Satellite

A-E Strand(s): Geometry and Number. Sample Courses:

Topic/Expectation

G.D.1 Triangle trigonometry.
- a. Know the definitions of sine, cosine, and tangent as ratios of sides in a right triangle and use trigonometry to calculate the length of sides, measure of angles, and area of a triangle.
- b. Show how similarity of right triangles allows the trigonometric functions sine, cosine, and tangent to be properly defined as ratios of sides.

G.C.1 Geometry of a circle.
- b. Recognize and apply the fact that a tangent to a circle is perpendicular to the radius at the point of tangency.
- d. Recognize, verify, and apply statements about the relationships between interior and exterior angles of a circle and the arcs and segments they define.
- e. Determine the length of line segments and arcs, the size of angles, and the area of shapes that they define in complex geometric drawings.

Other Topic/Expectation(s)

G.B.4 Circles.
- a. Identify and explain the relationships among the radius, diameter, circumference, and area of a circle.
- c. Know and apply formulas for the circumference and area of a circle.

N.A.4 Ratios, rates, and derived quantities.
- c. Solve data problems using ratios, rates, and product quantities.

Rationale

This task allows students to use trigonometry and properties of circles and triangles to find distances.

Instructional Task

A satellite orbiting the earth uses radar to communicate with two control stations on the earth's surface. The satellite's orbit maintains a 10-degree angle of separation between the two stations. Knowing that the earth's radius is 3,963 miles, answer the following questions. Round all answers to the nearest whole number.
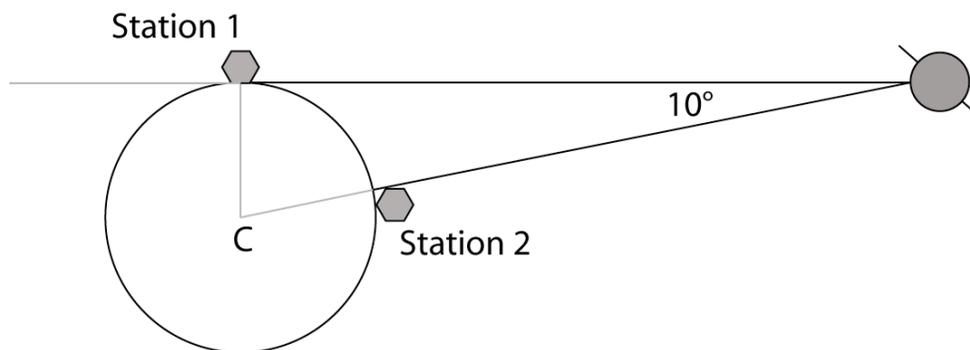
1. How many miles will a signal sent from Station 1 to the satellite and then to Station 2 have to travel? Explain your answer.

2. A satellite technician is traveling from one station to the other in a direct path along the earth's surface. What is the distance she will have to travel? If she travels an average of 50 mph, how long will the trip take? Explain your answer.

3. If a signal could travel through the earth's surface from one station to the other, what is the shortest distance the signal could travel to get from Station 1 to Station 2? Explain your answer.

Discussion/Guiding Questions/Extensions

Students might be asked to solve this problem in metric units (the radius of the earth is approximately 6378 km). Students could also use the law of sines or the law of cosines to answer question 3.

Sample Solutions

1. How many miles will a signal sent from Station 1 to the satellite and then to Station 2 have to travel? Explain your answer.

   The line from the satellite to Station 1 is tangent to the Earth, therefore it forms a right angle with the radius of the earth at that point. Because the triangle formed by the satellite, Station 1, and the center of the earth is a right triangle, then trigonometric ratios can be used to solve the problem. Let x equal the length from Station 1 to the satellite.

   $$tan10^o = \frac{3963}{x} \qquad x = 22{,}475 \; miles$$

   Using trigonometric ratios and the right triangle formed by the satellite, Station 1, and the center of the earth, the length of the hypotenuse (the center of the earth to the satellite) can be found. Since Station 2 is on the hypotenuse and on the earth's surface, its distance from the satellite can be found by subtracting the radius from the length of the hypotenuse. Let y equal the length from the center of the earth to the satellite.

$$\sin 10^{o} = \frac{3963}{y} \qquad\qquad y = 22{,}822$$

Distance from satellite to Station 2 = 22,822 – 3963 = 18,859 miles

The distance from Station 1 to the satellite is 22,475 miles and the distance from the satellite to Station 2 is 18,859 miles. Therefore the total distance from Station 1 to the satellite to Station 2 is 41,334.

2. A satellite technician is traveling from one station to the other in a direct path along the earth's surface. What is the distance she will have to travel? If she travels an average of 50 mph, how long will the trip take? Explain your answer.

Measure of central angle = 180 – (90 + 10) = 80°

$$2\pi(3963) \cdot \frac{80}{360} = 5{,}533$$

The distance from Station 1 to Station 2 is 5,533 miles.

$$t = \frac{D}{r} = \frac{5{,}533 \; mi}{50 \; mph} = 110.66$$

If the technician travels an average of 50 mph, it will take approximately 111 hours to get from Station 1 to Station 2.

3. If a signal could travel through the earth's surface from one station to the other, what is the shortest distance the signal could travel to get from Station 1 to Station 2? Explain your answer.

If the signal could travel through the earth's surface, the distance the signal travels would be approximately 5095 miles. To find this distance, use the isosceles triangle with vertices at the center of the earth (point C), Station 1, and Station 2. The angle at the center of the earth is 80° and the other two angles each measure 50°. Construct a perpendicular bisector from point C to the line connecting Station 1 and Station 2. This creates two right triangles.

Calculating $\cos 50° = \dfrac{z}{3{,}963}$, *you find the measure from the perpendicular bisector to Station 1 to be 2547.37. The distance between Station 1 and Station 2 is twice this distance, or approximately 5095 miles.*