Season Pass

A-E Strand(s): Algebra

Topic/Expectation

    A.B.1 Quadratic functions

        c. Recognize contexts in which quadratic models are appropriate; determine and interpret quadratic models that describe quadratic phenomena.

Other Topic/Expectation(s)

    A.B.3 Systems of linear equations and inequalities

        a. Solve systems of linear equations in two variables using algebraic procedures.

    A.C.2 Polynomial functions

        d. Recognize and solve problems that can be modeled using simple polynomial functions; interpret the solution(s) in terms of the context of the problem.

Core Algebra II EOC Content

    EOC: P1 Quadratic functions

        d. Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.

    EOC: E1 Linear equations and inequalities

        b. Express and solve systems of linear equations in three variables with and without the use of technology.

Rationale

This task asks students to analyze and fit a mathematical model to data in order to answer questions about maximizing revenue. Students may use various methods to determine a quadratic function that fits the mathematical model and helps answer the questions.

Instructional Task

Your school is trying to increase interest in its athletic program. It has decided to sell a season pass that will allow the holder to attend all athletic events at the school. The school surveyed families in the community and asked, "What is the most you would pay for an all-sports season pass?" The survey results are shown in the table below.

Maximum price of season pass	Number who chose this maximum amount
$50	146
$75	80
$90	45
$100	85
$115	120
$135	80
$150	160
$175	50

1. How many people would be willing to pay $115 for a season pass? Explain your answer.

2. Complete the third and fourth columns in the table below. Describe how you determined the projected revenue for each maximum price listed.

Maximum price of season pass	Number who chose this maximum amount	Number willing to pay this price	Projected revenue
$50	146		
$75	80		
$90	45		
$100	85		
$115	120		
$135	80		
$150	160		
$175	50		

3. On a graphing calculator make a scatter plot of projected revenue versus maximum price. Make a sketch of the scatter plot.

4. What type of function does this data suggest? Write a function rule whose graph might fit the data in the scatter plot.

5. Calculate the price of the season pass that yields the maximum projected revenue. Based on your calculations and any other relevant factors, make a recommendation to the school about what price they might choose to set for the season pass. Explain your recommendation.

Discussion/Further Questions/Extensions

If students are unsure how to begin, you may ask questions like those below to guide their thinking. The task assumes that students have worked with quadratic functions enough to find a maximum value and recognize and work with a quadratic graph. It also builds on students' experience with systems of equations and/or curve-fitting.

Once students determine that the data suggest a quadratic function, they may use technology to fit an appropriate quadratic function to the data. The instructional emphasis in this task is on using a model to analyze situations, make predictions, and solve problems.

You may use scaffolding questions to help students get started, but if you rely too heavily on scaffolding questions or use them too soon in the problem-solving process, it may reduce the cognitive load and the richness of the task. Students should have the opportunity to constructively and independently approach tasks like this before being guided toward a particular problem-solving method.

Scaffolding questions

1. What do you observe about the data that suggests a mathematical model might be helpful and appropriate?

2. What kinds of mathematical models have you worked with in the past?

If students choose to find a function rule that represents the data set by solving a system of equations, these scaffolding questions may be helpful:

3. a. How many equations will you need to find the function rule?

   b. How many points from the data set will you need to use to create the equations? (Note: it takes only two points to determine a line, but it takes three points to determine a parabola.)

Extension:

How does the maximum amount found by using a graphing calculator compare to what you might have calculated the maximum price to be using only the survey results?

Using the survey results, we might have overpriced the season pass. The maximum data point is a price of $100 resulting in a revenue of $49,500 as opposed to the actual maximum revenue of $49,775.52 (found using a graphing calculator) for a season pass price of $93.32. (See sample solution #5.)

Sample Solutions

1. How many people would be willing to pay $115 for a season pass? Explain your answer.

   *There are 410 people willing to pay $115 for a season pass. This number can be found by adding the number of people willing to pay $115 to the number of people willing to pay any amount greater than $115. That is, 120 + 80 + 160 + 50 =410.*

2. Complete the third and fourth columns in the table below. Describe how you determined the projected revenue for each maximum price listed.

Maximum price of season pass	Number who chose this maximum amount	Number willing to pay this price	Projected revenue
$50	146	766	$38,300
$75	80	620	$46,500
$90	45	540	$48,600
$100	85	495	$49,500
$115	120	410	$47,150
$135	80	290	$39,150
$150	160	210	$31,500
$175	50	50	$8,750

   To determine the revenue at each price, multiply the maximum price of the season pass by the number of buyers willing to pay that price.

3. On a graphing calculator, make a scatter plot of the revenue versus the maximum price as stated in the first column. Make a sketch of the scatter plot.
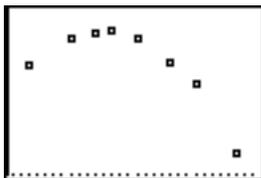
4. What type of function does this data suggest? Write a function rule whose graph might fit the data in the scatter plot.

    The data on the scatter plot suggest a quadratic function. Students may use several methods to fit a quadratic function to the data. Some students may choose to solve a system of equations in three variables algebraically, solve the system with matrices, or use technology to fit a quadratic curve to the data.

    To find a quadratic function, choose three possible points from the scatter plot and set up a system of three equations to solve for the unknowns a, b, and c. For example, choosing (50, 38300), (100, 49500), and (135, 39150) gives three equations with three unknowns. Solve algebraically or use matrices to find a, b, and c.

    $38300 = a(50^2) + b(50) + c$

    $49500 = a(100^2) + b(100) + c$

    $39150 = a(135^2) + b(135) + c$

    Solving the system of equations gives values for a, b, and c. Round these values to the nearest thousandth and substitute them into the quadratic equation to get the following revenue function R(x). (Note that the number of decimal places is specified as thousandths, but could be chosen differently.)

    $R(x) = -6.114 x^2 + 1141.143x - 3471.429$

Other examples:

Selecting the points (50, 38300), (135, 39150), and (90, 48600) yields

$R(x) = -5.500x^2 + 1027.500 x + 675.000$

Selecting the points (100, 49500), (135, 39150), and (150, 31500) yields

$R(x) = -4.286x^2 + 711.429x + 21214.286$

By regression:

Students may also use technology to determine a quadratic fit by performing a quadratic regression on maximum price and projected revenue. A possible regression equation calculated in this manner is $R(x) = -6.113x^2 + 1149.997x - 4588.255$.

5. Calculate the price of the season pass that produces the maximum projected revenue. Based on your calculations and any other relevant factors, make a recommendation to the school about what price they might choose to set for season tickets. Explain your recommendation.

Using the quadratic function found in the first example in solution 4 above, the maximum projected revenue can be determined by finding the x- and y-values of the maximum point of the parabola:

$R(x) = -6.114 x^2 + 1141.143x - 3471.429$

Use the equation $x = \dfrac{-b}{2a}$ to find the x-coordinate of the vertex, representing the price of the ticket:

$x = 93.322$

Substitute this value for x in the original function and solve for R(x):

$R(x) = 49,775.518$

Therefore, the maximum projected revenue is $49,775.52, obtained when the maximum price of the season pass is $93.32.

Many calculators allow direct calculation of a maximum. Using this feature, students can find the maximum point for the regression equation above:

$R(x) = -6.113x^2 + 1149.997x - 4588.255$

Maximum point in dollars: (94.059, 49495.361)

Therefore, using this method, the maximum projected revenue is $49,495.36, obtained when the maximum price of the season pass is $94.06.

Student recommendations may vary, as long as they are appropriately justified.