Precalculus Clarifying Lessons: Function Models for Real-Life Situations

OLD Resources. These resources have NOT yet been updated to align with the revised secondary mathematics TEKS. These revised TEKS were adopted by the Texas State Board of Education in 2005, with full implementation scheduled for 2006–07. These resources align with the original TEKS that were adopted in 1998 and should be used as a starting point only.

What is a Clarifying Lesson?

A model lesson teachers can implement in their classroom. Clarifying Lessons combine multiple TEKS statements and may use several Clarifying Activities in one lesson. Clarifying Lessons help to answer the question "What does a complete lesson look like that addresses a set of related TEKS statements, and how can these TEKS statements be connected to other parts of the TEKS?"

TEKS Addressed in This Lesson

c.1.B, D, E

c.3.A, B, C

Materials

- Graphing calculator
- Activity sheets A, B, and C
- Teacher's Solutions

Lesson Overview

Students use calculators and "by hand" techniques to compare models of real-life data situations, determine the best model for a situation, and use their models to make predictions.

Mathematics Overview

Students use calculator regression procedures to investigate functions as models of real-life data. They compare different regression models (e.g., logarithmic, exponential, power) to determine the apparent best model. Students use both technology and mathematical procedures to predict input values for given output values.

Set-up (to set the stage and motivate the students to participate)

1. Introduce the lesson by having students complete Activity A, emphasizing that the lesson focuses on using exponential, logarithmic and power functions as mathematical models of data.

2. Give students an overview of Activity B via the Guiding Questions. Review calculator procedures for making scatter plots, determining regression models and r- values, and graphing the regression model on the scatter plot.

3. Have students complete Activity B in their teams, followed by Whole Group Discussion via the Summary Questions.

Teacher Notes (to personalize the lesson for your classroom)

Guiding Questions (to engage students in mathematical thinking during the lesson)

- What is the difference between the domain for a (contextual) situation and the mathematical domain for a function that might model the situation? Why is it important to consider this? (c.1.B) The domain for the modeling function may include values for the independent variable that do not make sense in the contextual situation. By determining the domain for the situation, you know more about what restrictions there are on the independent variable.

- What decisions must you make in order to construct a scatter plot that accurately pictures the data? (c.1.B) You need to decide which variable you want to be independent and which to be dependent. The range of values you see for each variable gives you an idea about how to set the calculator ranges and scales.

- What do you think the scatter plot will look like? As crumple zone length increases, force of impact on crash victim decreases and in a non linear way.

- What mathematical models for the scatter plot would be appropriate to consider? (c.1.D, E; c.3.A) Exponential decreasing, Logarithmic decreasing, Inverse variation, i.e.,

  $y = ab^x$, $y = a + b\ln(x)$, $y = ax^b$.

  $b < 1$, $b < 0$, $b < 0$

- What does the regression correlation coefficient, r, tell you about a mathematical model for a scatter plot? (c.3.B) The r- value tells you how well the model fits the data. The closer abs(r) is to 1, the better the fit.

Teacher Notes (to personalize the lesson for your classroom)

Summary Questions (to direct students' attention to the key mathematics in the lesson)

- What did you determine about the domain for the situation? How does this compare with the domains of the models you generated? (c.1.B) Suppose crumple zone means distance, x meters, between passenger and, say, the windshield. Well, $x > 0$ and probably $x < 2$ (and that's stretching it!). This further restricts the mathematical domains of the models.

- What did you choose as your calculator window? Why? (c.1.B) Responses will vary. Some will use ZoomStat. Others might, based on the data table, choose [0,1.2] by [0,180], xscl = 0.2, yscl = 20

- Is it the class' consensus that the scatter plot looks like . . . ? Have a student model with view screen.

- How did you determine your regression models? What are your results, including r values? (c.3.B) Have students model procedure with view screen and record results on board.

  $y_1 = 21.37 - 78.58\ln(x)$, $r = -0.997$

  $y_2 = 196.36(0.143^x)$, $r = -0.974$

  $y_3 = 32x^{-0.993}$, $r = -0.9998$

- You have applied four different mathematical models to this data. (The 4th model is not regression generated.) Based on the graphs of the models, compared to the scatter plot, which model appears best? Why? (c.3.A, B) The Power model. It gives the best r value. Also the y-values for the function are closer to the y-values in the scatter plot than the other models. This can be seen by graph/trace or table comparison. Have students model with view screen.

- How did you determine distance traveled during impact so that passenger deceleration does not exceed 30 g's? What were your results and responses to Questions #6 and #7 in Activity B? (c.3.C) Have students model how they used the calculator. Expected responses:

  Graph $y_5 = 30$. Graph and Trace with each model. Graph and Calc - Intersect with each model. Use tables.

  Have students present their mathematical solutions as well and explain why they think it is important to be able to do the problem mathematically.

Crumple zone distances vary by as much as 0.34 meters. The average of the four distances is 1.03, which is closest to the distance given by the Power model.

Safety issues and decisions about design and manufacturing dictate that the most accurate model possible be determined and used.

Teacher Notes (to personalize the lesson for your classroom)

Assessment Task(s) (to identify the mathematics students have learned in the lesson)

Activity C may be used, with teams generating a Team Poster/Presentation for Whole Group Discussion.

Teacher Notes (to personalize the lesson for your classroom)

Extension(s) (to lead students to connect the mathematics learned to other situations, both within and outside the classroom)

Team presentations of Activity C

Students research how to use mathematics to linearize data sets and how calculators generate nonlinear regression models.

Teacher Notes (to personalize the lesson for your classroom)