Student Activity: Circular Functions

Overview: These activities develope radian measure through a hands-on activity. To understand most applications of sinusoidal functions, one must develop an understanding of the circular functions that have as their domains real numbers. These activities make the connection between the trigonometric functions and the circular functions.

Objective: **Algebra II TEKS**

(c.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representation functions, including polynomial, rational, radical, exponential, logarithmic, trigonometric, and piecewise-defined functions.

(c.1.A) The student is expected to describe parent functions symbolically and graphically, including $y = \sin x$.

Terms: trigonometric function, circular function, radian, central angle

Materials: 1” grid paper, circle, push pins, string, graphing calculator, one piece of cardboard for every pair of participants, paper plate for each pair, protractors or angle rulers

Procedure:

Activity: Circular Functions

Math Note: The trigonometric functions are generally defined with domain values that are measures of angles in degrees. These activities will extend that understanding first to measurement in radians and then to the circular functions. The domains of the circular functions are subsets of the real numbers.

Make a large model like the one described on the activity sheet.

Model and describe as follows. Attach a piece of string to the point, which is at the intersection of the circle with the x-axis. Use a marker to mark one radius of the circle on the string. Wrap the string around the circle until you have measured an arc that is the length of one radius. Wrap the string back around through the center of the circle. You have formed a central angle of the circle. The central angle of this circle is said to have a measure of one radian.

- What is the approximate degree measure of an angle of 1 radian? *Looking at the demonstration, one may say less than* 90°.

Ask a participant to show you central angles that measure 2 radians, 3 radians, 4 radians, 5 radians, 6 radians, and 7 radians.

- What is the relationship between the degree measure of an angle and the radian measure of an angle?
  Allow time to work on Exercises 1 through 6.

TF-1-1

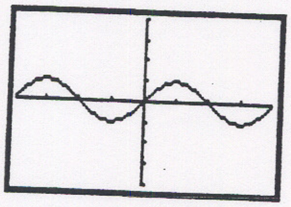
1. *One radian is about* 60°.

2. *The answers to the table may vary, but should be approximately as follows:*

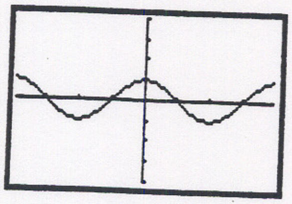
Radian measure	1	2	3	4	5	6	7	8	9	10	11	12	13
Degree Measure	60°	120°	180°	240°	300°	360°	420°	480°	540°	600°	660°	720°	780°

3. *An angle that measures 360° has an arc that measures the circumference of the circle (2π times the radius of the circle). This arc that measures 2π times the radius of the circle has a radian measure of 2π times one radian or 2π radians. One radian is equivalent to 360 divided by 2π or approximately 57.3°. One radian* $= \dfrac{360}{2\pi} \approx 57.3°$.

4. Help students see the periodic nature of the circle to lead them to the notion of that the period of trigonometric functions has to do with 2π.

5. 

6. *The domain is the set of angles measured in radians* $(-\infty, \infty)$. *The range is* $[-1, 1]$.

7. 

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Ask the students to follow the directions on the circular function sheet to model the association of every number on the number line with an arc on the circle and with a central angle.

- Describe how to associate a negative number with an angle. *The number line would be wrapped in the clockwise direction and be associated with a negative angle.*

- How would you find the number (radian measure) that is associated with a 120° angle? 120° *is one-third of a circle. One-third of the circumference is*

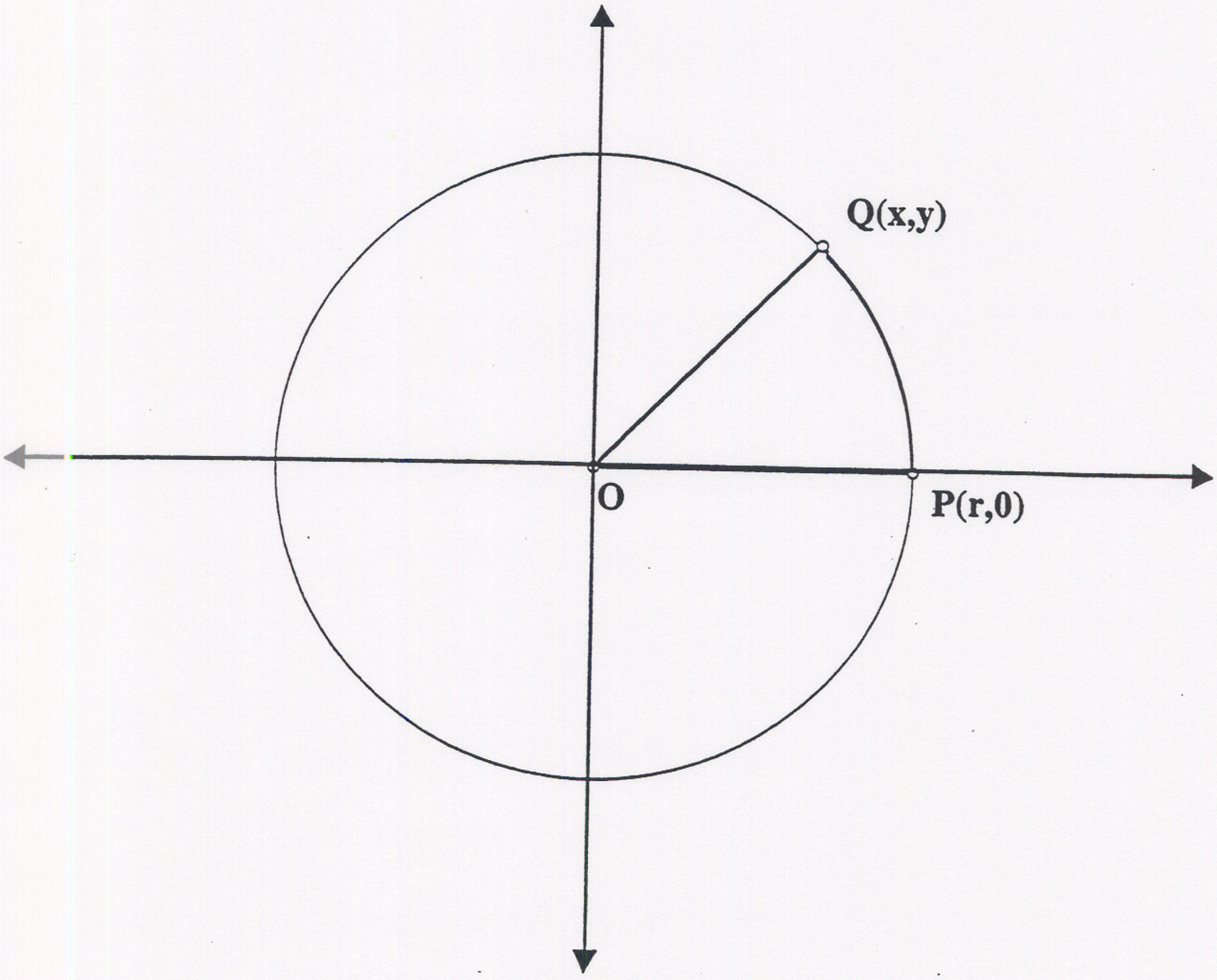
$$\frac{1}{3}2\pi = \frac{2\pi}{3} \approx \frac{2(3.14)}{3} \approx 2.09.$$

To approximate the values the calculator may be used.

3. *sin* 1 = 0.8415
4. *cos* 3 = -0.9900
5. *tan* 2 = -2.1850
6. *sin* 8 = 0.9894

- Describe how you determined each of the values. *Answers will vary.*
- Describe how to find another number that has the same sine as 8. *If* 1 *on the number line is added to one rotation of the unit circle* (2π), *it will end on the same point.* $\sin 1 = \sin(1+2\pi)$.
- Describe a real situation in which a sine wave may model the situation, but the domain is the set of real numbers, not the set of angle measures. *Many real world applications have domains that are units of time. For instance, the tidal movement situation has a domain that is hours of the day. A sound wave has a domain of seconds.*

Transparency 1: Radian Measure



1 radian is the measure of a central angle that intersects an arc of length one radius of the circle.

If the measure of arc *PQ* is *r*, the radian measure of central angle *O* is 1 radian.

TF-1-4