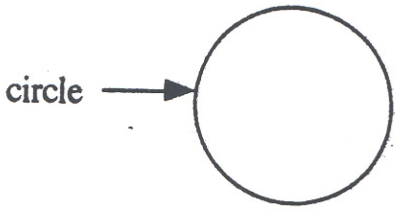
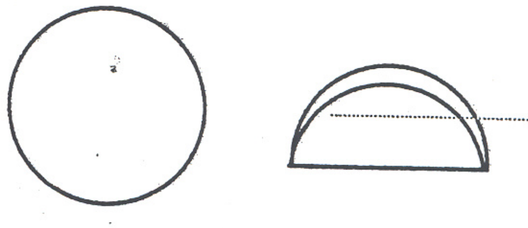
PAPER FOLDING WITH CIRCLES

Paper folding is not limited just to polygons. It can be used to develop and illustrate some of the theorems involving circles. We can use a circular region if we remember that the circle is the outside edge of the circular region.

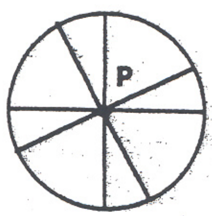




Fold the circular region so that one side coincides with the other.

Definition: One half of a circle is called a **semicircle.**

Definition: The line that divides the circle into two semicircles is called the **diameter**.

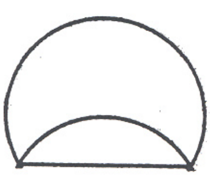


Fold in several diameters. All of the diameters are **congruent**. Notice that they are also **concurrent**.

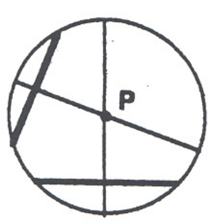
Definition: The point of concurrency of the diameters is called the **center.**

Definition: The segment from the center to the circle is called a **radius.** The length of the radius is one-half the length of the diameter.

Definition: The line joining two points of a circle is called a **chord.** The diameter is the longest chord in a circle.



Take a circular region and fold up a portion (less than half) of the region and crease. The crease is a **chord** of the circle.



Fold in a perpendicular bisector of this chord. Repeat with a second chord. Show that the perpendicular bisectors are diameters. The point of intersection of these diameters is the center of the circle.

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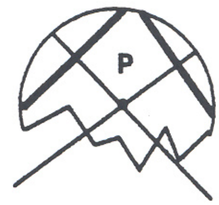
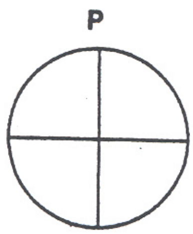
Theorem: The perpendicular bisector of a chord of a circle is a diameter of the circle.

Given a portion of a circular region, the center can be found by finding the intersection of the perpendicular bisectors of two chords.

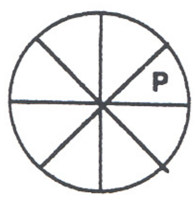


Note: The center may not be in the interior of the region.

The theorems about angles inscribed in a circle can also be illustrated using paper folding.

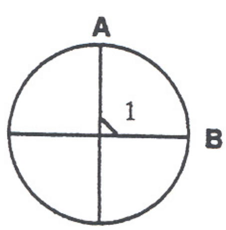


Take a circular region and fold a pair of diameters that are perpendicular bisectors of each other. Each angle in the figure is a right angle and measures 90°. The diameters cut off four congruent arcs, therefore the degree measure of each arc is 90°.

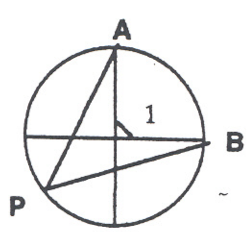


Fold in the bisectors of the four right angles. These diameters divide the circle into eight congruent arcs each measuring 45°.

Definition: The angles formed by two intersecting diameters are called **central angles.** Central angles can be formed by two radii with the center as a common endpoint.



Take a circular region and fold two perpendicular diameters. Label one of the right angle formed as $\angle 1$ and its intercepted arc AB.



Choose a point P not on arc AB and fold PA and PB. Notice that the measure of $\angle APB < 90^{\circ}$.

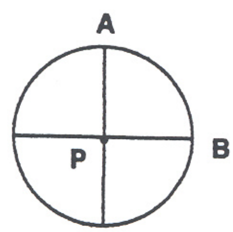
Cut out $\angle APB$ and show that it measures 45° by matching it with the 45° angles folded in an earlier activity.

Definition: ∠ APB is called an **inscribed angle** since its vertex is on the circle and its rays intersect the circle in two points.

Theorem: The measure of an inscribed angle is one-half the measure of its intercepted arc.

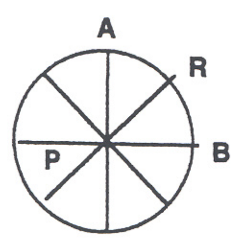
If we consider the circular region as an area then we can discuss sectors and segments of a circle and find some interesting theorems with regard to these concepts.

Definition: A **sector of a circle** is the area enclosed between two non-collinear radii and the circle itself.



Fold two perpendicular diameters. Label the arc AB and the center of the circle P. APB is a sector of the circle.

The two perpendicular diameters divide the circle into four congruent sectors, therefore each sector has an area that is 1/4 the area of the circle. There are 360° around the point P. Each of the angles of one of the sectors is 90°. The ratio of the measure of the angle of the sector to the complete circle is 90/360 or 1/4.



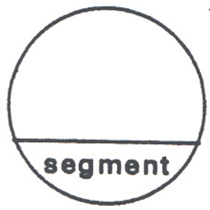
Fold the bisectors of the angles of the sectors. ∠ APR = 45° Sector APR is 1/8 of the circle. The ratio 45/360 = 1/8. The area of the sector is 1/8 the area of the circle

∠ RPB = 45° Sector RPB is 1/8 of the circle. The ratio 45/360 = 1/8. The area of the sector is 1/8 the area of the circle.

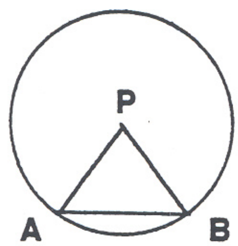
Theorem: The area of a sector of a circle can be found by finding the ratio of the measure of the angle of the sector to 360° and taking this ratio of the area of the circle.

$$\frac{\text{measure of the angle}}{360^{\circ}} \times \text{area of the circle}$$

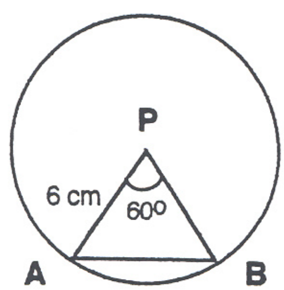
Definition: A **segment of a circle** is the area enclosed between a chord and the arc of the chord.



Fold a circle so that you have two diameters. They need not be perpendicular. Fold the chord that is determined by the endpoints of two radii. The area that you turned up is a segment of the circle.



To find the area of the segment you must find the area of the isosceles triangle, ΔABP, formed by the radii and the chord and subtract this area from the area of the sector ABP formed by the radii and the circle.



For example:

If the radius of the circle is 6 cm and the measure of $\angle APB = 60^\circ$, then the area of the sector is 1/6 the area of the circle and the triangle is an equilateral triangle.

area of the sector is 6π
area of the triangle is $9\sqrt{3}$
area of the segment is $6\pi - 9\sqrt{3}$