4.2 Periodic Models

Overview: We will "listen" to the pluck of a guitar string with a data collection device, and then using the graph of the data, we will find the period and the frequency of the sound. Based on this information, we will find a model to predict the relationship between the length of the string that is plucked and the frequency of the resulting sound.

Objective: **Mathematical Models with Applications TEKS:**
3A, 8B, 9B

Terms: Periodic phenomena, period, frequency, crest, transverse wave, longitudinal wave, and guitar fret

Materials: Graphing calculator, data collection device with microphone, guitar, Slinky®, and weight

Procedures: Set up a data collection device with a microphone and a viewscreen calculator, so the whole group can see the results of the experiment on a screen in front of the room.

Activity: The Sounds of Music
Explain to students that when a guitar string is plucked, the air particles next to the string vibrate.

1. Have two participants stretch a Slinky to a length of about 6 feet on a smooth floor. Have one of the participants give the Slinky a quick sideways jerk with his/her hand. The movement of the hand sends a wave traveling along the Slinky, but the Slinky itself moves side to side perpendicular to the direction of the wave movement. Such a wave is called a transverse wave. The guitar string works just like the Slinky.

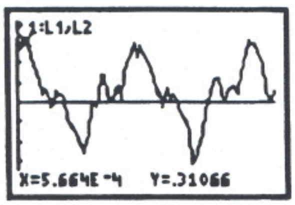
   The wave that travels along the guitar string when it is plucked is transverse, but the sound wave that the string transmits to the air is longitudinal. To illustrate a longitudinal wave, hang the Slinky from the ceiling with a weight attached to the end. If you pull on the weight and then let go, the whole system bobs up and down. The wave (created by pulling on the weight) and the medium (the Slinky) move parallel to each other. The motion is periodic with a new wave starting to travel along the Slinky every time the weight reaches its lowest point. As the wave travels along the Slinky, its coils will be close together at some places (called points of compression) and farther apart at others (called points of rarefaction). Sound waves are longitudinal. The longitudinal wave travels through the air, hits the eardrum, and allows us to hear the note.

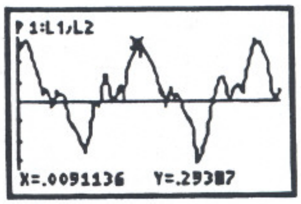
2. The microphone will measure the pressure of the longitudinal sound wave, record the time and pressure, and transmit the data to the graphing calculator. Once the data is stored in lists in the calculator, a graph of the data can be constructed.

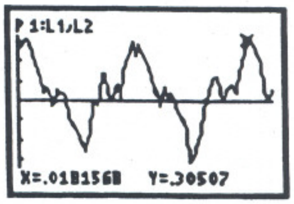
3. Connect the calculator, microphone, and data collection device. Pluck the fifth string (low A string) of the guitar and collect the data. Examine the graph of the data. The graph of the data should be periodic. If not, repeat the procedure.
   - What about the shape of the graph indicates that the vibration is periodic? [It repeats.]
   - If the sound were louder, how would the graph change? [The amplitude, distance from the x-axis, would be higher.]
   - If the pitch, or frequency, of the note were increased, what effect would this have on the graph? [The cycles would happen more often.]
   - How can we find the period of the graph? [The time to complete a cycle is called the period, *T*, of the graph. Use the trace function to determine the time of successive crests and calculate the difference between these crests to approximate the period of the graph. Fill in the table with the word "period" under "Difference between Crests" and then with the data.]
   - What is the relationship between the period and the frequency of the data? [The reciprocal of the period, $1/T$, is the frequency, *f*.]

   Have the participants find the reciprocal of each difference and find the average of the frequencies. Have them fill in the table.

   - How close did you get? [For the sample data shown below, we got *close*; the frequency of the A string should be 110. Note that since we only had 3 crests to measure between, we also measured between the 2 troughs.]





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X
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Ans-X
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Ans^-1
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X
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Ans-X
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Ans^-1
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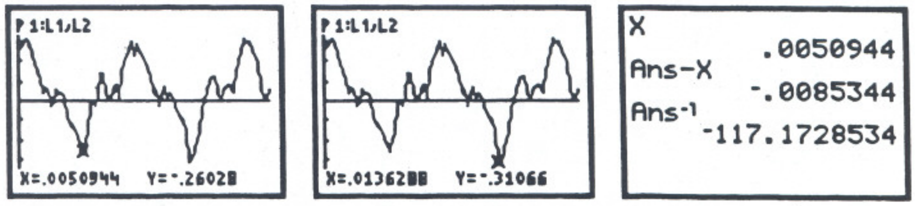
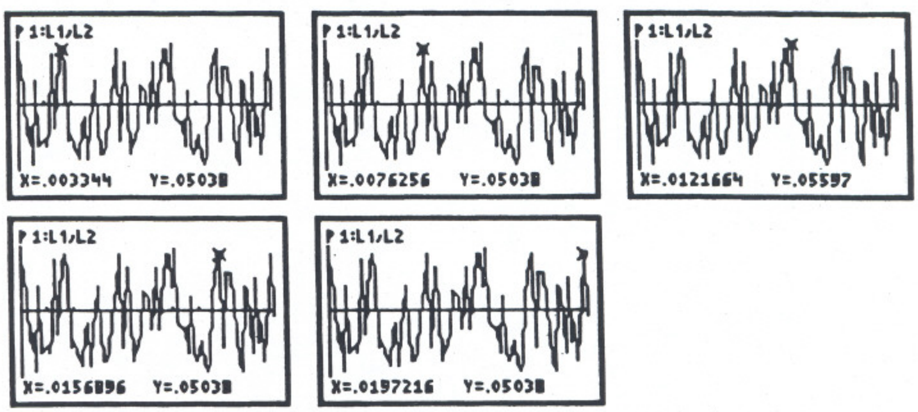


Difference between Crests PERIOD	FREQUENCY	AVERAGE FREQUENCY
.0085472	116.997	114.9
.0090432	110.580	
.0085344	117.173	

Have participants do the rest of the activity sheet in groups. Have one group discuss their findings with the whole group.

4. If you fret the guitar at the twelfth fret (push down on the string and pluck it), the vibrating string will be half the length of the original E string. Using the procedure above, calculate the frequency of this string. (The frequency should be double.) Note that the "vibrating string" is the part of the string from where you fret to the saddle. Also note that you find the twelfth fret by counting twelve frets from the top (nut) of the guitar.

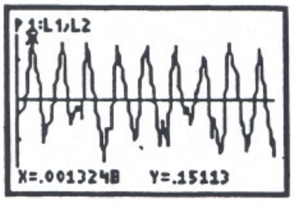


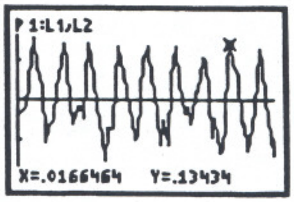
Difference between Crests PERIOD	FREQUENCY	AVERAGE FREQUENCY
.0042816	233.558	233.8
.0045408	220.223	
.0042816	233.558	
.0040320	248.016	

5. Based on your data, how do the frequencies appear to be related to the string length? [As the string length is cut in half, the frequency doubles. The A string should have been 110, and the octave higher A should have been 220. Our experimental results were still about double, both being a little sharp.]
   - What would you predict the frequency of a string one-third the length of the original string to be? One-fourth? [As the length of the string is

cut in half, the frequency doubles, so one-third the length of the string would give you three times the frequency. A string one-fourth the length of the original string would give you four times the frequency.]

Another way to find the frequency of the plucked string is shown below. This is the octave higher A. We trace to the first crest and then to the eighth crest. Find the average and then the reciprocal. We got about 457 cycles per second. We started with the 110 A (We got 115.), then the 220 A (We got 234.), and now the 440 A (We got 457.). Considering the precision (or lack of) of the data collection devices, these results are quite good. The microphone is only picking up 2 tenths of a second of sound.



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X
              .0013248
Ans-X
             -.0153216
Ans/7
             -.0021888
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              .0013248
Ans-X
             -.0153216
Ans/7
             -.0021888
Ans⁻¹
           -456.871345
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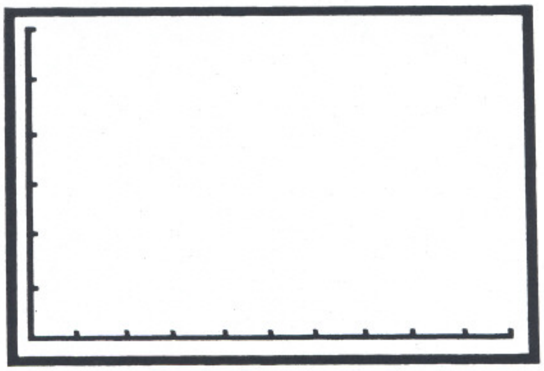
Note that the better the guitar, the more overtones are present. This makes for a fuller, more pleasing sound. It also makes for a graph that looks less sinusoidal because of the interference of the overtones. On a cheaper guitar, the overtones do not sound as well, and the microphone picks up more of a single tone. Hence, the graph looks more sinusoidal. This is also why a tuning fork, which produces one sound only, will produce a sinusoidal graph.

Summary: When a guitar string is plucked, the air particles next to the string vibrate. We can measure the pressure of the longitudinal sound wave with a microphone probe. The resulting periodic graph can be used to find the pitch of the note. Using the trace feature on the calculator, we can measure the time between crests or peaks of the periodic graph. The time to complete a cycle is called the period, T, of the graph. The reciprocal of the period, $1/T$, represents the number of cycles completed per second, cps, and is called the frequency, f, of the wave or pitch. By finding the frequencies of different lengths of a string, we found that there is an inverse relationship between the length of the string and its frequency.

Activity: The Sounds of Music

1. Briefly explain the difference between a transverse wave and a longitudinal wave.

2. The microphone will measure the pressure of the longitudinal sound wave, record the time and pressure, and transmit the data to the graphing calculator. Sketch your prediction of the graph of the sound data. Label your axes including units.



3.

Difference between Crests		

4. If you fret the guitar at the twelfth fret, the vibrating string will be half the length of the original A string. Using the procedure above, calculate the frequency of this string.

5. Based on your data, how do the frequencies appear to be related to the string length? Find a model. Using your model, what would you predict the frequency of a string one-third the length of the original string to be? One-fourth? Check your predictions.