

Talk Is Cheap

A-E Strand(s): Algebra

Topic/Expectation

A.A.2 Functions

- b. Represent and interpret functions using graphs, tables, words, and symbols.

A.A.3 Linear Functions

- a. Analyze and identify linear functions of one variable.
- e. Recognize contexts in which linear models are appropriate; determine and interpret linear models that describe linear phenomena.

Other Topic/Expectation(s)

N.A.1 Rational Numbers

- d. Know and employ effective methods of calculation with rational numbers.
- f. Solve practical problems involving rational numbers.

Rationale

This task can be used to introduce students to functions in a realistic setting—choosing a cell phone plan given certain conditions. Students gain experience working with decimals and translating among different representations of linear functions. They use the graphing calculator to find the intersection of two linear functions graphically and interpret the intersection in terms of the problem situation.

Instructional Task

To encourage communication between parents and their children and to prevent children from having extremely large monthly bills due to additional minute charges, two cell phone companies are offering special service plans for students.

Talk Fast cellular phone service charges \$0.10 for each minute the phone is used.

Talk Easy cellular phone service charges a basic monthly fee of \$18 plus \$0.04 for each minute the phone is used.

Your parents are willing to purchase for you one of the cellular phone service plans listed above. However, to help you become fiscally responsible they ask you to use the following questions to analyze the plans before choosing one.

1. How much would each company charge per month if you talked on the phone for 100 minutes in a month? If you talked for 200 minutes in a month?
2. Build a table, make a graph, and write a function rule, $f(x)$ or $g(x)$, to represent the cost of each cellular service in terms of the number of minutes, x .

Use the table, graph, and/or rule to help answer the following questions:

3. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.
4. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.
5. Depending on the number of minutes you talk on the phone each month, explain to your parents which cellular phone plan is more economical. Include in your explanation the point at which both cellular phone plans cost the same amount of money.
6. If you know the cost of each plan for 300 minutes, can you double this cost to find the cost for 600 minutes? Explain your answer.

Discussion/Further Questions/Extensions

Discussion

Students might benefit from a discussion about how to choose values for the independent variable in order to more visually notice patterns.

Extension Questions

- Look at your graph. Should the points be connected? Why or why not?
- Why is it reasonable for the graph of the cost of service for both companies to be linear?
- How might the graphs be affected if Talk Easy increased or decreased its basic fee, or if Talk Fast began charging a basic monthly fee?
- How might the graph of either Talk Fast or Talk Easy be affected if the company increased or decreased its cost per minute?

Sample Solutions

1. How much would each company charge per month if you talked on the phone for 100 minutes in a month? If you talked for 200 minutes in a month?

Talk Fast would charge \$10 for 100 minutes and \$20 for 200 minutes. Talk Easy would charge \$22 for 100 minutes and \$26 for 200 minutes.

2. Build a table, make a graph, and write a function rule, $f(x)$ or $g(x)$, to represent the cost of each cellular service in terms of the number of minutes, x .

Table:

(Note that students might provide either horizontal tables, like those shown, or vertical 'T-tables'. Also, the particular values selected for x might vary.)

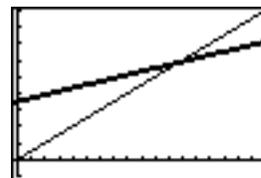
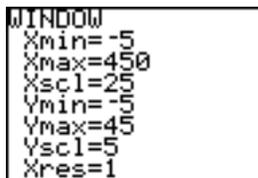
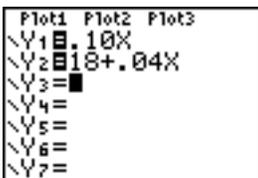
Talk Fast:

(number of minutes) x	0	50	100	150	200	250	300	350
(cost in dollars) $f(x)$	0	5	10	15	20	25	30	35

Talk Easy:

(number of minutes) x	0	50	100	150	200	250	300	350
(cost in dollars) $g(x)$	18	20	22	24	26	28	30	32

Graph:



- Talk Fast - Talk Easy

Function Rule:

Students should write a function rule to represent the monthly cost, $f(x)$ or $g(x)$, in dollars in terms of the number of minutes, x .

The cost for Talk Fast is 10 cents per minute times the number of minutes the phone is used: $f(x) = 0.10x$

The cost for Talk Easy is \$18 plus the product of 4 cents times the number of minutes the phone is used: $g(x) = 18 + 0.04x$

Use the table, graph, and/or rule to help answer the following questions:

3. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.

If you plan to use the phone for 200 minutes a month, Talk Fast is the better deal. Using the function rule, when I substitute 200 for the number of minutes, the monthly bill for Talk Fast is \$20 and the monthly bill for Talk Easy is \$26. (Students may also choose to justify their reasoning using a graph or table.)

$$f(x) = 0.10x$$

$$g(x) = 18 + 0.04x$$

$$f(x) = 0.10(200)$$

$$g(x) = 18 + 0.04(200)$$

$$f(x) = \$20$$

$$g(x) = \$26$$

4. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.

If you plan to use the phone for 500 minutes a month, Talk Easy is the better deal. I choose to use the table of values to help me answer this question. Using a table of values, if you look at the dollar values when $x = 500$, you can see that the Talk Fast plan will have a monthly bill of \$50 and Talk Easy will have a monthly bill of \$38. (Students may also choose to justify their reasoning using a graph or a symbolic approach as above.)

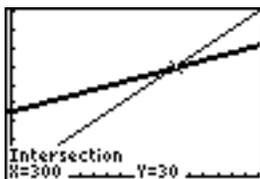
TABLE SETUP	
TblStart=500	
ΔTbl=50	
Indent: Ask	
Depend: Auto Ask	

X	Y ₁	Y ₂
500	50	38
550	55	40
600	60	42
650	65	44
700	70	46
750	75	48
800	80	50

X=500

5. Depending on the number of minutes you predict you will talk on the phone each month, explain to your parents which cellular phone plan is more economical for you. Include in your explanation the point at which both cellular phone plans cost the same amount of money.

(A student may choose to use any of the three following methods or an alternate method to find how many minutes of phone use will yield the same monthly costs for both plans. The graph shows lines for each of the two function rules; in the table from the graphing calculator Y₁ represents Talk Fast and Y₂ represents Talk Easy. The equation sets the two function rules equal to solve for the number of minutes, x.)



X	Y ₁	Y ₂
150	15	24
200	20	26
250	25	28
300	30	30
350	35	32
400	40	34
450	45	36

X=300

$$0.10x = 18 + 0.04x$$

$$0.06x = 18$$

$$x = 300$$

The intersection point of the two function rules shows the point at which the number of minutes the phone is used is equal to 300 and the monthly cost for both plans is \$30. This means that if I am going to use the phone for 300 minutes a month, the monthly cost for each service plan will be the same. If I am going to use the phone for fewer than 300 minutes, the Talk Fast service plan will be cheaper; if I am going to use the phone for more than 300 minutes a month, the Talk Easy plan will be cheaper. This can be easily seen by looking at the table of values. In the table for any number of minutes less than 300 the cost for Talk Fast is less than the cost for Talk Easy. At 300 minutes the cost is equal, and at any number of minutes above 300 the cost for Talk Easy is less than the cost for Talk Fast.

6. If you know the cost of each plan for 300 minutes, can you double this cost to find the cost for 600 minutes? Explain your answer.

As discussed earlier the cost for each plan at 300 minutes is \$30. For the Talk Fast plan the cost for 600 minutes is $f(x) = 0.10(600) = \$60$. For the Talk Easy plan, the cost for 600 minutes is $g(x) = 18 + 0.04(600) = \42 . It is possible to double the cost for 300 minutes on the Talk Fast plan to find the cost for 600 minutes, but it is not possible to do so for the Talk Easy plan. The reason it works for the Talk Fast plan is because the function $f(x) = 0.10x$ represents a proportional relationship in which the cost is proportional to the number of minutes used. For any situation with a constant rate of change—that is, one for which the graph of the rule passes through the origin—a proportional relationship exists. Since the graph of the rule for Talk Easy has a y -intercept of 18 (the basic monthly fee), this function rule does not represent a proportional relationship, so doubling the amount does not work.