

Unidentified Circular Objects (UCO's)

Overview: Students investigate the relationship between the diameter of a circular light on a surface produced by a flashlight and the distance of the flashlight from the surface.

Objective: **Algebra I TEKS**
 (b.1.B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.
 (b.1.E) The student interprets and makes inferences from functional relationships.
 (c.1.A) The student determines whether or not given situations can be represented by linear functions.
 (c.1.C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.
 (c.2.B) The student interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Terms: diameter, trend line, linear model, rate of change, slope, y-intercept

Materials: flashlights (one per group), rulers, yardstick or meter stick, graphing calculators

Procedures: Students should be seated at tables in groups of 3 – 4.

Activity 1: Unidentified Circular Objects

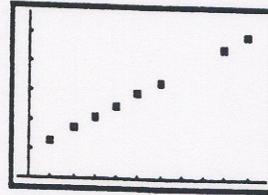
Briefly describe and/or demonstrate the experiment. Make sure students hold the meter stick perpendicular to the surface on which the light is shining.

1. Stress how important it is for students to predict the results of the experiment **before** they perform the experiment. Encourage students to think about and anticipate the results of the experiment before they begin collecting data.

2. *Sample data:*

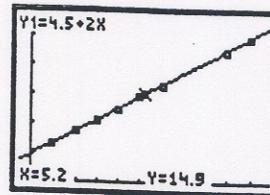
Distance (cm)	Diameter
1	6.5
2	8.5
3	10.6
4	12.4
5	14.5
6	16.3
9	22.3
10	24.6

3. Sample data:



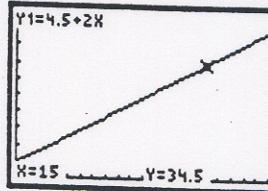
4. Rate of change is approximately 2 cm/cm
5. Estimated starting point is 4.5 cm.
6. $y = 4.5 + 2x$

7. Sample data:



8. The units of slope are centimeters per centimeters.
9. The real world meaning of the y -intercept is that if the flashlight was no centimeters from the surface, the circular light pattern would have the y -intercept as its diameter.
10. The equation is $y = 4.5 + 2(15) = 34.5$. Some solution methods:

$$4.5 + 2(15) = 34.5$$



X	Y ₁
10	24.5
11	26.5
12	28.5
13	30.5
14	32.5
15	34.5
16	36.5

Y₁ = 34.5

11. For our *sample data*, solve: $4.5 + 2x = 18$

Table:

X	Y ₁
6	16.5
6.7	18.5
8	20.5
9	22.5
10	24.5
11	26.5
12	28.5

X=7

X	Y ₁
6.7	17.9
6.8	18.1
6.9	18.3
7	18.5
7.1	18.7
7.2	18.9
7.3	19.1

X=6.7

X	Y ₁
6.7	17.9
6.71	17.92
6.72	17.94
6.73	17.96
6.74	17.98
6.75	18
6.76	18.02

X=6.75

Other Table:

X	Y ₁	Y ₂
5	14.5	18
6	16.5	18
7	18.5	18
8	20.5	18
9	22.5	18
10	24.5	18
11	26.5	18

X=6

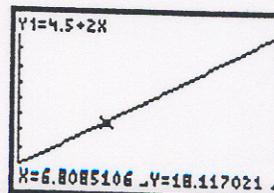
X	Y ₁	Y ₂
6.7	17.9	18
6.8	18.1	18
6.9	18.3	18
7	18.5	18
7.1	18.7	18
7.2	18.9	18
7.3	19.1	18

X=6.7

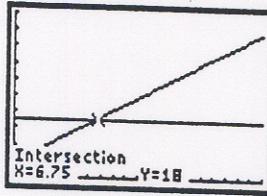
X	Y ₁	Y ₂
6.7	17.9	18
6.71	17.92	18
6.72	17.94	18
6.73	17.96	18
6.74	17.98	18
6.75	18	18
6.76	18.02	18

X=6.75

Trace:



Trace to the Intersection point.



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Plot1 Plot2 Plot3
Y1=4.5+2X
Y2=18
Y3=
Y4=
Y5=
Y6=
Y7=
    
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Guess and check.

$4.5+2(6)$	16.5
$4.5+2(7)$	18.5
$4.5+2(6.5)$	17.5

$4.5+2(6.8)$	17.5
$4.5+2(6.7)$	18.1
$4.5+2(6.75)$	17.9
	18

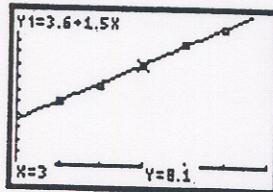
Solve algebraically, if you are at a place in your curriculum where it makes sense for students to do so:

$$\begin{aligned}
 4.5 + 2x &= 18 \\
 4.5 + 2x - 4.5 &= 18 - 4.5 \\
 \left(\frac{1}{2}\right)2x &= 13.5\left(\frac{1}{2}\right) \\
 x &= 6.75
 \end{aligned}$$

12. The further from the surface the flashlight is, the larger the circular light pattern produced.

Answers to Sample Assessment:

1.



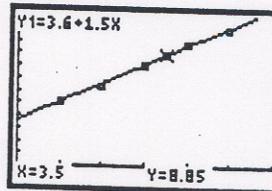
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WINDOW
Xmin=0
Xmax=6
Xscl=1
Ymin=-1
Ymax=12
Yscl=1
Xres=1
    
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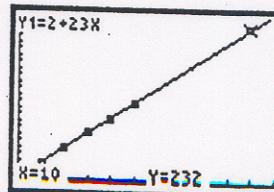
Plot1 Plot2 Plot3
Y1=3.6+1.5X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
    
```

2. Trace to $x = 3.5$

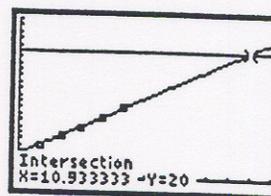


3. Two graphical methods:

Trace to $y = 20$



Trace to the intersection of $y = 3.6 + 1.5x$ and $y = 20$

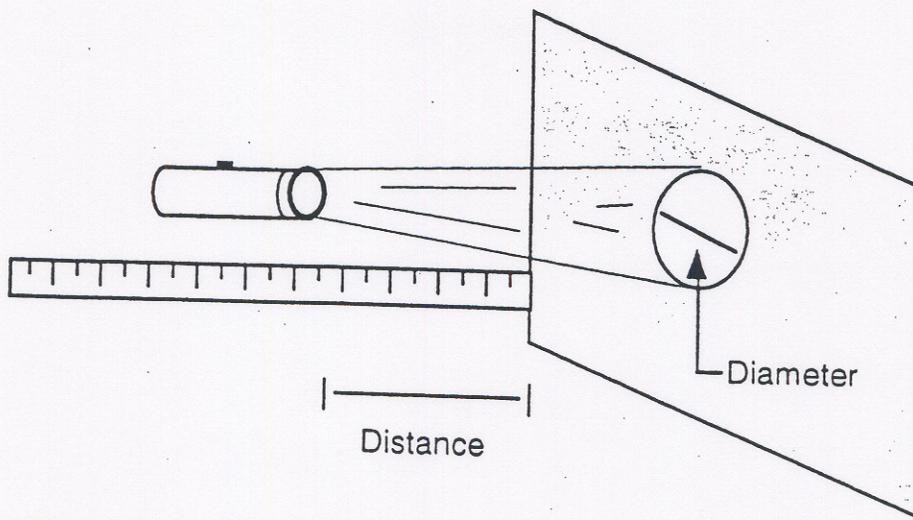


Summary:

By collecting data and finding a trend line, students investigate the relationship between the diameter of the circular light pattern produced by a flashlight at varying distances from the surface. Students use real data to further their conceptualization of the linear function.

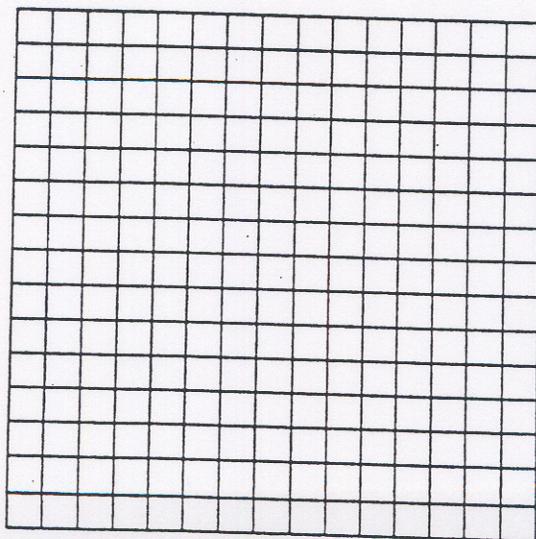
Activity 2: UCO's

What is the relationship between the diameter of the circular light pattern cast by a flashlight and the flashlight's distance from the circular light pattern?



Vary the distance of the flashlight from the surface and measure the diameter of the circular light pattern cast by the flashlight.

1. Sketch a graph predicting the relationship between the diameter of the circular light pattern cast by a flashlight and the flashlight's distance from the circular light pattern.

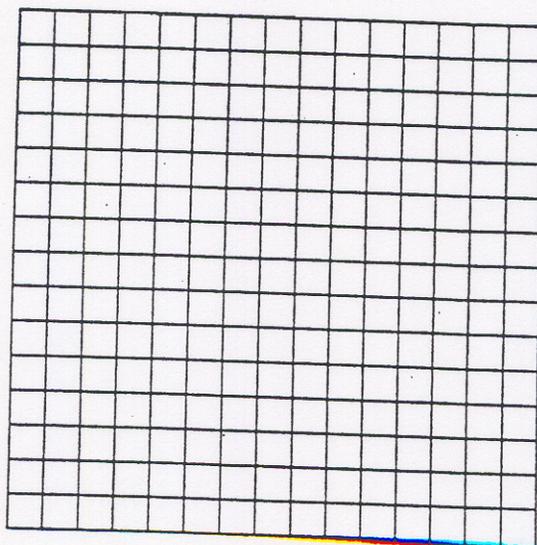


2. Data Collection

Hold a yardstick perpendicular to a flat surface, such as a table, with the end starting at 0 on the flat surface. Hold a flashlight next to the meter stick so that it will cast light on the flat surface. Place the rim of the flashlight (light source end) at 1 cm and measure the diameter of the distinct circular pattern formed on the flat surface. Record in the table below. Continue to vary the distance of the flashlight from the table and record the diameter of the circle formed.

Distance (cm)	Diameter
1	
2	
3	
4	
5	
6	
9	
10	

3. Make a scatter plot using a graphing calculator. Sketch below.



4. Use first differences to estimate a rate of change.
5. Estimate the y -intercept (*starting point*.)
6. Find a trend line for the data using the estimated rate and y -intercept.
7. Graph your trend line over the scatter plot and adjust the parameters y -intercept and *rate of change*, if necessary, for a better fit.
8. What are the units of slope for the trend line?
9. What is the meaning of the y -intercept in the trend line?
10. Use the trend line to determine the diameter of the circle when the flashlight is 15 cm from the flat surface. Write the equation and solve in at least three ways.

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11. Use the trend line to determine how far the rim of the flashlight is from the flat surface if the diameter of the circle is 18 cm. Write the equation and solve in at least four ways.
12. Make a general statement about the relationship between the distance of the flashlight from the surfaces and the diameter of the circular light pattern produced on the surface.

Sample Assessment

A group collected the following data for Unidentified Circular Objects.

Distance (cm)	Diameter (cm)
1	5
2	6.4
3	8
4	9.6
5	11

1. Create a scatter plot and find a trend line. Sketch both in an appropriate window.
2. Use the graph to determine what the diameter of the circular light pattern is when the flashlight is 3.5 cm from the surface. Show on the graph how you found the answer.
3. Use the graph to determine how far the flashlight is from the surface when the diameter of the circular light pattern is 20 cm. Show on the graph how you found the answer.