Have You Lost Your Marbles?

Overview: Students investigate the relationship between the height of an object and the distance the object rolls.

Objective:

Algebra I TEKS

(b.1.B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

(b.1.E) The student interprets and makes inferences from functional relationships.

(c.1.A) The student determines whether or not given situations can be represented by linear functions.

(c.1.C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(c.2.B) The student interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

(b.2.D) In solving problems, the student collects and organizes data, makes and interprets scatter plots and models, predicts, and makes decisions and critical judgments.

Terms: rate, slope

Materials: each group needs 5 – 6 building blocks, 39 cm of PVC pipe, marble or steel ball bearing that will fit inside and roll freely through the PVC pipe, metric tape measure, graphing calculators

**Procedures:**   Students should be seated at tables in groups of 3 – 4.

Activity: Have You Lost Your Marbles?

Briefly describe and/or demonstrate the experiment. Make sure students measure the distance the marble rolls once it leaves the end of the pipe.

1. Stress how important it is for students to predict the results of the experiment **before** they perform the experiment. Encourage students to think about and anticipate the results of the experiment before they begin collecting data.

2. *Sample data*:

Height (blocks)	Distance (cm)
1	19
2	53.5
3	74
4	100
5	137.5

3. *Sample data*:
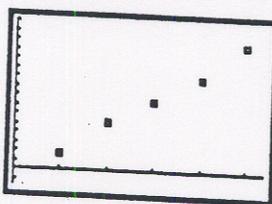

```
WINDOW
 Xmin=0
 Xmax=5.4
 Xscl=1
 Ymin=-16
 Ymax=160
 Yscl=10
 Xres=1█
```

4. *Sample data*:

```
L1    L2     █3      3
1     19     34.5
2     53.5   20.5
3     74     26
4     100    37.5
5     137.5  -----
----- -----
L3 =ΔList(L2)█
```
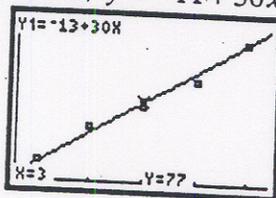
```
mean(L3
              29.625
█
```

An estimate for the rate of change is about 30 cm/block.

5. Since the rate of change is about 30, then the *y*-intercept is about $19 - 30 = -11$.

6. Using our *sample data*, $y = -11 + 30x$

7. *Sample data*:


```
Plot1 Plot2 Plot3
\Y1⊟-13+30X
\Y2=█
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```
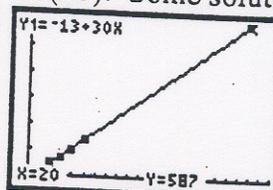
8. The units of slope are centimeters per block.

9. The real world meaning of the *y*-intercept is that for zero blocks the marble does not roll out of the pipe at all, it covers no distance.

10. The equation is $y = -13 + 30(20)$. Some solution methods:

```
-13+30(20)
             587
█
```



```
X    Y1
18   527
19   557
20   587
21   617
22   647
23   677
24   707
Y1=587
```

11. For our *sample data*, solve: $-13 + 30x = 60$. Some solution methods:

Table:

```
X    Y1
0    -13
1    17
2    47
3    77
4    107
5    137
6    167
X=2
```

```
X    Y1
2    47
2.1  50
2.2  53
2.3  56
2.4  59
2.5  62
2.6  65
X=2.4
```

```
X    Y1
2.4  59
2.41 59.3
2.42 59.6
2.43 59.9
2.44 60.2
2.45 60.5
2.46 60.8
X=2.43
```

Other Table:

```
X    Y1    Y2
0    -13   60
1    17    60
2    47    60
3    77    60
4    107   60
5    137   60
6    167   60
X=2
```
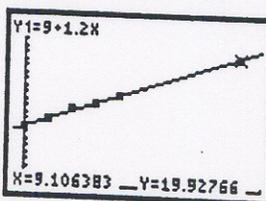
```
X    Y1    Y2
2    47    60
2.1  50    60
2.2  53    60
2.3  56    60
2.4  59    60
2.5  62    60
2.6  65    60
X=2.4
```

```
X    Y1    Y2
2.4  59    60
2.41 59.3  60
2.42 59.6  60
2.43 59.9  60
2.44 60.2  60
2.45 60.5  60
2.46 60.8  60
X=2.43
```

Trace:

```
Y1=9+1.2X




X=9.106383 __ Y=19.92766 __
```

Trace to the
Intersection point.

```
Intersection
X=2.4333333   Y=60
```

```
Plot1 Plot2 Plot3
\Y1⊟-13+30X
\Y2⊟60
\Y3=■
\Y4=
\Y5=
\Y6=
\Y7=
```
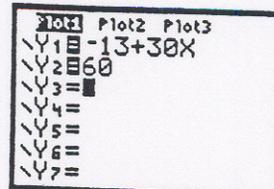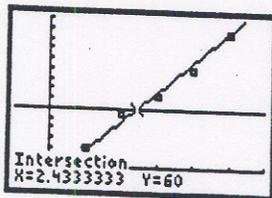
Guess and check.

```
-13+30(2)
                47
-13+30(2.5
                62
-13+30(2.3
                56
■
```

```
                56
-13+30(2.4)
                59
-13+30(2.45)
                60.5
-13+30(2.43)
                59.9
■
```

Solve algebraically, if you are at a place in your curriculum where it makes sense for students to do so: $-13 + 30x = 60$

$$-13 + 30x + 13 = 60 + 13$$
$$\left(\frac{1}{30}\right)30x = 73\left(\frac{1}{30}\right)$$
$$x = 2.4\overline{3}$$

12. The more blocks placed under the pipe, the farther the marble travels.
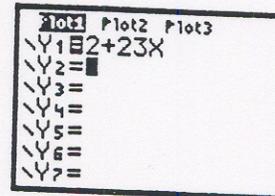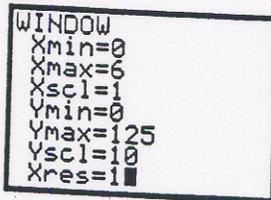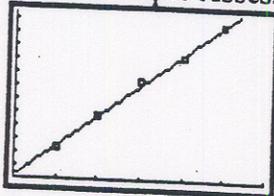
- Did the height of the pipe increase the distance the marble rolled? [Yes.]
- Why or why not? [The marble has more potential (stored) energy with greater height.]
- If the slope keeps increasing, will the marble roll farther each time? Why or why not? [Up to a point, yes. When the slope is completely vertical, the marble will not roll far because its energy is absorbed by the ground.]

Extensions. Find the trend lines for the following and compare.
- Use different sized marbles (as long as they still travel freely through the pipe),
- Use spheres with different masses (i.e., golf balls, ping pong balls, steel ball bearings),
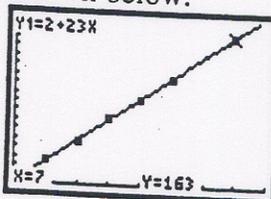- Use different surfaces (i.e., rug, cement, dirt, table top, sheets).
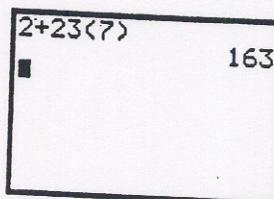
Answers to Sample Assessment

1.

WINDOW
Xmin=0
Xmax=6
Xscl=1
Ymin=0
Ymax=125
Yscl=10
Xres=1■

```
Plot1 Plot2 Plot3
\Y1■2+23X
\Y2=■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

2. With 7 blocks, the marble will roll approximately 163 cm. Some solution methods are shown below.

Trace to $x = 7$

```
Y1=2+23X

X=7          Y=163
```

Solve using the trend line and arithmetic.

```
2+23(7)
                    163
■
```

Use a table.

X	Y1
1	25
2	48
3	71
4	94
5	117
6	140
7	163

Y1=163

3. $x \approx 10$. Students' answers should be close, depending on their trend lines. Some solution methods are shown below:

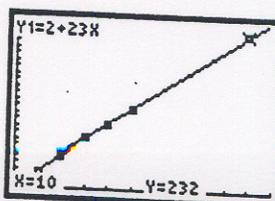Trace to the intersection of $y = 2 + 23x$ and $y = 232$

```
Plot1 Plot2 Plot3
\Y1■2+23X
\Y2■232
\Y3=■
\Y4=
\Y5=
\Y6=
\Y7=
```

```
Intersection
X=10          Y=232
```
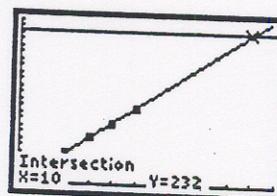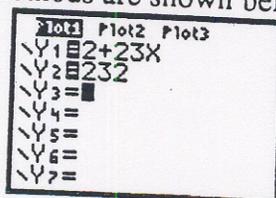
Use a table, in two ways

X	Y1
7	163
7.5	174.5
8	186
8.5	197.5
9	209
9.5	220.5
10	232

X=10

X	Y1	Y2
7	163	232
7.5	174.5	232
8	186	232
8.5	197.5	232
9	209	232
9.5	220.5	232
10	232	232

X=10

Trace to $y = 232$

```
Y1=2+23X

X=10          Y=232
```

Summary: By collecting data and finding a trend line, students investigate the relationship between the height of an object and the distance it rolls. Students use real data to further their conceptualization of the linear function.

Student Activity 1: Have You Lost Your Marbles?

What is the relationship between the height of the pipe and the distance the marble rolls?
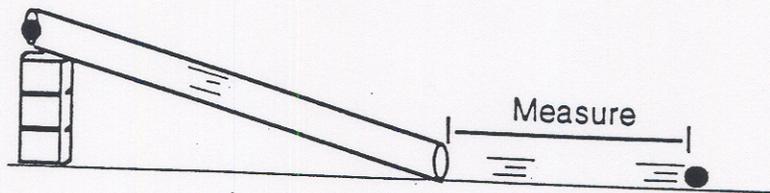
Roll the marble from heights of 1, 2, 3, 4, and 5 blocks. Release the marble at the opening of the pipe. Measure the distance the marble rolls from the end of the pipe.

1. Sketch a graph predicting the relationship between the height of the pipe and the distance the marble rolls.
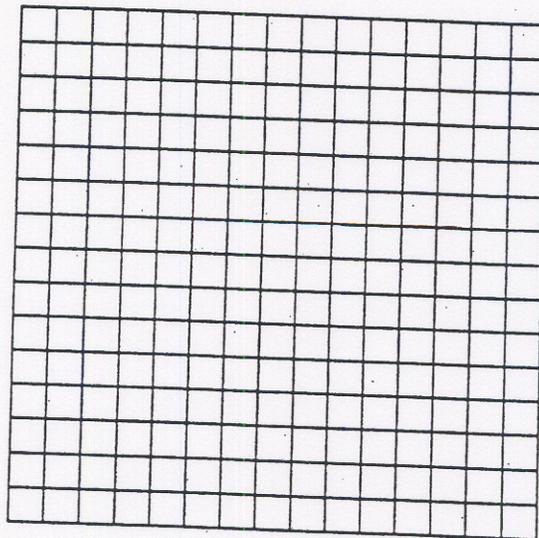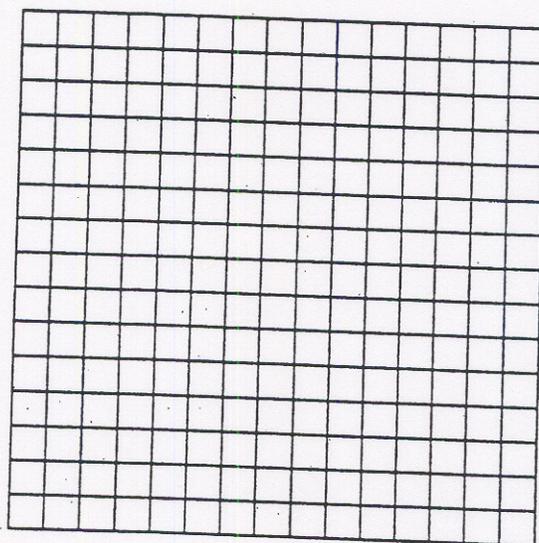
2. Data Collection
    Tasks:

   - one person rolls the marble,
   - one person holds the blocks and pipe,
   - one person marks where the marble stops,
   - one person measures the distance the marble traveled.

Height (blocks)	Distance (cm)

3. Make a scatter plot using a graphing calculator. Sketch below.

4.   Use first differences to estimate a rate of change.

5.   Estimate the y-intercept (*starting point.*)

6.   Find a trend line for the data using the estimated rate and y-intercept.

7.   Graph your trend line over the scatter plot. Adjust the parameters *y-intercept* and *rate of change*, if necessary, for a better fit.

8.   What are the units of slope for the trend line?

9.   What is the meaning of the y-intercept in the trend line?

10. Use your trend line to determine how far the marble would roll if you placed 20 blocks under the pipe. Write an equation and solve in at least four ways.

11. Use your trend line to determine how many blocks are needed for the marble to roll 60 cm. Write an equation and solve in at least four ways.

12. Make a general statement about the relationship between the number of blocks and the distance the marble travels.

Sample Assessment

A group collected the following data for "Have You Lost Your Marbles?"

Height (blocks)	Distance (cm)
1	25
2	47.5
3	73.75
4	92
5	117

1. Create a scatter plot and find a trend line. Sketch both in an appropriate window.

2. Use the graph to determine how far the marble would roll with 7 blocks. Solve in two ways. Show your work.

3. Use the graph to determine how many blocks it would take for the marble to roll 232 cm. Solve in two ways. Show your work.