Mathematical Models with Applications Clarifying Lessons: The Sounds of Music

OLD Resources. These resources have NOT yet been updated to align with the revised secondary mathematics TEKS. These revised TEKS were adopted by the Texas State Board of Education in 2005, with full implementation scheduled for 2006–07. These resources align with the original TEKS that were adopted in 1998 and should be used as a starting point only.

What is a Clarifying Lesson?

A model lesson teachers can implement in their classroom. Clarifying Lessons combine multiple TEKS statements and may use several Clarifying Activities in one lesson. Clarifying Lessons help to answer the question "What does a complete lesson look like that addresses a set of related TEKS statements, and how can these TEKS statements be connected to other parts of the TEKS?"

TEKS Addressed in This Lesson

2.A (interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, and scatterplots to draw conclusions from data)

8.C (use direct and inverse variation to describe physical laws such as Hooke's, Newton's, and Boyle's laws)

9.B (use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music)

Materials

- Graphing calculator
- Electronic data-collection device with microphone
- Guitar
- Slinky™
- Activity sheet

Lesson Resources

Brueningsen, et. al. *Real-World Math with the CBL™ System*. Texas Instruments, 1995.

Related Resources

Garland, Trudi Hammel and Charity Vaughn Kahn. *Math and Music: Harmonious Connections*. Palo Alto, CA: Dale Seymour Publications, 1995.

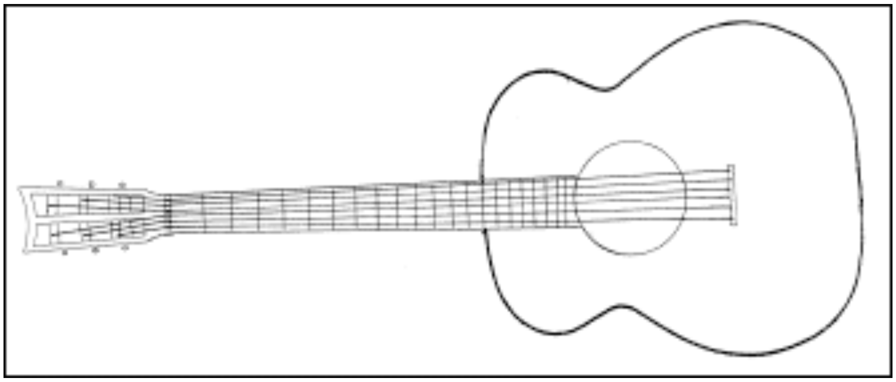
Lesson Overview

Students use an electronic data-collection device to model the sound produced by a guitar string.

Mathematics Overview

Students identify the graph of a periodic situation and use the graph to determine the period and frequency of sound waves.

Set-up (to set the stage and motivate the students to participate)



1. Ask any students with guitars to bring them to class, along with tuners. Ask them to demonstrate the strings effects of fretting on pitch.

2. Explain to students that when a guitar string is plucked the air particles next to the string vibrate.

3. To illustrate the guitar strings' vibration, have two students stretch a Slinky to a length of about 6 feet on a smooth floor. Have one of the students give the Slinky a quick sideways jerk with his/her hand. The movement of the hand sends a wave traveling along the Slinky, while the Slinky itself moves side to side perpendicular to the direction of the wave movement. Such a wave is called a transverse wave. The guitar string works just like the Slinky.

4. The wave that travels along the string of the guitar string when it is plucked is transverse, but the sound wave that the string transmits to the air is longitudinal. To illustrate a longitudinal wave, hang the Slinky from the ceiling with a weight attached to the end. If you pull on the weight and then let go, the whole system bobs up and down. The wave (created by pulling on the weight) and the medium (the Slinky) move parallel to each other. The motion is periodic with a new wave starting to travel along the Slinky every time the weight reaches its lowest point. As the wave travels along the Slinky, its coils will be close together at some places (called points of compression) and farther apart at others (called points of rarefaction). Sound waves are longitudinal. The longitudinal wave travels through the air, hits the eardrum, and allows us to hear the note.

5. The microphone will measure the pressure of the longitudinal sound wave: the data-collection device will record the time and pressure, and transmit the data to the graphing calculator. Once the data is stored in lists in the calculator, a graph of the data can be constructed.

6. Using the TRACE function on the calculator we can measure the time between crests or peaks of the periodic graph. The time to complete a cycle is called the period, T, of the graph and is measured in seconds per cycle. The reciprocal of the period, 1/T, represents the number of cycles completed per second, cps, and is called the frequency, f, or pitch of the wave.

7. Have students complete the Activity Sheet.

Teacher Notes (to personalize the lesson for your classroom)

Guiding Questions (to engage students in mathematical thinking during the lesson)

- What about the shape of the graph indicates that the vibration is periodic? (2.A) (Responses include repeating, cyclic, up and down, sinusoidal, sine-like, curvy, wavy, etc.)
- If the pitch, i.e. frequency, of the note is increased what effect would this have on the graph? (2.A, 9.B) (The period decreases inversely and proportionally.)
- What is the relationship between the graphs when the frequency is doubled? (9.B) (The period is halved.)
- What other type of real-world situation might result in a similar graph? Why? (2.A, 8.C) (Examples include a dot on a wheel, position on a merry-go-round, seasons, pendulums, tides, time of day, sundial, the stages of the moon, radio waves (i.e., FM (frequency modulation) and AM (amplitude modulation).)

Teacher Notes (to personalize the lesson for your classroom)

Summary Questions (to direct students' attention to the key mathematics in the lesson)

- What property does the graph of a periodic phenomena have? (2.A) (It is repeating.)
- What is meant by the period of such a graph? How can you calculate the period of the graph? (2.A, 8.C, 9.B) (The period is the shortest time it takes to start repeating one cycle. It is calculated by counting units on the x-axis of one cycle, with the x-axis indicating time.)
- What is meant by the frequency of a periodic graph? How is it related to the period? (8.C, 9.B) (The frequency is how many cycles that can be completed in a certain length of time; i.e., how compressed or sparse the waves are.)

Teacher Notes (to personalize the lesson for your classroom)

Assessment Task(s) (to identify the mathematics students have learned in the lesson)

- Give students a frequency and have them find the note on the guitar.
- Have students develop general relationships between time of vibration, length, period, frequency, etc. in an algebraic context.
- Have students draw a Cartesian coordinate graph and label the axes, for example, x-axis, y-axis, time, sine wave, period, frequency, loudness, amplitude, etc.

Teacher Notes (to personalize the lesson for your classroom)

Extension(s) (to lead students to connect the mathematics learned to other situations, both within and outside the classroom)

- Have students predict combinations of notes (in terms of frequency, time, etc.)
- Have students investigate amplitude.

Teacher Notes (to personalize the lesson for your classroom)