

# Out of the Swimming Pool

A-E Strand(s): Algebra

**Topic/Expectation**

A.A.1 Variables and expressions

e. Determine whether two algebraic expressions are equivalent.

**Other Topic/Expectation(s)**

A.A.1 Variables and expressions

d. Identify and transform expressions into equivalent expressions.

A.A.2 Functions

b. Represent and interpret functions using graphs, tables, words and symbols.

## Rationale

This task illustrates that different but equivalent expressions can be used in a function rule to provide valuable information about the context of the problem.

## Instructional Task

Sandra and Tim have a circular above-ground pool in their backyard that is 5 meters in diameter. They decide to have the water drained from the pool before the winter season. They hire Paul's Pool Service to pump the water from the pool.

The water is pumped from the swimming pool at a constant rate. Below is one representation of the amount of water (in liters) remaining in the pool after  $n$  minutes have passed since the pump started emptying the full pool:

$$f(n) = 20,100 - 100n$$

1. In the function  $f(n) = 20,100 - 100n$  (where  $n$  is the number of minutes that have passed since the pump started emptying the full pool), what might the values 20,100 and 100 represent in the context of the problem?
2. How much water is in the pool after 10 minutes of pumping? After 25 minutes of pumping?
3. How many minutes will it take to empty the pool?

Paul from Paul's Pool Service uses the function  $f(n) = 100(201 - n)$  to represent the same quantity—that is, the amount of water (in liters) remaining in the pool after  $n$  minutes have passed since the pool was full.

4. Are the two functions  $f(n) = 20,100 - 100n$  and  $f(n) = 100(201 - n)$  equivalent? How can you use the graphing feature and the table feature on a graphing calculator to show that these two functions either represent the same function or do not represent the same function?

- How can you show algebraically that these two functions represent either the same function or different functions?
- Why might it be helpful for Paul to write the function as  $f(n) = 100(201 - n)$ ?
- Write two equivalent functions for the following situation: a pool that started with 38,500 liters of water is being emptied at a rate of 100 liters per minute.
- Write two equivalent functions for the following situation: a pool starts with 20,100 liters of water and can be completely emptied in 167 minutes and 30 seconds.

## Discussion/Further Questions/Extensions

This task illustrates that sometimes when we ask students to “simplify” their answers, we lose valuable information about the context. It also provides an opportunity to illustrate the value of using the distributive property to show equivalent expressions.

## Sample Solutions

- In the function  $f(n) = 20,100 - 100n$  (where  $n$  is the number of minutes that have passed since the pump started emptying the full pool), what might the values 20,100 and 100 represent in the context of the problem?

*The value 20,100 represents the number of liters of water in the pool before any water has been removed. The value 100 represents the rate at which water is being pumped from the pool—100 liters per minute.*

- How much water is in the pool after 10 minutes of pumping? After 25 minutes of pumping?

*Solution methods may vary.*

*Substituting 10 for  $n$  in the function,*

$$f(n) = 20,100 - 100n = 20,100 - 100(10) = 19,100$$

*After 10 minutes there were 19,100 liters of water remaining in the pool.*

*Use the table of values from the graphing calculator to find the number of liters when  $n = 25$ .*

X	Y1	Y2
22	17900	17900
23	17800	17800
24	17700	17700
25	17600	17600
26	17500	17500
27	17400	17400
28	17300	17300
X=25		

*After 25 minutes there were 17,600 liters of water remaining in the pool.*

3. How many minutes will it take to empty the pool?

When  $f(n) = 0$ , the pool will be empty. Therefore, set the function  $f(n) = 20,100 - 100n$  equal to 0 and solve for  $n$ .

$$0 = 20,100 - 100n$$

$$100n = 20,100$$

$$n = 201$$

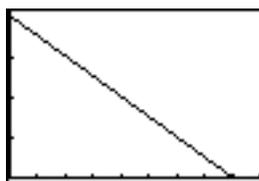
After 201 minutes or 3 hours and 21 minutes the pool will be empty.

4. Are the two functions  $f(n) = 20,100 - 100n$  and  $f(n) = 100(201 - n)$  equivalent? How can you use the graphing feature and the table feature on a graphing calculator to show that these two functions either represent the same function or do not represent the same function?

The two functions are equivalent. When the functions are graphed simultaneously, only one line appears on the screen, indicating that the two functions represent the same line. Additionally, the table of values shows that both functions generate the same output ( $Y_1$  and  $Y_2$ ) for the same input ( $x$ ).

Plot1	Plot2	Plot3
Y1=20100-100X		
Y2=100(201-X)		
Y3=		
Y4=		
Y5=		
Y6=		
Y7=		

WINDOW
Xmin=0
Xmax=225
Xscl=25
Ymin=0
Ymax=20500
Yscl=5000
Xres=1



X	Y1	Y2
0	20100	20100
1	20000	20000
2	19900	19900
3	19800	19800
4	19700	19700
5	19600	19600
6	19500	19500

5. How can you show algebraically that these two functions represent either the same function or different functions?

The distributive property can be used to show the two functions are equivalent.

$$\begin{aligned} f(n) &= 100(201 - n) \\ &= 100(201) - 100(n) = 20,100 - 100n \end{aligned}$$

6. Why might it be helpful for Paul to write the function as  $f(n) = 100(201 - n)$ ?

Writing the function as  $f(n) = 100(201 - n)$  will allow Paul to quickly see how many minutes it will take to empty all the water from the pool. This may help Paul figure out how much time to schedule for an appointment.

7. Write two equivalent functions for the following situation: a pool that started with 38,500 liters of water is being emptied at a rate of 100 liters per minute.

$$f(n) = 38500 - 100n$$

$$f(n) = 100(385 - n)$$

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8. Write two equivalent functions for the following situation: a pool starts with 20,100 liters of water and can be completely emptied in 167 minutes and 30 seconds.

$$g(n) = 20100 - 120n$$

$$g(n) = 120(167.5 - n)$$