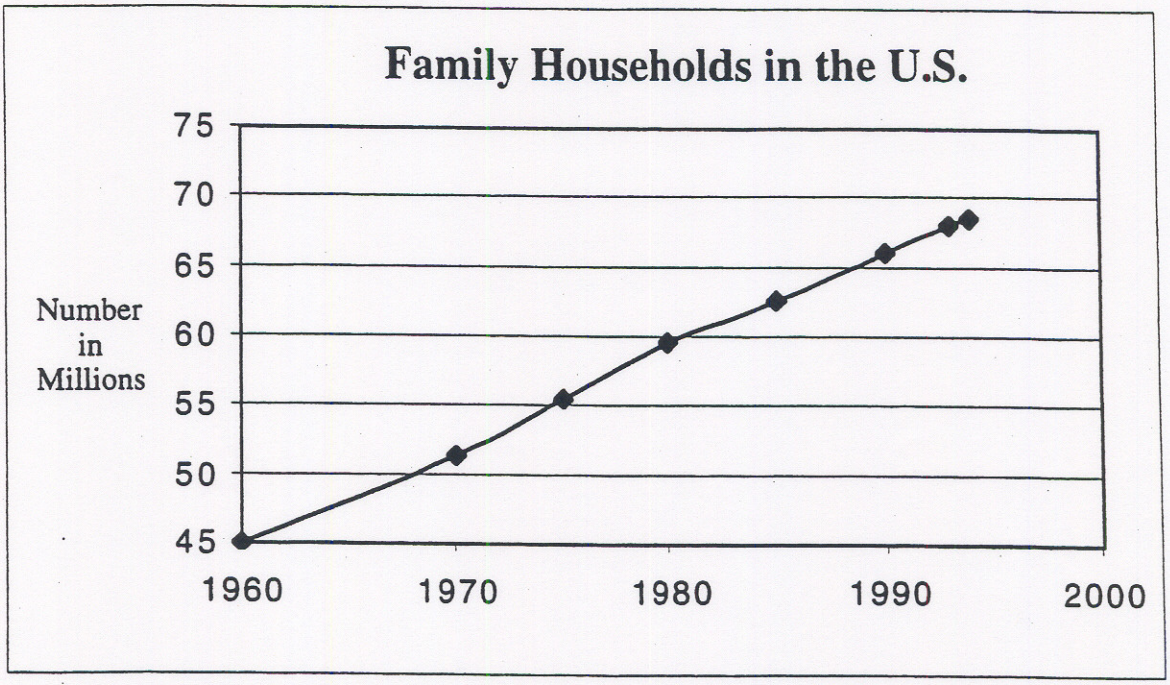


a. Where is the graph changing the fastest? The slowest?

b. What could have caused the changes in the rate?

2. For each of the graphs below:
   - Identify the variables
   - Describe the situation in a sentence or two
   - Use a trend line to predict the value in the year 2000.

a.



b.